Computation of the Lax matrix L in the oper gauge from a general Lax matrix \td{L} and application to the gl_3 case of one pole at infinity

```
> restart:
  with(LinearAlgebra):
  with(algcurves):
> L:=Matrix(3,3,0):
  L[1,2]:=1:
  L[2,3]:=1:
  L[3,1]:=L31(lambda):
  L[3,2]:=L32(lambda):
  L[3,3]:=L33(lambda):
  L;
```

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ L31(\lambda) & L32(\lambda) & L33(\lambda) \end{bmatrix}$$

(1.1)

```
> tdL:=Matrix(3,3,0):
  tdL[1,1]:=tdL11(lambda):
  tdL[1,2]:=tdL12(lambda):
  tdL[1,3]:=tdL13(lambda):
  tdL[2,1]:=tdL21(lambda):
  tdL[2,2]:=tdL22(lambda):
  tdL[2,3]:=tdL23(lambda):
  tdL[3,1]:=tdL31(lambda):
  tdL[3,2]:=tdL32(lambda):
  tdL[3,3]:=tdL33(lambda):
  tdL;
```

$$\begin{bmatrix} tdL11(\lambda) & tdL12(\lambda) & tdL13(\lambda) \\ tdL21(\lambda) & tdL22(\lambda) & tdL23(\lambda) \\ tdL31(\lambda) & tdL32(\lambda) & tdL33(\lambda) \end{bmatrix}$$

(1.2)

```
> G:=Matrix(3,3,0):
  G[1,1]:=G11(lambda):
  G[1,2]:=G12(lambda):
  G[1,3]:=G13(lambda):
  G[2,1]:=G21(lambda):
  G[2,2]:=G22(lambda):
  G[2,3]:=G23(lambda):
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```
G[3,1]:=G31(lambda):
G[3,2]:=G32(lambda):
G[3,3]:=G33(lambda):
G;
dGdlambda:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do dGdlambda[i,j]:=diff
(G[i,j],lambda): od: od:
dGdlambda;
```

$$
\begin{bmatrix}
G11(\lambda) & G12(\lambda) & G13(\lambda) \\
G21(\lambda) & G22(\lambda) & G23(\lambda) \\
G31(\lambda) & G32(\lambda) & G33(\lambda)
\end{bmatrix}
\tag{1.3}
$$

$$
\begin{bmatrix}
\dfrac{d}{d\lambda} G11(\lambda) & \dfrac{d}{d\lambda} G12(\lambda) & \dfrac{d}{d\lambda} G13(\lambda) \\[2mm]
\dfrac{d}{d\lambda} G21(\lambda) & \dfrac{d}{d\lambda} G22(\lambda) & \dfrac{d}{d\lambda} G23(\lambda) \\[2mm]
\dfrac{d}{d\lambda} G31(\lambda) & \dfrac{d}{d\lambda} G32(\lambda) & \dfrac{d}{d\lambda} G33(\lambda)
\end{bmatrix}
$$

```
> ToCancel:=simplify(L-Multiply(Multiply(G,tdL),G^(-1))-h*
  Multiply(dGdlambda,G^(-1))):
> G11:=unapply(1,lambda):
  G12:=unapply(0,lambda):
  G13:=unapply(0,lambda):
  G23:=unapply(tdL13(lambda),lambda):
  G22:=unapply(tdL12(lambda),lambda):
  G21:=unapply(tdL11(lambda),lambda):
  G31:=unapply(tdL12(lambda)*tdL21(lambda)+tdL13(lambda)*tdL31
  (lambda)+tdL11(lambda)^2+h*diff(tdL11(lambda), lambda),lambda):
  G32:=unapply(tdL12(lambda)*tdL22(lambda)+tdL11(lambda)*tdL12
  (lambda)+tdL13(lambda)*tdL32(lambda)+h*diff(tdL12(lambda),
  lambda),lambda):
  G33:=unapply(tdL12(lambda)*tdL23(lambda)+tdL13(lambda)*tdL33
