Titre: Combinatoire des tableaux et polynômes orthogonaux

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There are several recent works connecting tableaux and orthogonal polynomials:

- Permutation tableaux are new objects that come from the enumeration of the totally positive Grassmannian cells [11, 15]. A permutation tableau [12] is a Ferrers diagram together with a filling of the cells with 0's and 1's such that the following properties hold: Each column contains at least one 1 and there is no 0 which has a 1 above it in the same column and a 1 to its left in the same row. They are called permutation tableaux because they are in bijection with permutations [12]. Different statistics on permutation tableaux were defined in [5, 12] and they have counter part in permutations. Surprisingly they are also connected to a statistical physics model called the Partially ASymmetric Exclusion Process (PASEP) [4, 5]. This connection was made thanks to previous work by Duchi and Schaeffer [6]. In particular, it was shown in [4] that the probability of finding the PASEP in configuration tau in the steady state is the probability generating function of the tableau of shape tau, according to the number of superfluous ones, the number of ones in the first row and the number of unrestricted rows minus one. Moreover, the partition function Z_n for the PASEP is equal to the weight-generating function for all permutation tableaux of length $n + 1$. In [5], we defined a Markov chain on the permutation tableaux that gave a combinatorial proof of this result. Moreover this Partially ASymmetric Exclusion Process has more generalizations than the ones studied in [4, 5].

  Firstly we want to better understand all the combinatorics of the ASymmetric Exclusion Process [13]. Secondly the solutions given by physicist for those models [13] use orthogonal polynomials and the more general model uses the Askey-Wilson polynomials. Therefore it would be possible to use these tableaux to construct some theory of orthogonal polynomials as done for certain polynomials by Viennot and others [14]. In this context the combinatorics of the Al-Salam-Chihara polynomials has been worked out recently and it is also known that the generating function of some statistics on tableaux can be written as Al-Salam-Chihara polynomials [7, 8].

- There has been a series of papers on the number of fillings of cell diagrams with restrictions on their increasing and decreasing chains, most notably the paper by Chen, Deng, Du, Stanley and Yan on matchings and set partitions with restrictions on their ”crossings” and ”nestings”, the paper by Bousquet-Mélou and Stengrimsson on involutions with restrictions on their increasing subsequences, and the paper by Jonsson on fillings of ”moon polyominoes” with restrictions on their increasing chains. Krattenthaler has shown that a unifying way to approach these problems is by the diagrammatic versions of the Robinson-Schensted algorithm (due to Fomin) and of Schtzenberger's jeu de taquin (due to van Leeuwen). Several problems are still open in this big picture, though Rubey has made some progress for problems related to moon polyominoes.

- The above two themes are not disconnected. Actually the original motivation of Chen et al. was to generalize the classical result of equi-distribution of crossings and nestings of two edges in matchings and partitions. A natural extension has been worked out by Kasraoui and Zeng [9], and a similar result for permutations has been obtained by Corteel [3] while studing problems related to permutation tableaux. It would be interesting to find the similar notions of k-crossings and k-nestings in permutations. Finally the generating function of the permutation tableaux can be written by continued fractions [3] or hypergeometric series [15]. The explanation for these two different expressions may be given by identifying the corresponding orthogonal polynomials and then using the known explicit formula for the moment sequence.

The candidate should be familiar with the basic theory of enumerative combinatorics.

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