Math2B - 17 février 2023 - Questions sur le CC1

① Surfaces de niveau

1) $\phi(x,y,z) = e^{z-x^2-y^2}$

$\phi : \mathbb{R}^3 \longrightarrow \mathbb{R}$ champ scalaire sur $\mathbb{R}^3$

S_a surface de niveau $a \in \mathbb{R}$

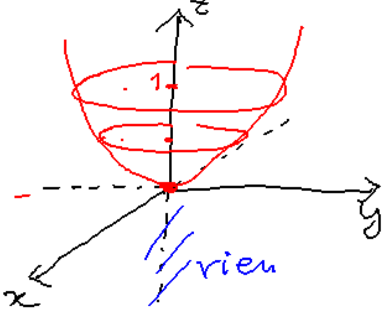
$S_1(\phi) = \{(x,y,z) \in \mathbb{R}^3 \mid \phi(x,y,z) = 1\}$

$\boxed{\ln(e^a) = a}$

$\ln\left(e^{z-x^2-y^2}\right) = 1 = \ln(e^0) \iff z-x^2-y^2 = 0 \iff \boxed{z = x^2+y^2}$

$\Rightarrow S_1(\phi) = \{(x,y,z) \in \mathbb{R}^3 \mid \underset{\geq 0}{z} = \underbrace{x^2+y^2}_{\geq 0}\}$

éq. cartésienne d'un paraboloïde



2) $\phi(x,y,z) = \dfrac{y}{\sqrt{x^2+z^2+2}}$ $\qquad D_\phi = \mathbb{R}^3$

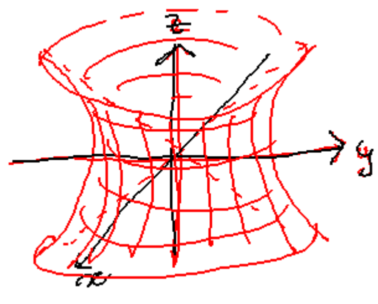
$S_1(\phi) = \left\{(x,y,z) \mid \dfrac{y}{\underbrace{\sqrt{x^2+z^2+2}}_{\neq 0}} = 1\right\}$ ⟵ on multiplie à gauche et à droite par $\sqrt{x^2+z^2+2}$

$\iff \underset{\geq 0}{y} = \underbrace{\sqrt{x^2+z^2+2}}_{\geq 0} \iff y^2 = x^2+z^2+2 \iff \boxed{y^2-x^2-z^2 = 2}$

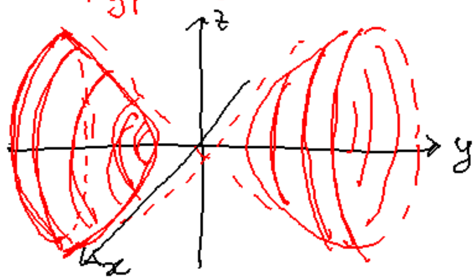
$\Leftarrow$

$\iff -\left(\frac{x}{\sqrt{2}}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 - \left(\frac{z}{\sqrt{2}}\right)^2 = 1$ $\qquad \boxed{-X^2+Y^2-Z^2 = 1}$

hyperboloïde à 2 nappes



hyperboloïde à 1 nappe

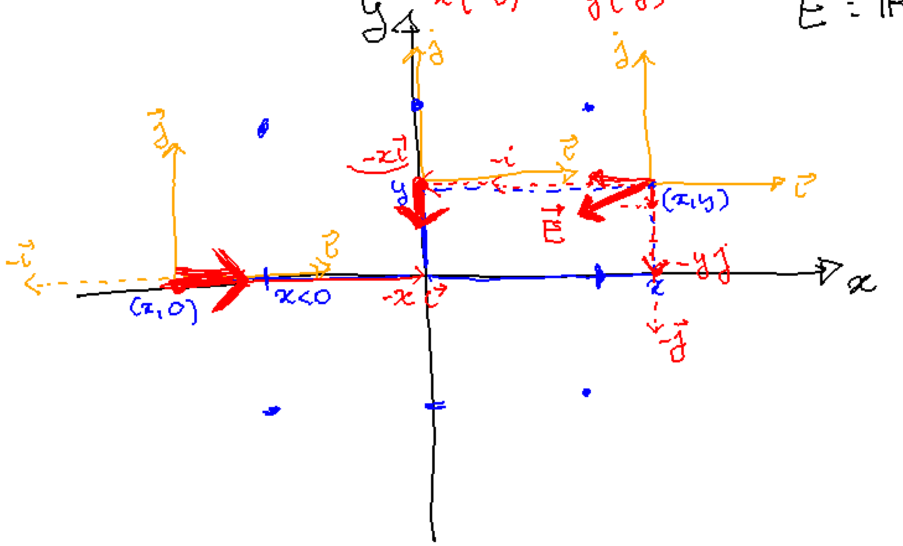


(2) Dessin des champs de vecteurs

1) $\vec{E}(x,y) = \frac{1}{5}\left(\underbrace{-x\vec{i}}_{x(-\vec{i})} - \underbrace{y\vec{j}}_{y(-\vec{j})}\right)$