  (lambda)+tdL11(lambda)*tdL13(lambda)+h*diff(tdL13(lambda),
  lambda),lambda):
  L32:=unapply((-2*(diff(tdL11(lambda), lambda))*tdL13(lambda)*
  tdL33(lambda)*tdL12(lambda)*h+2*(diff(tdL11(lambda), lambda))*
  tdL13(lambda)*tdL12(lambda)*tdL22(lambda)*h-tdL11(lambda)*(diff
  (tdL22(lambda), lambda))*tdL12(lambda)*tdL13(lambda)*h+tdL11
  (lambda)*(diff(tdL33(lambda), lambda))*tdL12(lambda)*tdL13
  (lambda)*h-2*tdL11(lambda)*tdL22(lambda)*(diff(tdL12(lambda),
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```
lambda))*tdL13(lambda)*h+2*tdL11(lambda)*tdL12(lambda)*tdL33
(lambda)*(diff(tdL13(lambda), lambda))*h-(diff(tdL22(lambda),
lambda))*tdL12(lambda)*tdL13(lambda)*tdL33(lambda)*h+(diff
(tdL23(lambda), lambda))*tdL32(lambda)*tdL12(lambda)*tdL13
(lambda)*h-(diff(tdL32(lambda), lambda))*tdL12(lambda)*tdL23
(lambda)*tdL13(lambda)*h+(diff(tdL33(lambda), lambda))*tdL22
(lambda)*tdL12(lambda)*tdL13(lambda)*h+tdL22(lambda)*tdL32
(lambda)*tdL12(lambda)*tdL23(lambda)*tdL13(lambda)-tdL22
(lambda)*tdL32(lambda)*tdL13(lambda)*(diff(tdL13(lambda),
lambda))*h+tdL22(lambda)*(diff(tdL12(lambda), lambda))*tdL12
(lambda)*tdL23(lambda)*h-2*tdL22(lambda)*(diff(tdL12(lambda),
lambda))*tdL13(lambda)*tdL33(lambda)*h+2*tdL22(lambda)*tdL12
(lambda)*tdL33(lambda)*(diff(tdL13(lambda), lambda))*h+3*tdL32
(lambda)*(diff(tdL12(lambda), lambda))*tdL23(lambda)*tdL13
(lambda)*h-tdL32(lambda)*tdL12(lambda)*tdL23(lambda)*tdL13
(lambda)*tdL33(lambda)-3*tdL32(lambda)*tdL12(lambda)*tdL23
(lambda)*(diff(tdL13(lambda), lambda))*h-tdL32(lambda)*tdL13
(lambda)*tdL33(lambda)*(diff(tdL13(lambda), lambda))*h+(diff
(tdL12(lambda), lambda))*tdL12(lambda)*tdL23(lambda)*tdL33
(lambda)*h+(diff(tdL13(lambda), lambda, lambda))*(diff(tdL12
(lambda), lambda))*h^3-(diff(tdL12(lambda), lambda, lambda))*
(diff(tdL13(lambda), lambda))*h^3-tdL21(lambda)*tdL12(lambda)
^3*tdL23(lambda)-tdL32(lambda)*tdL12(lambda)^2*tdL23(lambda)^2+
tdL32(lambda)^2*tdL23(lambda)*tdL13(lambda)^2-2*tdL32(lambda)*
(diff(tdL13(lambda), lambda))^2*h^2+tdL31(lambda)*tdL32(lambda)
*tdL13(lambda)^3+2*(diff(tdL12(lambda), lambda))^2*tdL23
(lambda)*h^2+(diff(tdL12(lambda), lambda))*tdL13(lambda)*tdL33
(lambda)^2*h+2*(diff(tdL12(lambda), lambda))*tdL33(lambda)*
(diff(tdL13(lambda), lambda))*h^2+2*(diff(tdL11(lambda),
lambda))*(diff(tdL12(lambda), lambda))*tdL13(lambda)*h^2-2*
(diff(tdL11(lambda), lambda))*tdL12(lambda)*(diff(tdL13
(lambda), lambda))*h^2+tdL21(lambda)*tdL22(lambda)*tdL12
