Fast Marching Methods for Hamilton-Jacobi Equations

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MAIN EQUATION

\[
\begin{align*}
H(x, u, \nabla u) & = 0 \quad x \in \Omega \\
\partial_t u - \nabla \cdot (c(x) \nabla u) & = 0 \quad x \in \Omega, t > 0
\end{align*}
\]

Fast Marching Methods (FMM) were introduced by J. A. Sethian in [6] to solve equation (1) by a finite difference discretization only in the case of monotone front propagation problems. We present some improvements of FMM and extensions of this technique to non-monotone front propagation and non-convex Hamiltonians.

Theorem 1. Let \( c > 0, c \in L^\infty(\mathbb{R}^n) \) and
\[
\Delta x \leq (\sqrt{5} - 1) \max \frac{1}{c(x)}
\]
where \( L_c \) is the Lipschitz constant of \( c \) and \( \epsilon_{	ext{min}} = \min_{x \in \Omega} c(x) \). Then, FMM computes a real approximate viscosity solution of (2).

**Numerical result** \( \epsilon_0 = (0.0, c(x,y)) \approx 1.1 \). Exact solution. \( T(x,y) = \sqrt{x^2 + y^2} \). Note the error!

<table>
<thead>
<tr>
<th>Method</th>
<th>( \epsilon_0 )</th>
<th>( \epsilon_0 \text{ error} )</th>
<th>( T \text{ time (sec)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SL-FMM</td>
<td>0.081</td>
<td>0.0329</td>
<td>7.15</td>
</tr>
<tr>
<td>FMM</td>
<td>0.081</td>
<td>0.0329</td>
<td>7.13</td>
</tr>
</tbody>
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References