Rings of definable scalars of Verma modules

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This a joint work with Mike Prest [1].

Let sl_2k be the Lie algebra of trace zero 2×2 matrices over an algebraically closed field k of characteristic zero and let $U = U(sl_2k)$ the universal enveloping algebra. We consider the corresponding Verma module $M(\lambda)$ for any $\lambda \in k$. We show that the ring of definable scalars of $M(\lambda)$, $R_{M(\lambda)}$, is von Neumann regular by regarded $M(\lambda)$ as a module over a suitable generalized Weyl Algebra. We also obtain some information about the Ziegler spectrum of $R_{M(\lambda)}$. The proofs make use of ideas from the model theory of modules.

The work was inspired by Herzog's paper [2] in which some remarkable results about the ring of definable scalars are described for the set of finite-dimensional representations of U. It is natural to ask what happens if we replace the set of finite-dimensional representations by the set of all Verma modules $M(\lambda)$ for all $\lambda \in k$.

References

[1] S. L'Innocente, M. Prest, Rings of definable scalars of Verma modules, *Journal of Algebra and its applications*, to appear.

[2] I. Herzog, The pseudo-finte dimensional representations of $sl(2, k)$, *Selecta Mathematica*, 7 (2001), 241-290