(lambda)^2*tdL13(lambda)+tdL21(lambda)*tdL32(lambda)*tdL12
(lambda)*tdL13(lambda)^2-tdL21(lambda)*tdL12(lambda)^2*tdL13
(lambda)*tdL33(lambda)+tdL31(lambda)*tdL22(lambda)*tdL12
(lambda)*tdL13(lambda)^2-tdL31(lambda)*tdL12(lambda)^2*tdL23
(lambda)*tdL13(lambda)-tdL31(lambda)*tdL12(lambda)*tdL13
(lambda)^2*tdL33(lambda)+tdL11(lambda)*(diff(tdL23(lambda),
lambda))*tdL12(lambda)^2*h-tdL11(lambda)*(diff(tdL32(lambda),
lambda))*tdL13(lambda)^2*h-tdL11(lambda)*tdL22(lambda)^2*tdL12
(lambda)*tdL13(lambda)-tdL11(lambda)*tdL22(lambda)*tdL32
(lambda)*tdL13(lambda)^2+tdL11(lambda)*tdL22(lambda)*tdL12
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```
(lambda)^2*tdL23(lambda)-tdL11(lambda)*tdL32(lambda)*tdL13
(lambda)^2*tdL33(lambda)+tdL11(lambda)*tdL12(lambda)^2*tdL23
(lambda)*tdL33(lambda)+tdL11(lambda)*tdL12(lambda)*tdL13
(lambda)*tdL33(lambda)^2-(diff(tdL22(lambda), lambda))*tdL12
(lambda)^2*tdL23(lambda)*h-(diff(tdL22(lambda), lambda))*tdL12
(lambda)*(diff(tdL13(lambda), lambda))*h^2+(diff(tdL23(lambda),
lambda))*tdL22(lambda)*tdL12(lambda)^2*h+(diff(tdL23(lambda),
lambda))*(diff(tdL12(lambda), lambda))*tdL12(lambda)*h^2-(diff
(tdL32(lambda), lambda))*tdL13(lambda)^2*tdL33(lambda)*h-(diff
(tdL32(lambda), lambda))*tdL13(lambda)*(diff(tdL13(lambda),
lambda))*h^2+(diff(tdL33(lambda), lambda))*tdL32(lambda)*tdL13
(lambda)^2*h+(diff(tdL33(lambda), lambda))*(diff(tdL12(lambda),
lambda))*tdL13(lambda)*h^2-tdL22(lambda)^2*tdL12(lambda)*tdL13
(lambda)*tdL33(lambda)-tdL22(lambda)^2*tdL12(lambda)*(diff
(tdL13(lambda), lambda))*h-tdL22(lambda)*tdL32(lambda)*tdL13
(lambda)^2*tdL33(lambda)-2*tdL22(lambda)*(diff(tdL12(lambda),
lambda))*(diff(tdL13(lambda), lambda))*h^2+tdL22(lambda)*tdL12
(lambda)^2*tdL23(lambda)*tdL33(lambda)+tdL22(lambda)*tdL12
(lambda)*tdL13(lambda)*tdL33(lambda)^2+(diff(tdL12(lambda),
lambda))*tdL13(lambda)*tdL12(lambda)*tdL21(lambda)*h+2*(diff
(tdL12(lambda), lambda))*tdL23(lambda)*tdL12(lambda)*tdL11
(lambda)*h-(diff(tdL13(lambda), lambda))*tdL13(lambda)*tdL12
(lambda)*tdL31(lambda)*h-2*(diff(tdL13(lambda), lambda))*tdL13
(lambda)*tdL32(lambda)*tdL11(lambda)*h+(diff(tdL12(lambda),
lambda))*tdL13(lambda)^2*tdL31(lambda)*h-(diff(tdL13(lambda),
lambda))*tdL12(lambda)^2*tdL21(lambda)*h+2*(diff(tdL11(lambda),
lambda))*tdL13(lambda)^2*tdL32(lambda)*h-2*(diff(tdL11(lambda),
lambda))*tdL23(lambda)*tdL12(lambda)^2*h+(diff(tdL13(lambda),
lambda, lambda))*tdL32(lambda)*tdL13(lambda)*h^2-(diff(tdL12
(lambda), lambda, lambda))*tdL11(lambda)*tdL13(lambda)*h^2-
