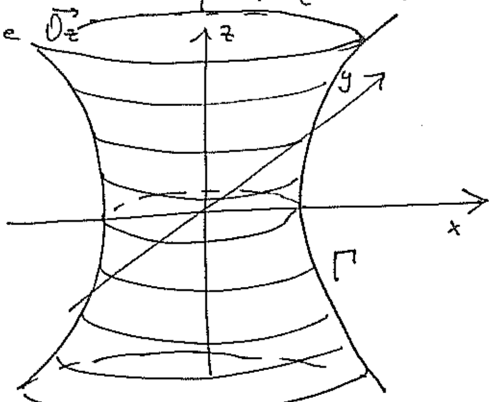
Exercice 1 S = caténoïde = surf. de révolution de $\Gamma = \{ \gamma(t) = (\operatorname{ch} t, 0, t) \mid t \in \mathbb{R} \}$ autour de l'axe $\vec{Oz}$

1. Dessin :

$$\Gamma : \begin{cases} x = \operatorname{ch} t \\ y = 0 \\ z = t \end{cases} \Longrightarrow \begin{cases} x = \operatorname{ch} z \\ y = 0 \end{cases}$$



2. Paramétrisation :

$$S = \{ f(t,\varphi) = R^{Oz}_{\varphi}\, \gamma(t) \mid t \in \mathbb{R},\ \varphi \in [0, 2\pi[\}$$

$$f(t,\varphi) = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \operatorname{ch} t \\ 0 \\ t \end{pmatrix} = (\operatorname{ch} t \cos\varphi,\ \operatorname{ch} t \sin\varphi,\ t)$$

3. Points réguliers :

$$\left[\begin{array}{l} \dfrac{\partial f}{\partial t} = (\operatorname{sh} t \cos\varphi,\ \operatorname{sh} t \sin\varphi,\ 1) \\ \dfrac{\partial f}{\partial \varphi} = (-\operatorname{ch} t \sin\varphi,\ \operatorname{ch} t \cos\varphi,\ 0) \end{array}\right. \Longrightarrow \frac{\partial f}{\partial t} \wedge \frac{\partial f}{\partial \varphi} = (-\operatorname{ch} t \cos\varphi,\ -\operatorname{ch} t \sin\varphi,\ \operatorname{ch} t \operatorname{sh} t) = \operatorname{ch} t\,(-\cos\varphi,\ -\sin\varphi,\ \operatorname{sh} t)$$

$$\left\| \frac{\partial f}{\partial t} \wedge \frac{\partial f}{\partial \varphi} \right\| = \operatorname{ch} t \sqrt{1 + \operatorname{sh}^2 t} = (\operatorname{ch} t)^2 \neq 0 \quad \forall t \in \mathbb{R} \Longrightarrow \text{tous les points de } S \text{ sont réguliers.}$$

$$\vec{N}_S(t,\varphi) = \frac{1}{\operatorname{ch} t} (-\cos\varphi,\ -\sin\varphi,\ \operatorname{sh} t).$$

4. Parallèle à hauteur $t = \ln(2 + \sqrt{3})$:

$$\operatorname{ch}(\ln(2+\sqrt{3})) = \frac{1}{2}\left(e^{\ln(2+\sqrt{3})} + e^{\ln \frac{1}{2+\sqrt{3}}} \right) = \frac{1}{2}\left(2 + \sqrt{3} + \frac{1}{2+\sqrt{3}} \right) = \frac{4 + 4\sqrt{3} + 3 + 1}{2(2+\sqrt{3})} = \frac{4(2+\sqrt{3})}{2(2+\sqrt{3})} = 2$$

$$\Longrightarrow \Gamma_t = \{ \alpha(\varphi) = (2\cos\varphi,\ 2\sin\varphi,\ \ln(2+\sqrt{3})) \mid \varphi \in [0, 2\pi[\}$$

$$L_0^{2\pi}(\alpha) = \int_0^{2\pi} \| \alpha'(\varphi) \| \, d\varphi = \int_0^{2\pi} \| (-2\sin\varphi,\ 2\cos\varphi,\ 0) \| \, d\varphi = \int_0^{2\pi} 2 \, d\varphi = 2[\varphi]_0^{2\pi} = 4\pi.$$

5. Équation cartésienne de S :

$$\begin{cases} x = \operatorname{ch} t \cos\varphi \\ y = \operatorname{ch} t \sin\varphi \\ z = t \end{cases} \Longrightarrow \begin{cases} x^2 + y^2 = \operatorname{ch}^2 t \\ z = t \end{cases} \Longrightarrow x^2 + y^2 = \operatorname{ch}^2 z \quad \text{avec } z = t \in \mathbb{R} \text{ et } x, y \in \mathbb{R}$$

$$\Longrightarrow S = \{ (x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 = \operatorname{ch}^2 z \}.$$

Exercice 2 : $S = \{ f(u,v) = (u(1+v),\ u^2+v,\ uv) \mid u,v \in \mathbb{R} \}$

1. <u>M.q. S est une surface réglée</u>, i.e. $f(u,v) = \gamma(u) + v\,\alpha(u)$ avec $v \in \mathbb{R}$ et $\alpha(u) \neq \vec{0}$ $\forall u \in \mathbb{R}$.

$$f(u,v) = (u(1+v), u^2+v, uv) = (u, u^2, 0) + (uv, v, uv) = (u,u^2,0) + v\,(u,1,u)$$

$\Longrightarrow$ $\gamma(u) = (u, u^2, 0)$, $u \in \mathbb{R}$ et $\alpha(u) = (u,1,u) \neq \vec{0}$ $\forall u \in \mathbb{R}$.