champ de vecteurs sur $\mathbb{R}^2$

$\vec{E} : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$



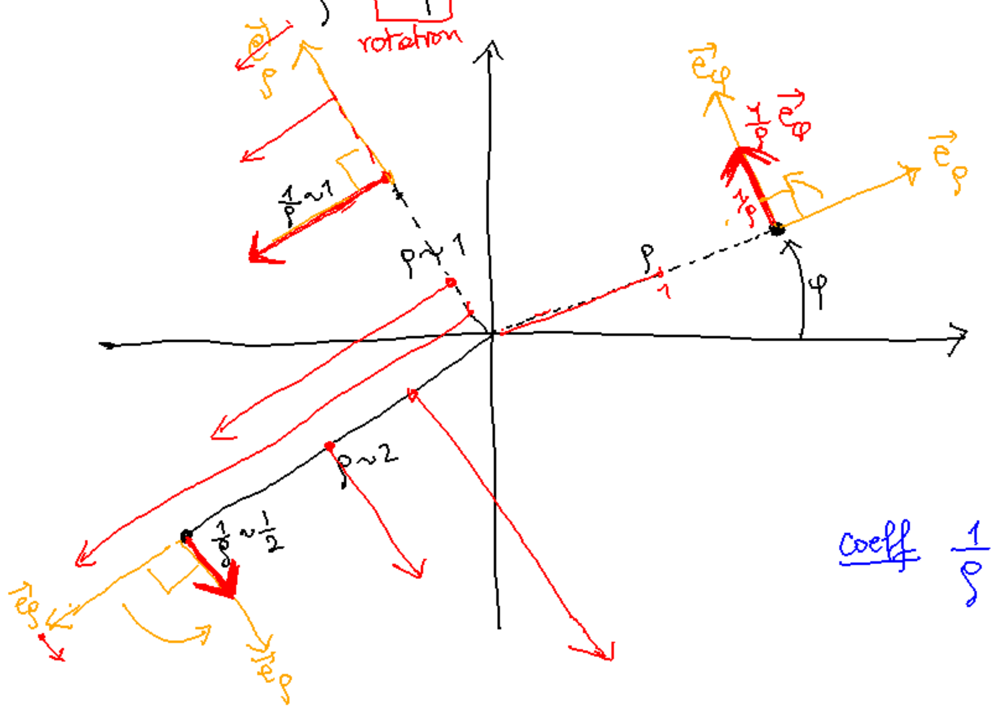
$\vec{E}(1,0)$ $\quad$ $\vec{E}(1,1)$

$\vec{E}(0,1)$ $\quad$ $\vec{E}(-1,1)$

$\vec{E}(-1,0)$ $\quad$ $\vec{E}(-1,-1)$

$\vec{E}(0,-1)$ $\quad$ $\vec{E}(1,-1)$

2) $\vec{V}(\rho,\varphi) = \frac{1}{\rho}\underbrace{\vec{e}_\varphi}_{\text{rotation}}$ $\leftarrow$ repère mobile polaire $(\vec{e}_\rho, \vec{e}_\varphi)$ o.n.



$\frac{1}{\rho} < 1$ si $\rho > 1$

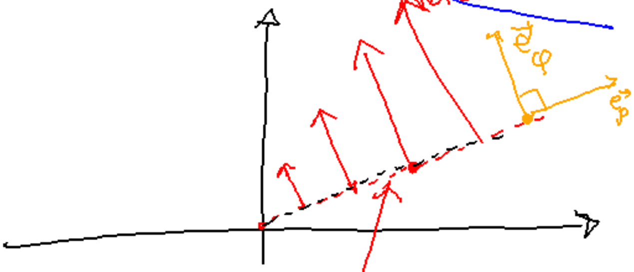
champ de rotation

(seule flèche $\vec{e}_\varphi$)

coeff $\frac{1}{\rho}$ donne la longueur de la flèche de $\vec{V}$

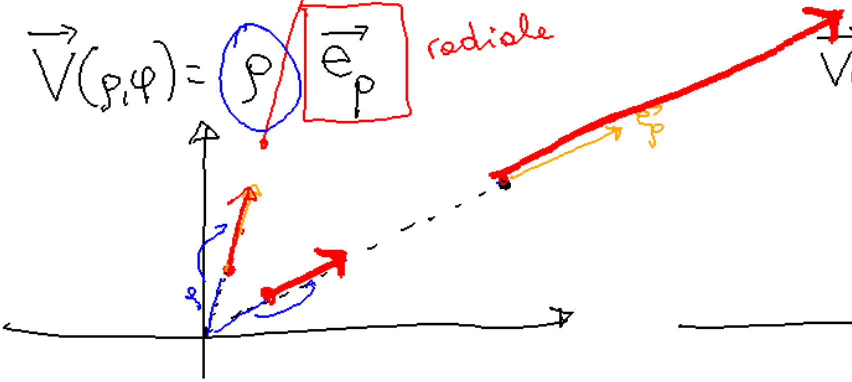
3) $\vec{V}(\rho,\varphi) = \rho \, \boxed{\vec{e}_\varphi}$ rotation

longueur de la flèche



$\vec{V}(\rho,\varphi) = \frac{1}{\rho} \, \boxed{\vec{e}_\varphi}$ rotation

$\vec{V}(\rho,\varphi) = \rho \, \boxed{\vec{e}_\rho}$ radiale



$\vec{V}(\rho,\varphi) = \frac{1}{\rho} \, \boxed{\vec{e}_\rho}$ radiale