

Meeting Lyon – São Paulo

Algebra, Group and Logic

Lyon, April 22–26, 2013

Abstracts

Monday, April 22, 2013:

1. *Weight Representations of Kac-Moody Algebras*, by V. Futorny (USP).
Abstract: Classification problem of weight irreducible representations is open even for finite-dimensional simple Lie algebras (except \mathfrak{sl}_2). S. Fernando and O. Mathieu classified such representations with finite-dimensional weight spaces. This is the largest subcategory of modules where classification of irreducible objects is known. On the other hand, Y. Drozd, V. Futorny and S. Ovsienko introduced a larger category of so-called Gelfand-Tsetlin modules where classification is known up to a certain finiteness. Recently, a classification of irreducible Gelfand-Tsetlin modules was completed for \mathfrak{sl}_3 by D. Grantcharov, V. Futorny and E. Ramires. We will discuss these results and also comment on the current situation with the same problem for affine Kac-Moody algebras.
2. *Groups Definable in Orthogonal Sets*, by F. Wagner (UdL, Lyon 1).
Abstract: Two sets X and Y are orthogonal if every definable subset of $X \times Y$ is a finite union of rectangles $A \times B$ with $A \subseteq X$ and $B \subseteq Y$. I shall study groups definable in the product $X \times Y$. Examples show that they need not be isomorphic to the product of a group definable in X and one definable in Y — in fact there need not be any groups definable in X or Y separately. I shall give a structure theorem under an additional hypothesis on X , and make a conjecture for the general case involving local and approximate groups.
3. *Noncommutative Algebras and Poisson Algebras*, by A. Pichereau (UdL, St. Étienne).
Abstract: This is a joint work with R. Berger (St-Étienne) Following the idea that classical mechanics should be a limit case of quantum

mechanics, P.A.M. Dirac explained that the commutator of dynamical variables in quantum mechanics should be the analogue of the symplectic Poisson bracket of \mathbb{R}^{2r} in classical mechanics. Working in a mathematical setting, we consider a non-commutative algebra B , which can be seen as a deformation of a Poisson algebra T . This algebra B belongs to a family of 3-Calabi-Yau algebras defined by potentials and depending on a natural integer n and the algebra B is for us the most interesting example in the case $n = 2$. We give cohomological links between B and T , as we obtain the Poisson cohomology of T and prove that the Hochschild cohomology of B is isomorphic to the Poisson cohomology of T .

4. *Problems and Conjectures in Division Rings: new Advances*, by J.Z. Gonçalves (USP).

Abstract: Let D be a division ring with center k and multiplicative group $D^* = D \setminus \{0\}$. We will discuss the history and progress obtained trying to prove the following conjectures:

Conjecture(G) (Lichtman, 1977): D^* contains a free (noncyclic) subgroup.

Conjecture(A) (Makar-Limanov, 1982): If D is finitely generated and infinite-dimensional over k then D contains a free (noncyclic) algebra.

Conjecture(GA): Under the same conditions as in (A), D contains the k -group algebra of a free group of rank 2.

We also address the problem of constructing free groups and free algebras in D .

5. *Cubic Forms over \mathbb{F}_2 and the Hurwitz-Radon Function*, by V. Ovsienko (UdL, Lyon 1).

Abstract: We consider cubic forms over the field with 2 elements (sometimes called "boolean cubic functions"). The classification problem of such forms is widely open. Our goal is to define an invariant that can be understood as an analog of the classical Arf invariant of quadratic forms, and show its relation to the Hurwitz-Radon numbers. We will show a combinatorial consequence of this relation.

Tuesday, April 23, 2013:

1. *On Idempotents in Algebraic Semigroups*, by M. Brion (Grenoble).

Abstract: The objects of the talk are algebraic semigroups, that is, algebraic varieties equipped with an associative composition law which is a morphism of varieties. We shall show that every such semigroup S admits idempotent elements; moreover, the set $E(S)$ of idempotents has an unexpectedly rich structure under additional assumptions on S . For example, if S is commutative then $E(S)$ is finite; if in addition S is irreducible (as a variety), then $E(S)$ is the set of faces of some rational polyhedral convex cone.

