$$\iint_D (1+x+x^3)(y^2+y^4)\,dx\,dy, \quad \text{où } D=[0,1]\times[0,1].$$

$$= \int_0^1 \left((1+x+x^3) \left[\int_0^1 (y^2+y^4)\,dy \right] \right) dx$$

constante % x

$$= \left(\int_0^1 (y^2+y^4)\,dy \right) \times \left(\int_0^1 (1+x+x^3)\,dx \right)$$

$$= \left[\frac{y^3}{3} + \frac{y^5}{5} \right]_0^1 \times \left[x + \frac{x^2}{2} + \frac{x^4}{4} \right]_0^1$$

$$= \left(\frac{1}{3} + \frac{1}{5} \right) \times \left(1 + \frac{1}{2} + \frac{1}{4} \right) = \frac{8 \times 7}{15 \times 4} = \frac{14}{15}$$

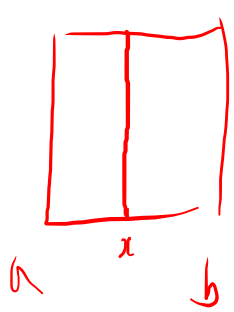
$$= \left(\frac{5}{15} + \frac{3}{15} \right) \times \left(\frac{4}{4} + \frac{2}{4} + \frac{1}{4} \right)$$

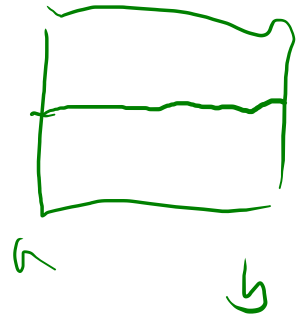
$$\iint_{[a,b]\times[c,d]} f(x,y)\,dx\,dy = \int_a^b \left(\int_c^d f(x,y)\,dy \right) dx$$

x est constant % y

$$= \int_c^d \left(\int_a^b f(x,y)\,dx \right) dy$$

y est constant % x





b) $\iint_D (1 + x + x^3 + y^2 + y^4)\, dx\, dy, \quad$ où $D = [0,1] \times [0,1]$.

$$= \iint_D (1 + x + x^3)\, dx\, dy + \iint_D (y^2 + y^4)\, dx\, dy$$

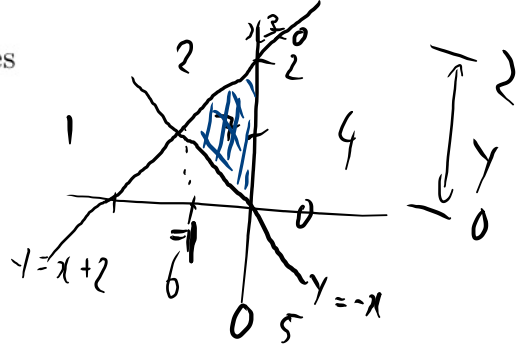
$$= \int_0^1 dy \times \int_0^1 (1 + x + x^3)\, dx + \int_0^1 dx \times \int_0^1 (y^2 + y^4)\, dy$$

$$= 1 \times \left[x + \frac{x^2}{2} + \frac{x^4}{4} \right]_0^1 + 1 \times \left[\frac{y^3}{3} + \frac{y^5}{5} \right]_0^1$$



$$= \frac{7}{4} + \frac{8}{15} = \frac{137}{60}$$

c) $\iint_D (1 + x + x^3 + y^2 + y^4)\, dx\, dy$, où D est la partie bornée du plan délimitée par les droites $x = 0$, $y = x + 2$ et $y = -x$.



$$= \int_{x=-1}^{x=0} \left(\int_{y=-x}^{y=x+2} (1+x+x^3+y^2+y^4)\, dy \right) dx$$

$$= \int_{x=-1}^{x=0} \left[(1+x+x^3)y + \frac{y^3}{3} + \frac{y^5}{5} \right]_{y=-x}^{y=x+2} dx$$

$$= \int_{x=-1}^{x=0} \left((1+x+x^3)(2x+2) + \frac{(x+2)^3 + x^3}{3} + \frac{(x+2)^5 + x^5}{5} \right) dx$$

$$= \int_{x=-1}^{x=0} \left(2 + 4x + 2x^2 + 2x^3 + \frac{(x+2)^3 + x^3}{3} + \frac{(x+2)^5 + x^5}{5} \right) dx$$

$$= \left[2x + \frac{4x^2}{2} + \frac{2x^3}{3} + \frac{2x^4}{4} + \frac{(x+2)^4}{12} + \frac{x^4}{12} + \frac{(x+2)^6}{30} + \frac{x^6}{30} \right]_{x=-1}^{x=0}$$



$$= 0 - \left(-2 + 2 - \frac{2}{3} + \frac{1}{2} + \frac{1}{12} + \frac{1}{12} + \frac{1}{30} + \frac{1}{30} \right)$$

$$= \frac{-2}{30} = \frac{-1}{15} \; !$$

c) $\iint_D (1 + x + x^3 + y^2 + y^4)\, dx\, dy$, où D est la partie bornée du plan délimitée par les droites $x = 0$, $y = x + 2$ et $y = -x$.

$$= \int_{y=0}^{y=2} \left(\int_{x=-y}^{x=0} (1 + x + x^3 + y^2 + y^4)\, dx \right) dy = \int_{y=0}^{y=1} \left[(1 + y^2 + y^4)x + \frac{x^2}{2} + \frac{x^4}{4} \right]_{x=-y}^{x=0} dy$$

$$+ \int_{y=1}^{y=2} \left[(1 + y^2 + y^4)x + \frac{x^2}{2} + \frac{x^4}{4} \right]_{x=y-2}^{x=0} dy$$



$$= \int_{y=0}^{y=1} + \left((1 + y^2 + y^4) \times y - \frac{y^2}{2} - \frac{y^4}{4} \right) dy$$

$$+ \int_{y=1}^{y=2} - \left(-(1 + y^2 + y^4)(y - 2) + \frac{(y-2)^2}{2} + \frac{(y-2)^4}{4} \right) dy$$

$2 - y + 2y^2 - y^3 + 2y^4$

$\neq -40/3$!

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{6} - \frac{1}{6} - \frac{1}{20} - \left(4 - 2 + \frac{16}{3} - 4 + \frac{64}{5}\right)$$
$$+ \left(2 - \frac{1}{2} + \frac{2}{3} - \frac{1}{4} + \frac{2}{5} - \frac{1}{6} - \frac{1}{20}\right) = \left[\frac{y^2}{2} + \frac{y^4}{4} + \frac{y^6}{6} - \frac{y^3}{6} - \frac{y^5}{20} \right]_0^1 - \left[2y - \frac{y^2}{2} + 2\frac{y^3}{3} - \frac{y^4}{4} + 2\frac{y^5}{5} + \frac{(y-2)^3}{6} + \frac{(y-2)^5}{20} \right]_{y=1}^{y=1}$$