REPÈRES MOBILES ET CHAMPS DE VECTEURS

CONVENTION STANDARD ISO 80000-2

`http://en.wikipedia.org/wiki/ISO 80000-2`

	Coordonnées cartesiennes	**Coordonnées cylindriques**	**Coordonnées sphériques**
Point P	$(x,y,z) \quad x,y,z\in\mathbb{R}$	$(\rho,\varphi,z) \quad \left[\begin{array}{l} \rho\geq 0 \\ \varphi\in[0,2\pi[\\ z\in\mathbb{R}\end{array}\right.$	$(r,\varphi,\theta) \quad \left[\begin{array}{l} r\geq 0 \\ \varphi\in[0,2\pi[\\ \theta\in[0,\pi]\end{array}\right.$
Changement coordonnées	$\begin{cases} \rho=\sqrt{x^2+y^2} \\ \varphi=\begin{cases}\arctan\frac{y}{x} & \text{si } x,y>0 \\ \text{etc} & \text{sinon}\end{cases} \\ z=z\end{cases}$ $\begin{cases} r=\sqrt{x^2+y^2+z^2} \\ \varphi=\text{idem} \\ \theta=\arccos\dfrac{z}{\sqrt{x^2+y^2+z^2}}\end{cases}$	$\begin{cases} x=\rho\ \cos\varphi \\ y=\rho\ \sin\varphi \\ z=z\end{cases}$ $\begin{cases} r=\sqrt{\rho^2+z^2} \\ \varphi=\varphi \\ \theta=\arccos\dfrac{z}{\sqrt{\rho^2+z^2}}\end{cases}$	$\begin{cases} x=r\ \cos\varphi\ \sin\theta \\ y=r\ \sin\varphi\ \sin\theta \\ z=r\ \cos\theta\end{cases}$ $\begin{cases} \rho=r\ \sin\theta \\ \varphi=\varphi \\ z=r\ \cos\theta\end{cases}$
Repère en P	$(P;\ \vec{\imath},\vec{\jmath},\vec{k}) \quad \left[\begin{array}{l} \vec{\imath}=\dfrac{\partial}{\partial x} \\ \vec{\jmath}=\dfrac{\partial}{\partial y} \\ \vec{k}=\dfrac{\partial}{\partial z}\end{array}\right.$	$(P;\ \vec{e_\rho},\vec{e_\varphi},\vec{k}) \quad \left[\begin{array}{l} \vec{e_\rho}=\dfrac{\partial}{\partial\rho} \\ \vec{e_\varphi}=\dfrac{1}{\rho}\dfrac{\partial}{\partial\varphi} \\ \vec{k}=\dfrac{\partial}{\partial z}\end{array}\right.$	$(P;\ \vec{e_r},\vec{e_\varphi},\vec{e_\theta}) \quad \left[\begin{array}{l} \vec{e_r}=\dfrac{\partial}{\partial r} \\ \vec{e_\varphi}=\dfrac{1}{r\sin\theta}\dfrac{\partial}{\partial\varphi} \\ \vec{e_\theta}=\dfrac{1}{r}\dfrac{\partial}{\partial\theta}\end{array}\right.$
Changement repère	$\begin{cases} \vec{e_\rho}=\cos\varphi\ \vec{\imath}+\sin\varphi\ \vec{\jmath} \\ \vec{e_\varphi}=-\sin\varphi\ \vec{\imath}+\cos\varphi\ \vec{\jmath} \\ \vec{k}=\vec{k}\end{cases}$ $\begin{cases} \vec{e_r}=\cos\varphi\ \sin\theta\ \vec{\imath}+\sin\varphi\ \sin\theta\ \vec{\jmath} \\ \qquad +\cos\theta\ \vec{k} \\ \vec{e_\varphi}=-\sin\varphi\ \vec{\imath}+\cos\varphi\ \vec{\jmath} \\ \vec{e_\theta}=\cos\varphi\ \cos\theta\ \vec{\imath}+\sin\varphi\ \cos\theta\ \vec{\jmath} \\ \qquad -\sin\theta\ \vec{k}\end{cases}$	$\begin{cases} \vec{\imath}=\cos\varphi\ \vec{e_\rho}-\sin\varphi\ \vec{e_\varphi} \\ \vec{\jmath}=\sin\varphi\ \vec{e_\rho}+\cos\varphi\ \vec{e_\varphi} \\ \vec{k}=\vec{k}\end{cases}$ $\begin{cases} \vec{e_r}=\sin\theta\ \vec{e_\rho}+\cos\theta\ \vec{k} \\ \vec{e_\varphi}=\vec{e_\varphi} \\ \vec{e_\theta}=\cos\theta\ \vec{e_\rho}-\sin\theta\ \vec{k}\end{cases}$	$\begin{cases} \vec{\imath}=\cos\varphi\sin\theta\ \vec{e_r}-\sin\varphi\ \vec{e_\varphi} \\ \qquad +\cos\varphi\cos\theta\ \vec{e_\theta} \\ \vec{\jmath}=\sin\varphi\sin\theta\ \vec{e_r}+\cos\varphi\ \vec{e_\varphi} \\ \qquad +\sin\varphi\cos\theta\ \vec{e_\theta} \\ \vec{k}=\cos\theta\ \vec{e_r}-\sin\theta\ \vec{e_\theta}\end{cases}$ $\begin{cases} \vec{e_\rho}=\sin\theta\ \vec{e_r}+\cos\theta\ \vec{e_\theta} \\ \vec{e_\varphi}=\vec{e_\varphi} \\ \vec{k}=\cos\theta\ \vec{e_\rho}-\sin\theta\ \vec{e_\theta}\end{cases}$
Vitesse du repère sur une courbe	$\begin{cases} \dot{\vec{\imath}}(t)=0 \\ \dot{\vec{\jmath}}(t)=0 \\ \dot{\vec{k}}(t)=0\end{cases}$	$\begin{cases} \dot{\vec{e_\rho}}(t)=\dot\varphi(t)\,\vec{e_\varphi}(t) \\ \dot{\vec{e_\varphi}}(t)=-\dot\varphi(t)\,\vec{e_\rho}(t) \\ \dot{\vec{k}}(t)=0\end{cases}$	$\begin{cases} \dot{\vec{e_r}}(t)=\dot\varphi\ \sin\theta\,\vec{e_\varphi}+\dot\theta\,\vec{e_\theta} \\ \dot{\vec{e_\varphi}}(t)=-\dot\varphi\ \sin\theta\,\vec{e_r}-\dot\varphi\ \cos\theta\,\vec{e_\theta} \\ \dot{\vec{e_\theta}}(t)=-\dot\theta\,\vec{e_r}+\dot\varphi\cos\theta\,\vec{e_\varphi}\end{cases}$
Vecteur position $\vec{x}(t)$	$x(t)\ \vec{\imath}+y(t)\ \vec{\jmath}+z(t)\ \vec{k}$	$\rho(t)\ \vec{e_\rho}(t)+z(t)\ \vec{k}$	$r(t)\ \vec{e_r}(t)$
Vecteur vitesse $\dot{\vec{x}}(t)$	$\dot x(t)\,\vec{\imath}+\dot y(t)\,\vec{\jmath}+\dot z(t)\,\vec{k}$	$\dot\rho(t)\,\vec{e_\rho}(t)+\rho(t)\dot\varphi(t)\,\vec{e_\varphi}(t)+\dot z(t)\,\vec{k}$	$\dot r(t)\,\vec{e_r}(t)+r(t)\ \sin\theta(t)\ \dot\varphi(t)\,\vec{e_\varphi}(t)$ $+r(t)\dot\theta(t)\,\vec{e_\theta}(t)$

