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PCSI L1 S1 - UE TMB
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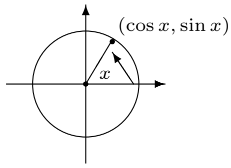
FORMULAIRE SUR LES FONCTIONS CIRCULAIRES



1. **Définition :** $(\cos x, \sin x)$ = coordonnées cartesiennes du point du cercle de rayon 1 à angle x :
   $D = \mathbb{R}, \qquad I = [-1, 1].$

   $$\tan x = \frac{\sin x}{\cos x} \qquad D = \bigcup_{k \in \mathbb{Z}} \left] -\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi \right[, \qquad I = \mathbb{R}.$$

   $$\cot x = \frac{\cos x}{\sin x} \qquad D = \bigcup_{k \in \mathbb{Z}} \left] k\pi, (k+1)\pi \right[, \qquad I = \mathbb{R}.$$

2. **Valeurs particulières :**

$\cos(0) = 1,$	$\sin(0) = 0,$	$\tan(0) = 0,$	$\cot(0) = \pm\infty$
$\cos(\pi/6) = \sqrt{3}/2,$	$\sin(\pi/6) = 1/2,$	$\tan(\pi/6) = \sqrt{3}/3,$	$\cot(\pi/6) = \sqrt{3}$
$\cos(\pi/4) = \sqrt{2}/2,$	$\sin(\pi/4) = \sqrt{2}/2,$	$\tan(\pi/4) = 1,$	$\cot(\pi/4) = 1$
$\cos(\pi/3) = 1/2,$	$\sin(\pi/3) = \sqrt{3}/2,$	$\tan(\pi/3) = \sqrt{3},$	$\cot(\pi/3) = \sqrt{3}/3$
$\cos(\pi/2) = 0,$	$\sin(\pi/2) = 1,$	$\tan(\pi/2) = \pm\infty,$	$\cot(\pi/2) = 0$
$\cos(\pi) = -1,$	$\sin(\pi) = 0,$	$\tan(\pi) = 0,$	$\cot(\pi) = \pm\infty$
$\cos(3\pi/2) = 0,$	$\sin(3\pi/2) = -1,$	$\tan(3\pi/2) = \pm\infty,$	$\cot(3\pi/2) = 0.$

3. **Periodicité :** pour tout $k \in \mathbb{Z}$ on a :

   $$\cos(x + 2k\pi) = \cos x \qquad \tan(x + k\pi) = \tan x$$
   $$\sin(x + 2k\pi) = \sin x \qquad \cot(x + k\pi) = \cot x$$

4. **Egalité :**

   $$\cos x = \cos y \iff x = y + 2k\pi \quad \text{ou} \quad x = -y + 2k\pi, \quad \forall k \in \mathbb{Z}$$
   $$\sin x = \sin y \iff x = y + 2k\pi \quad \text{ou} \quad x = -y + (2k+1)\pi, \quad \forall k \in \mathbb{Z}$$
   $$\tan x = \tan y \iff x = y + k\pi, \quad \forall k \in \mathbb{Z}$$
   $$\cot x = \cot y \iff x = y + k\pi, \quad \forall k \in \mathbb{Z}$$

5. **Identité circulaire :** $\cos^2 x + \sin^2 x = 1\,;$

6. **Expression de** $\sin x$ **et** $\tan x$ **en fonction de** $\cos x$ **et de** $\cos x$ **et** $\cot x$ **en fonction de** $\sin x$ **:**

   $$\sin x = \pm\sqrt{1 - \cos^2 x} \qquad \cos x = \pm\sqrt{1 - \sin^2 x}$$
   $$\tan x = \pm\sqrt{\frac{1}{\cos^2 x} - 1} \qquad \cot x = \pm\sqrt{\frac{1}{\sin^2 x} - 1}$$

7. **Formule de puissance (Moivre) :** $(\cos x + \sin x)^n = \cos(nx) + \sin(nx)$ pour tout $n \in \mathbb{N}$.

8. **Formules d'addition :**

$$\begin{aligned} \cos(x+y) &= \cos x \cos y - \sin x \sin y & \cos(x-y) &= \cos x \cos y + \sin x \sin y \\ \sin(x+y) &= \sin x \cos y + \cos x \sin y & \sin(x-y) &= \sin x \cos y - \cos x \sin y \\ \tan(x+y) &= \frac{\tan x + \tan y}{1 - \tan x \tan y} & \tan(x-y) &= \frac{\tan x - \tan y}{1 + \tan x \tan y}. \end{aligned}$$

9. **Formules de duplication :**

$$\begin{aligned} \cos(2x) &= \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1 \\ \sin(2x) &= 2 \sin x \cos x \\ \tan(2x) &= \frac{2\tan x}{1 - \tan^2 x}. \end{aligned}$$

10. **Expression de $\cos x$, $\sin x$ et $\tan x$ en fonction de $t = \tan(x/2)$:**

$$\cos x = \frac{1-t^2}{1+t^2}, \qquad \sin x = \frac{2t}{1+t^2}, \qquad \tan x = \frac{2t}{1-t^2}.$$

11. **Formules de linéarisation :**

$$\begin{aligned} &\cos x \cos y = \frac{1}{2}(\cos(x+y) + \cos(x-y)) & &\sin x \sin y = \frac{1}{2}(\cos(x-y) - \cos(x+y)) \\ &\cos x \sin y = \frac{1}{2}(\sin(x+y) - \sin(x-y)) & &\sin x \cos y = \frac{1}{2}(\sin(x+y) + \sin(x-y)) \\ &\cos^2 x = \frac{1+\cos(2x)}{2} & &\sin^2 x = \frac{1-\cos(2x)}{2} \end{aligned}$$

12. **Formules de factorisation :**

$$\begin{aligned} &\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) & &\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right) \\ &\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) & &\sin x - \sin y = 2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right) \end{aligned}$$

13. **Formules relatives aux angles associés :**
    (a) **Angles opposés :**

    $$\cos(-x) = \cos x \qquad \sin(-x) = -\sin x \qquad \tan(-x) = -\tan x.$$

    Donc la fonction cos est paire et la fonction sin est impaire.

    (b) **Angles supplémentaires :**

    $$\cos(\pi - x) = -\cos x \qquad \sin(\pi - x) = \sin x \qquad \tan(\pi - x) = -\tan x.$$

    (c) **Angles complémentaires :**

    $$\cos\left(\frac{\pi}{2} - x\right) = \sin x \qquad \sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \tan\left(\frac{\pi}{2} - x\right) = \frac{1}{\tan x}.$$

    (d) **Angles "de différence π" :**

    $$\cos(x + \pi) = -\cos x \qquad \sin(x + \pi) = -\sin x \qquad \tan(x + \pi) = \tan x.$$