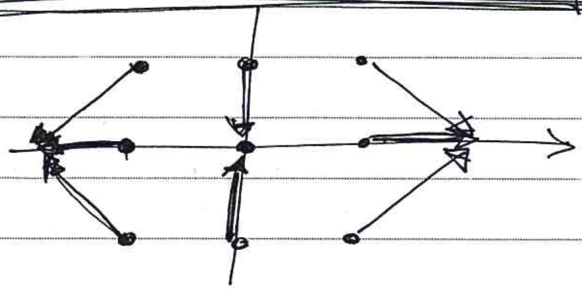
Numéro de la feuille d'examen
à reporter ci-dessous :
N°

CORRIGÉ DU CC3 — 12 décembre 2013

Ex. 1 (3pts)

$\vec{V}(x,y) = \begin{pmatrix} x \\ -y \end{pmatrix}$



Ex. 2 (4pts)

$$\operatorname{div}\begin{pmatrix} f(x) \\ -xy \\ -z \end{pmatrix} = f'(x) - x - 1 = 0 \iff f'(x) = x+1$$
$$\iff f(x) = \frac{1}{2}x^2 + x + c$$

Ex. 3 (5pts)

1 $\overrightarrow{\text{rot}}\,\vec{U} = \det\begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ 2xy^2 - z^3 & 2x^2y & 3xz^2 \end{pmatrix} = \begin{pmatrix} 0-0 \\ -[3z^2 - \cancel{(-3)}3z^2] \\ 4xy - 4xy \end{pmatrix} = \begin{pmatrix} 0 \\ -6z^2 \\ 0 \end{pmatrix} \neq \vec{0}$

1 $\overrightarrow{\text{rot}}\,\vec{V} = \begin{pmatrix} 0-0 \\ -[-3z^2 + 3z^2] \\ 4xy - 4xy \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \implies \vec{V}$ est un champ gradient

$$\vec{V} = \vec{\nabla} f \iff \begin{cases} \dfrac{\partial f}{\partial x} = 2xy^2 - z^3 \implies f(x,y,z) = \displaystyle\int (2xy^2 - z^3)\,dx + g(y,z) = x^2y^2 - z^3x + g(y,z) \\ \dfrac{\partial f}{\partial y} = 2x^2y \qquad \dfrac{\partial}{\partial y} = 2x^2y + \dfrac{\partial g}{\partial y} \overset{!}{=} 2x^2y \\ \dfrac{\partial f}{\partial z} = -3xz^2 \end{cases}$$

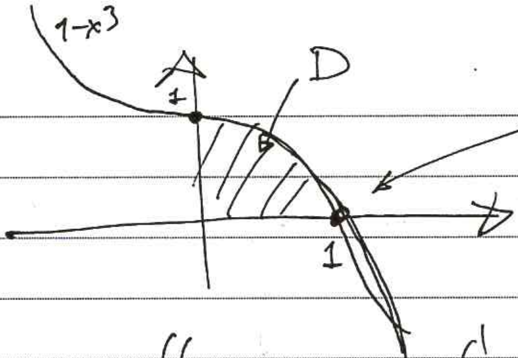
3 $\iff \dfrac{\partial g}{\partial y} = 0 \iff g(y,z) = h(z) \iff f(x,y) = x^2y^2 - xz^3 + h(z)$

$\dfrac{\partial}{\partial z} = -3xz^2 + h'(z) \overset{!}{=} -3xz^2 \iff h'(z) = 0 \iff h(z) = c$

Conclusion : $f(x,y,z) = x^2y^2 - xz^3 + c$, $c \in \mathbb{R}$.

Ex. 4

4 pts



$$\begin{cases} y = 1-x^3 \\ y = 0 \end{cases} \Leftrightarrow 1-x^3=0$$

$$x^3 = 1 \Rightarrow x = 1$$

$$D = \{(x,y) \in \mathbb{R}^2 \mid 0 \le x \le 1 ,\ 0 \le y \le 1-x^3\}$$

$$\text{Aire}(D) = \iint_D dx\,dy = \int_0^1 dx \int_0^{1-x^3} dy = \int_0^1 [y]_0^{1-x^3} dx$$

$$= \int_0^1 (1-x^3)\,dx = \left[x - \frac{1}{4}x^4\right]_0^1 = 1 - \frac{1}{4} = \frac{3}{4}.$$

Ex. 5

4 pts

$$\text{Masse} = \iiint_{[0,1]\times[0,1]\times[0,1]} x y^2 z^3 \, dx\,dy\,dz = \int_0^1 x\,dx \int_0^1 y^2 dy \int_0^1 z^3 dz$$

$$= \left[\frac{1}{2}x^2\right]_0^1 \cdot \left[\frac{1}{3}y^3\right]_0^1 \cdot \left[\frac{1}{4}z^4\right]_0^1 = \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{24}.$$