	Coordonnées cartesiennes (x, y, z)	**Coordonnées cylindriques** (ρ, φ, z)	**Coordonnées sphériques** (r, φ, θ)
Champ de vecteurs $\vec{A}$	$A_x\ \vec{\imath} + A_y\ \vec{\jmath} + A_z\ \vec{k}$	$A_\rho\ \vec{e_\rho} + A_\varphi\ \vec{e_\varphi} + A_z\ \vec{k}$	$A_r\ \vec{e_r} + A_\varphi\ \vec{e_\varphi} + A_\theta\ \vec{e_\theta}$
Gradient $\overrightarrow{\text{grad}}\, f = \vec{\nabla} f$	$\frac{\partial f}{\partial x}\ \vec{\imath} + \frac{\partial f}{\partial y}\ \vec{\jmath} + \frac{\partial f}{\partial z}\ \vec{k}$	$\frac{\partial f}{\partial \rho}\ \vec{e_\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \varphi}\ \vec{e_\varphi} + \frac{\partial f}{\partial z}\ \vec{k}$	$\frac{\partial f}{\partial r}\ \vec{e_r} + \frac{1}{r \sin\theta}\frac{\partial f}{\partial \varphi}\ \vec{e_\varphi} + \frac{1}{r}\frac{\partial f}{\partial \theta}\ \vec{e_\theta}$
Divergence $\text{div}\, \vec{A} = \nabla \cdot \vec{A}$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r \sin\theta}\frac{\partial(\sin\theta\ A_\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi}$
Rotationnel $\overrightarrow{\text{rot}}\, \vec{A} = \vec{\nabla} \times \vec{A}$	$\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\ \vec{\imath}$ $+\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\ \vec{\jmath}$ $+\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\ \vec{k}$	$\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\ \vec{e_\rho}$ $+\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\ \vec{e_\varphi}$ $+\frac{1}{\rho}\left(\frac{\partial(\rho A_\varphi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \varphi}\right)\ \vec{k}$	$\frac{1}{r\ \sin\theta}\left(\frac{\partial(\sin\theta\ A_\varphi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \varphi}\right)\ \vec{e_r}$ $+\frac{1}{r}\left(\frac{\partial(r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\ \vec{e_\varphi}$ $+\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial(r A_\varphi)}{\partial r}\right)\ \vec{e_\theta}$
Laplacien $\Delta f = \vec{\nabla} \cdot \vec{\nabla} f$ $= \text{div}\,(\overrightarrow{\text{grad}}\, f)$	$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \cos\theta}\frac{\partial}{\partial \theta}\left(\cos\theta\ \frac{\partial f}{\partial \theta}\right)$ $+\frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}$
Laplacien vectoriel $\Delta\vec{A}$	$\Delta A_x\ \vec{\imath} + \Delta A_y\ \vec{\jmath} + \Delta A_z\ \vec{k}$	(affreux...)	(horrible !)