2. *On Some Pro- p Groups from Infinite-Dimensional Lie Theory*, (joint work with Bertrand Rémy), by I. Capdeboscq (Warwick).
Abstract: We initiate the study of some pro- p -groups arising from infinite-dimensional Lie theory. These groups are completions of some subgroups of incomplete Kac-Moody groups over finite fields, with respect to various completions of algebraic or geometric origin. We show topological finite generation for the pro- p Sylow subgroups in many complete Kac-Moody groups. This implies abstract simplicity of the latter groups. We also discuss with the question of (non-)linearity of these pro- p groups.
3. *Finitely Presented Groups with Infinitely Many non-Homeomorphic Asymptotic Cones*, by A. Ould Houcine (UdL, Lyon 1).
Abstract: We construct a finitely presented group with infinitely many non-homeomorphic asymptotic cones. We also show that the existence of cut points in asymptotic cones of finitely presented groups does, in general, depend on the choice of scaling constants and ultrafilters. This is a joint work with D. Osin.
4. *Satake Isomorphism for Kac-Moody groups*, by S. Gaussent (UdL, St-Étienne).
Abstract: We use a Kac-Moody version of the Bruhat-Tits building to define the spherical Hecke algebra H for a Kac-Moody group over a local non-archimedean field. Then, we prove the Satake isomorphism between H and the algebra of Weyl-invariant elements of the Looijenga's coweight algebra. The proof involves a parabolic retraction of the hovel onto an extended tree inside the generalization of the building. This is a joint work with G. Rousseau.
5. *Trisections in Module Categories and Quasi-Directed Components*, by F.U. Coelho (USP).
Abstract: The purpose of this talk is to discuss new results concerning the existence of trisections in the module categories in relation to the components of the Auslander-Reiten quiver called quasi-directed. Such components appear naturally in several classes of algebras defined by homological properties.

Wednesday, April 24, 2013:

1. *Definable Subgroups of Minimal Simple Groups of Finite Morley Rank*, by T. Altinel (UdL, Lyon 1).
Abstract: Morley rank is an ordinal dimension that generalizes the Zariski dimension in algebraic geometry over algebraically closed fields. Simple groups that admit a finite rank are conjectured to be linear algebraic. This conjecture has frequently been attacked using inductive methods inspired by finite group theory. Such a strategy necessitates a better understanding of the minimal simple groups of finite Morley rank.

In a joint work with Jeffrey Burdges and Olivier Frecon, we have obtained a fairly precise structural description of the definable subgroups of most minimal simple groups of finite Morley rank. The proof establishes an abstract Jordan decomposition and yields an answer to an open question about the splitting of Borel subgroups in one of the known configurations.

2. *A Banachic Version of a Theorem of Delorme and Applications*, by R. Tessera (UdL, ENS Lyon).

Abstract: Delorme proved in 1977 that a unitary representation with non-trivial first cohomology of a connected solvable Lie group has a dimension 1 subrepresentation. In 2000, Shalom proved that a slightly weaker property passes to lattices. With Cornulier, we give a new geometric proof of Delorme's result, which allows us to extend it to isometric actions on any reflexive Banach spaces. We also enlarge the class of locally compact groups satisfying the conclusion.

3. *The Profinite Topology on Groups*, by P. Zalesskii (Brasilia).

Abstract: We shall start with a discussion of residual properties of groups and their interpretation in connection with the profinite completion. According to J-P. Serre a group G is *good* if the cohomology groups of G and its profinite completion \hat{G} are naturally isomorphic on finite coefficients. The talk will be about residual properties and goodness for arithmetic groups and groups of geometric nature.

Thursday, April 25, 2013:

1. *Nonassociative Lie Theory*, by I. Shestakov (USP).

Abstract: Non-associative Lie theory, that is, Lie theory for non-associative products, appeared as a subject of its own in the works of A.I. Malcev who constructed the tangent structures corresponding to Moufang loops. Its general development has been slow; nevertheless, by now many of the basic features of the theory have been understood, with much of the progress happening in the last ten years. In the present talk we outline the non-associative Lie theory in general and review the recent developments, related with the notions of Sabinin algebras and non-associative Hopf algebras.

2. *On the Classification of \mathbb{Z}^N -Graded Lie Algebras* (joint work with K. Iohara), by O. Mathieu (UdL, Lyon 1).

Abstract: In this talk, we will explain recent advances on the classification of simple \mathbb{Z}^N -graded Lie algebras. Here we consider Lie algebras whose homogenous components are one-dimensional.

Although this looks like a very abstract problem of algebra, the result involves concrete Lie algebras, namely some affine lie algebras and some sub-algebras of symbols of twisted PDO on the circle. Some parts of the proof are based on classical topics, like Jordan algebra theory or the question of non local equivariant binary maps between tensor densities modules over the circle.

3. *Algebraic Methods in Parameter Estimation*, by R. Ushirobira (INRIA, Lille).

Abstract: In this talk, we will review some basic concepts of the theory of automatic control and the algebraic structures related to it. We will show how the problem of parameter estimation can be treated by algebraic methods in an efficient manner.

4. *A Global Version of Grozman's Theorem* (joint work with O. Mathieu), by K. Iohara (UdL, Lyon 1).

Abstract: The Witt algebra \mathbf{W} is the Lie algebra of algebraic vector fields on \mathbb{C}^* whose central extension is known as the Virasoro algebra. The classification of the \mathbb{Z} -graded indecomposable \mathbf{W} -module with multiplicity 1 has been achieved by I. Kaplansky and L. J. Santharoubane in 1984. In this talk, we explain the classification of the \mathbf{W} -equivariant bilinear maps $M \times N \rightarrow P$ where M, N, P are \mathbb{Z} -graded indecomposable \mathbf{W} -modules with multiplicity 1.