(diff(tdL12(lambda), lambda, lambda))*tdL12(lambda)*tdL23
(lambda)*h^2-(diff(tdL12(lambda), lambda, lambda))*tdL13
(lambda)*tdL33(lambda)*h^2+(diff(tdL13(lambda), lambda, lambda)
)*tdL11(lambda)*tdL12(lambda)*h^2+(diff(tdL13(lambda), lambda,
lambda))*tdL22(lambda)*tdL12(lambda)*h^2)/(tdL13(lambda)*tdL12
(lambda)*tdL22(lambda)+tdL13(lambda)^2*tdL32(lambda)+h*tdL13
(lambda)*(diff(tdL12(lambda), lambda))-tdL23(lambda)*tdL12
(lambda)^2-tdL13(lambda)*tdL33(lambda)*tdL12(lambda)-h*tdL12
(lambda)*(diff(tdL13(lambda), lambda))),lambda):
L33:=unapply(((diff(tdL12(lambda), lambda, lambda))*tdL13
(lambda)*h^2-(diff(tdL13(lambda), lambda, lambda))*tdL12
(lambda)*h^2+tdL13(lambda)*tdL12(lambda)*tdL22(lambda)*tdL11
```

```
(lambda)+tdL13(lambda)^2*tdL32(lambda)*tdL11(lambda)+(diff
(tdL12(lambda), lambda))*tdL13(lambda)*tdL11(lambda)*h-tdL23
(lambda)*tdL12(lambda)^2*tdL11(lambda)-tdL13(lambda)*tdL33
(lambda)*tdL12(lambda)*tdL11(lambda)-(diff(tdL13(lambda),
lambda))*tdL12(lambda)*tdL11(lambda)*h+(diff(tdL22(lambda),
lambda))*tdL13(lambda)*tdL12(lambda)*h-(diff(tdL23(lambda),
lambda))*tdL12(lambda)^2*h+(diff(tdL32(lambda), lambda))*tdL13
(lambda)^2*h-(diff(tdL33(lambda), lambda))*tdL13(lambda)*tdL12
(lambda)*h+tdL13(lambda)*tdL12(lambda)*tdL22(lambda)^2+tdL13
(lambda)^2*tdL22(lambda)*tdL32(lambda)+2*(diff(tdL12(lambda),
lambda))*tdL13(lambda)*tdL22(lambda)*h-tdL23(lambda)*tdL12
(lambda)^2*tdL22(lambda)+tdL13(lambda)^2*tdL33(lambda)*tdL32
(lambda)+2*(diff(tdL13(lambda), lambda))*tdL13(lambda)*tdL32
(lambda)*h-2*(diff(tdL12(lambda), lambda))*tdL23(lambda)*tdL12
(lambda)*h-tdL23(lambda)*tdL33(lambda)*tdL12(lambda)^2-tdL13
(lambda)*tdL33(lambda)^2*tdL12(lambda)-2*(diff(tdL13(lambda),
lambda))*tdL33(lambda)*tdL12(lambda)*h)/(tdL13(lambda)*tdL12
(lambda)*tdL22(lambda)+tdL13(lambda)^2*tdL32(lambda)+h*tdL13
(lambda)*(diff(tdL12(lambda), lambda))-tdL23(lambda)*tdL12
(lambda)^2-tdL13(lambda)*tdL33(lambda)*tdL12(lambda)-h*tdL12
(lambda)*(diff(tdL13(lambda), lambda))),lambda):
L31:=unapply(-(-(diff(tdL11(lambda), lambda))*(diff(tdL23
(lambda), lambda))*tdL12(lambda)^2*h^2+(diff(tdL11(lambda),
lambda))*(diff(tdL32(lambda), lambda))*tdL13(lambda)^2*h^2-
tdL21(lambda)*(diff(tdL23(lambda), lambda))*tdL12(lambda)^3*
h-2*tdL21(lambda)*(diff(tdL12(lambda), lambda))^2*tdL13(lambda)
*h^2-tdL21(lambda)*tdL12(lambda)^3*tdL23(lambda)*tdL33(lambda)-
tdL21(lambda)*tdL12(lambda)^2*tdL13(lambda)*tdL33(lambda)^2+
tdL31(lambda)*(diff(tdL32(lambda), lambda))*tdL13(lambda)^3*h+