Notations : $\vec{\nabla} = \frac{\partial}{\partial x}\ \vec{\imath} + \frac{\partial}{\partial y}\ \vec{\jmath} + \frac{\partial}{\partial z}\ \vec{k}$ $\qquad \Delta \equiv \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ $\qquad \vec{\nabla} \times \vec{A} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$

$\vec{A} \cdot \vec{\nabla} = A_x \frac{\partial}{\partial x} + A_y \frac{\partial}{\partial y} + A_z \frac{\partial}{\partial z}$ $\qquad (\vec{A} \cdot \vec{\nabla})\vec{B} = A_x \frac{\partial B_x}{\partial x}\ \vec{\imath} + A_y \frac{\partial B_y}{\partial y}\ \vec{\jmath} + A_z \frac{\partial B_z}{\partial z}\ \vec{k}$

Proprietés :

$\overrightarrow{\text{grad}}\,(fg) = (\overrightarrow{\text{grad}}\, f)g + f(\overrightarrow{\text{grad}}\, g)$ $\qquad \overrightarrow{\text{grad}}\,(\vec{B} \cdot \vec{A}) = (\vec{B} \cdot \vec{\nabla})\vec{A} + (\vec{A} \cdot \vec{\nabla})\vec{B} + \vec{B} \wedge \overrightarrow{\text{rot}}\, \vec{A} + \vec{A} \wedge \overrightarrow{\text{rot}}\, \vec{B}$

$\text{div}\,(f\vec{A}) = (\overrightarrow{\text{grad}}\, f)\vec{A} + f(\text{div}\, \vec{A})$ $\qquad \text{div}\,(\vec{B} \wedge \vec{A}) = (\overrightarrow{\text{rot}}\, \vec{B}) \cdot \vec{A} - \vec{B} \cdot (\overrightarrow{\text{rot}}\, \vec{A})$

$\overrightarrow{\text{rot}}\,(f\vec{A}) = (\overrightarrow{\text{grad}}\, f) \wedge \vec{A} + f\ (\overrightarrow{\text{rot}}\, \vec{A})$ $\qquad \overrightarrow{\text{rot}}\,(\vec{B} \wedge \vec{A}) = (\text{div}\, \vec{A})\ \vec{B} - \vec{A}\ (\text{div}\, \vec{B}) + (\vec{A} \cdot \vec{\nabla})\ \vec{B} - (\vec{B} \cdot \vec{\nabla})\ \vec{A}$

$\Delta(fg) = f\, \Delta g + 2\, \overrightarrow{\text{grad}}\, f \cdot \overrightarrow{\text{grad}}\, g + \Delta f\ g$

Identités : $\overrightarrow{\text{rot}}\,(\overrightarrow{\text{grad}}\, f) = \vec{\nabla} \times (\vec{\nabla} f) = \vec{0}$ $\qquad \text{div}\,(\overrightarrow{\text{rot}}\, \vec{A}) = \vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$

$\overrightarrow{\text{rot}}\,(\overrightarrow{\text{rot}}\, \vec{A}) = \vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \Delta\vec{A} = \overrightarrow{\text{grad}}\,(\text{div}\, \vec{A}) - \Delta\vec{A}$

Théorème de Poincaré : Sur $D \subset \mathbb{R}^3$ *simplement connexe* : $\quad \vec{A} = \overrightarrow{\text{grad}}\, f \iff \overrightarrow{\text{rot}}\, \vec{A} = 0$

Sur $D \subset \mathbb{R}^3$ *contractile* : $\quad \vec{A} = \overrightarrow{\text{rot}}\, \vec{B} \iff \text{div}\, \vec{A} = 0$

Théorème de Stokes : Si $\vec{A} = \overrightarrow{\text{rot}}\, \vec{B}$: $\quad \iint_{S^+} \vec{A} \cdot d\vec{S} = \oint_{\partial S^+} \vec{B} \cdot d\vec{\ell}$

Théorème de Gauss : Si $S^+ = \partial\Omega$ est une surface fermée : $\quad \oiint_{S^+} \vec{A} \cdot d\vec{S} = \iiint_\Omega \text{div}\, \vec{A}\ dx\ dy\ dz$

Corollaires : Si $\vec{A} = \overrightarrow{\text{grad}}\, f$ et C^+ est une courbe fermée : $\quad \oint_{C^+} \vec{A} \cdot d\vec{\ell} = 0$

Si $\vec{A} = \overrightarrow{\text{grad}}\, f$ et C^+ est une courbe qui relie P à Q : $\quad \int_{C^+} \vec{A} \cdot d\vec{\ell} = f(Q) - f(P)$