5. *Free Symmetric Algebras in Skew Fields Generated by Orderable*, by V.O. Ferreira (USP).

Abstract: The canonical involution on a non-abelian ordered group can be extended to an involution on the division ring of fractions of the group algebra. With respect to this involution, a pair of symmetric elements generating a free subalgebra is shown to exist. (This is a joint work with J. Goncalves and J. Sanchez at IME-USP, Brazil.)

Friday, April 26, 2013:

1. *Jordan Algebras and Their One-sided Representations*, by I. Kashuba (USP).

Abstract: This talk is a survey on the joint results with S. Ovsienko, V. Serganova and I. Shestakov. It is devoted to the problem of the classification of indecomposable Jordan bimodules over finite-dimensional Jordan algebras when squared radical is zero.

Recall, that for a Jordan algebra \mathcal{J} the category \mathcal{J} -bimod of \mathbb{k} -finite dimensional \mathcal{J} -bimodules is equivalent to the category \mathcal{U} -mod of (left) finitely dimensional modules over an associative algebra $\mathcal{U} = \mathcal{U}(\mathcal{J})$, which is called the universal multiplication envelope of \mathcal{J} . If \mathcal{J} has finite dimension the algebra \mathcal{U} is finite dimensional as well. In particular, in accordance with the representation type of the algebra \mathcal{U} one can define Jordan algebras of the finite, tame and wild representation types.

From the other side to each Jordan algebra corresponds a Lie algebra $\mathcal{LKH}(\mathcal{J})$. Moreover there is a correspondence between the finite dimensional Jordan modules over \mathcal{J} and finite dimensional Lie modules over $\mathcal{LKH}(\mathcal{J})$.

This allows us to apply to the category \mathcal{J} -bimod all the machinery developed in the representation theory of finite dimensional algebras as well as the representation theory of Lie algebras.

2. *Full Groups of Minimal Homeomorphisms of the Cantor Space*, by J. Melleray (UdL, Lyon 1).

Abstract: Full groups of minimal homeomorphisms of the Cantor space are useful objects, which remember the structure of the orbit decomposition of the space under the action of the homeomorphism. These groups have counterparts in ergodic theory which can be endowed with a Polish group topology, enabling one to use Baire category methods to study them. I will explain why such a topology cannot exist in the topological context, and discuss some related problems.

3. *A Reduction Theorem for the Computation of the Yoneda Algebra*, by E. N. Marcos (USP).

Abstract: (This is a joint work in progress with Ed Green and Dag Madsen.) We take in an Artin Algebra Λ an idempotents e and f with $e+f=1$. Let $\Lambda^* = f\Lambda f$, we show that, with some strong hypothesis, the Yoneda algebra $\bigoplus_{i \geq 0} (\Lambda / (r^i \Lambda))$ is eventually isomorphic, as rings without unit, to $\bigoplus_{i \geq 0} (\Lambda^* / (r^i(\Lambda^*)))$. We also show that this can be generalized for the graded case.

4. *Unimodularity and Calabi-Yau Duality for the Algebras of Differential Operators*, by P. Le Meur (Clermont-Ferrand).

Abstract: The algebra of differential operators, or enveloping algebra, associated with a compatible action by derivations of a Lie algebra L on a commutative algebra S was defined by Rinehart (1963). The most classical examples arise from Lie algebras, from Poisson structures and from Lie algebroids. Such a pair (S, L) is now called a Lie-Rinehart algebra. In his study of Lie-Rinehart algebras, Huebschmann established (1999) some Poincaré duality involving the (co)homology groups associated with (S, L) . This duality is related to the existence of a suitable modular class which he defines, and which is tightly related to the modular class introduced by Weinstein. On the other hand, in many examples, the enveloping algebra of (S, L) appears to be Calabi-Yau in the sense of Ginzburg. This is the case for Weyl algebras and for enveloping algebras of finite-dimensional and unimodular Lie algebras. The talk shall present some links between the duality and the modular class on (S, L) , and the Calabi-Yau duality on the corresponding algebra of differential operators.

5. *Gradings and Fundamental Group of a Linear Category*, by C. Cibils (CIMPA).

Abstract: The intrinsic fundamental group *à la Grothendieck* of a category over a field is obtained through its connected gradings, which are related to its Galois coverings. In some cases there exists a universal (or a versal) grading. After considering the theoretical tools, I will concentrate in the Schurian generated case, that is on linear cat-

egories which are generated by their spaces of morphisms which are one-dimensional : an universal grading exists. The results are based on the joint paper on arXiv with Maria-Julia Redondo and Andrea Solotar, "On universal gradings, versal gradings and Schurian generated categories" to appear in J. Noncommut. Geom.