Erratum for the article "Multidimensional Potential Burgers Turbulence"

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In our contibution [2] we overlooked that the space $L_\infty/\mathbb{R}$ on which we consider the dual-Lipschitz metric is not separable: thus we are not in the standard setting in which stationary measures for stochastic PDEs are studied (cf. [3]). In particular, the semigroup S_t^ω is well-defined on $L_\infty/\mathbb{R}$ for every $\omega \in \Omega$, but the dual semigroup S_t^* is not well-defined. Thus Lemma 8.4. does not hold. However, we have the following result, the proof of which is almost word-to-word the same as the proof of the corresponding 1d result in [1].

Lemma *There exist positive constants C', δ such that for $\psi_1^0, \psi_2^0 \in L_\infty$ we have*

$$\mathbf{E}|S_t^\omega \delta_{\psi_1^0} - S_t^\omega \delta_{\psi_2^0}|_{L(\infty)/\mathbb{R}} \leq C' t^{-\delta}, \qquad t \geq 1. \tag{1}$$

In particular, these constants do not depend on ψ_1^0, ψ_2^0.

For all ω the solution of the stochastic Burgers equation is C^∞-smooth in space for $t > 0$: this is proved in Appendix 1. This allows us to define the semigroups $\tilde{S}_t^\omega$ and $\tilde{S}_t^*$, acting respectively on $L(1)$ and on the space of probability measures on $L(1)$. We consider two solutions ψ_1, ψ_2 to the stochastic Hamilton-Jacobi equation with the same noise and different initial conditions, as well as the corresponding solutions $\mathbf{u}_1, \mathbf{u}_2$ to the stochastic Burgers equation. By the Gagliardo-Nirenberg inequality we get

$$\begin{aligned} |\mathbf{u}_1 - \mathbf{u}_2|_1 &\leq C|\psi_1 - \psi_2 - \textstyle\int_{\mathbf{T}^d} (\psi_1 - \psi_2)|_1 |\nabla(\psi_1 - \psi_2)|_{1,1} \\ &\leq C|\psi_1 - \psi_2 - \textstyle\int_{\mathbf{T}^d} (\psi_1 - \psi_2)|_\infty |\mathbf{u}_1 - \mathbf{u}_2|_{1,1}. \end{aligned}$$

Thus, using Theorem 6.2. and the lemma stated above we obtain the existence of a u_1^0, u_2^0-independent constant C' such that:

$$\mathbf{E}|\tilde{S}_t^\omega \delta_{u_1^0} - \tilde{S}_t^\omega \delta_{u_2^0}|_{L_1} \leq C' t^{-\delta/2}, \qquad t \geq 1, \tag{2}$$

with the same δ as above. This inequality allows us to obtain Theorem 8.5.

Thus, Lemma 8.4. does not hold, but all other results remain valid.

References

[1] A. Boritchev, *Sharp estimates for turbulence in white-forced generalised Burgers equation*, GAFA 23 (2013), 6, 1730-1771.

[2] A. Boritchev, *Multidimensional Potential Burgers Turbulence*, Communications in Mathematical Physics, http://dx.doi.org/10.1007/s00220-015-2521-7, 2015.

[3] S. Kuksin and A. Shirikyan, *Mathematics of two-dimensional turbulence*, Cambridge tracts in mathematics, vol. 194, Cambridge University Press, 2012.