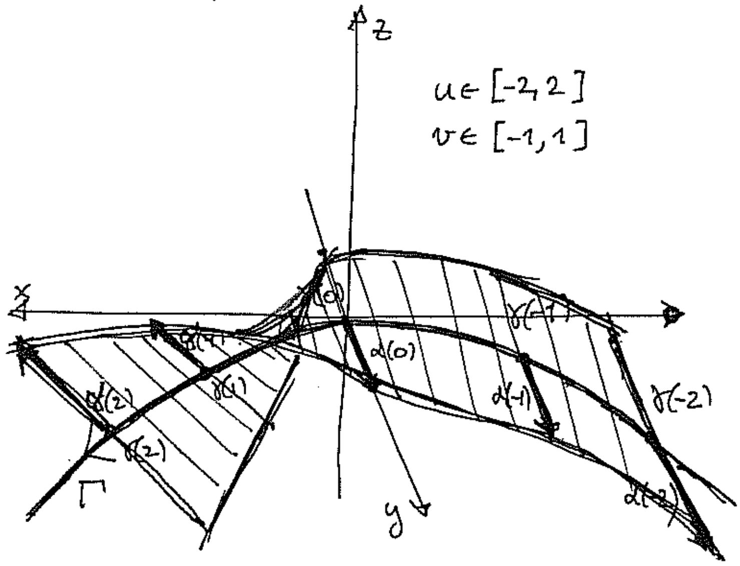
2. <u>Dessin</u> :

$u \in [-2,2]$
$v \in [-1,1]$

$\Gamma = \{ \gamma(u) = (u,u^2,0) \mid u \in \mathbb{R} \}$

parabole $y = x^2$
sur le plan $z = 0$

$\alpha(0) = (0,1,0)$
$\alpha(1) = (1,1,1)$ $\qquad \alpha(-1) = (-1,1,-1)$
$\alpha(2) = (2,1,2)$ $\qquad \alpha(-2) = (-2,1,-2)$



3. <u>Points réguliers</u> :

$$\begin{cases} \dfrac{\partial f}{\partial u} = (1+v, 2u, v) \\ \dfrac{\partial f}{\partial v} = (u,1,u) \end{cases} \Longrightarrow \frac{\partial f}{\partial u} \wedge \frac{\partial f}{\partial v} = \Big(2u^2 - v,\ -\big(\overbrace{u(1+v) - uv}^{u + uv - uv}\big),\ 1+v-2u^2\Big)$$
$$= (2u^2 - v,\ -u,\ 1+v-2u^2)$$

$$\frac{\partial f}{\partial u} \wedge \frac{\partial f}{\partial v} = (0,0,0) \Longleftrightarrow \begin{cases} 2u^2 - v = 0 \\ -u = 0 \\ 1+v-2u^2 = 0 \end{cases} \Longleftrightarrow \begin{cases} u = 0 \\ v = 0 \\ 1+v = 0 \end{cases} \text{)) impossible}$$

Donc $\dfrac{\partial f}{\partial u} \wedge \dfrac{\partial f}{\partial v} \neq \vec{0}$ $\forall (u,v) \in \mathbb{R}^2$ et S est régulière partout.

$$\left\| \frac{\partial f}{\partial u} \wedge \frac{\partial f}{\partial v} \right\| = \sqrt{(2u - v)^2 + u^2 + (1 - v - 2u^2)^2} \quad \text{horrible}$$

Le vecteur normal unitaire $\vec{n}_S(u,v) = \dfrac{\frac{\partial f}{\partial u} \wedge \frac{\partial f}{\partial u}}{\left\| \frac{\partial f}{\partial u} \wedge \frac{\partial f}{\partial u} \right\|}$ est horrible, mais un vecteur normal non nulle est $\dfrac{\partial f}{\partial u} \wedge \dfrac{\partial f}{\partial v} = (2u^2 - v, -u, 1+v-2u^2)$.

4. <u>Éq. cartesienne</u> :

$$\begin{cases} x = u(1+v) \\ y = u^2 + v \\ z = uv \end{cases} \Longrightarrow \begin{array}{l} x - z = u + uv - uv = u \\ (x-z)y = u(u^2+v) = u^3 + uv = (x-z)^3 + z \quad \text{avec } x,y,z \in \mathbb{R} \end{array}$$

$$\Longrightarrow S = \{ (x,y,z) \in \mathbb{R}^3 \mid (x-z)y = (x-z)^3 + z \}.$$