tdL31(lambda)*tdL22(lambda)^2*tdL12(lambda)*tdL13(lambda)^2+
tdL31(lambda)*tdL22(lambda)*tdL32(lambda)*tdL13(lambda)^3+2*
tdL31(lambda)*tdL12(lambda)*(diff(tdL13(lambda), lambda))^2*
h^2+tdL11(lambda)*tdL32(lambda)^2*tdL23(lambda)*tdL13(lambda)^2
-tdL11(lambda)*tdL32(lambda)*tdL12(lambda)^2*tdL23(lambda)^2-2*
tdL11(lambda)*tdL32(lambda)*(diff(tdL13(lambda), lambda))^2*
h^2+2*tdL11(lambda)*(diff(tdL12(lambda), lambda))^2*tdL23
(lambda)*h^2+(diff(tdL21(lambda), lambda))*tdL12(lambda)^3*
tdL23(lambda)*h+(diff(tdL21(lambda), lambda))*tdL12(lambda)^2*
(diff(tdL13(lambda), lambda))*h^2-(diff(tdL31(lambda), lambda))
*tdL32(lambda)*tdL13(lambda)^3*h-(diff(tdL31(lambda), lambda))*
(diff(tdL12(lambda), lambda))*tdL13(lambda)^2*h^2-(diff(tdL11
(lambda), lambda, lambda))*tdL32(lambda)*tdL13(lambda)^2*h^2+
```

```
(diff(tdL13(lambda), lambda, lambda))*tdL11(lambda)*(diff(tdL12
(lambda), lambda))*h^3+tdL31(lambda)*(diff(tdL12(lambda),
lambda, lambda))*tdL13(lambda)^2*h^2-(diff(tdL12(lambda),
lambda, lambda))*tdL11(lambda)*(diff(tdL13(lambda), lambda))*
h^3-tdL21(lambda)*(diff(tdL13(lambda), lambda, lambda))*tdL12
(lambda)^2*h^2-(diff(tdL11(lambda), lambda))*(diff(tdL13
(lambda), lambda, lambda))*tdL12(lambda)*h^3+(diff(tdL11
(lambda), lambda))*(diff(tdL12(lambda), lambda, lambda))*tdL13
(lambda)*h^3+tdL21(lambda)*(diff(tdL12(lambda), lambda, lambda)
)*tdL12(lambda)*tdL13(lambda)*h^2-tdL31(lambda)*(diff(tdL13
(lambda), lambda, lambda))*tdL12(lambda)*tdL13(lambda)*h^2+
(diff(tdL11(lambda), lambda, lambda))*tdL12(lambda)*(diff(tdL13
(lambda), lambda))*h^3-(diff(tdL11(lambda), lambda, lambda))*
(diff(tdL12(lambda), lambda))*tdL13(lambda)*h^3+(diff(tdL11
(lambda), lambda, lambda))*tdL12(lambda)^2*tdL23(lambda)*h^2+
tdL31(lambda)*tdL12(lambda)^3*tdL23(lambda)^2-tdL21(lambda)*
tdL32(lambda)^2*tdL13(lambda)^3-tdL21(lambda)*tdL22(lambda)*
(diff(tdL12(lambda), lambda))*tdL12(lambda)*tdL13(lambda)*h+3*
tdL21(lambda)*tdL32(lambda)*tdL12(lambda)*tdL13(lambda)*(diff
(tdL13(lambda), lambda))*h+2*tdL21(lambda)*(diff(tdL12(lambda),
lambda))*tdL12(lambda)*tdL13(lambda)*tdL33(lambda)*h-2*tdL31
(lambda)*tdL22(lambda)*tdL12(lambda)*tdL13(lambda)*(diff(tdL13
(lambda), lambda))*h-3*tdL31(lambda)*(diff(tdL12(lambda),
lambda))*tdL12(lambda)*tdL23(lambda)*tdL13(lambda)*h+tdL31
(lambda)*tdL12(lambda)*tdL13(lambda)*tdL33(lambda)*(diff(tdL13
(lambda), lambda))*h-tdL11(lambda)*(diff(tdL22(lambda), lambda)
)*tdL12(lambda)*tdL13(lambda)*tdL33(lambda)*h+tdL11(lambda)*
(diff(tdL23(lambda), lambda))*tdL32(lambda)*tdL12(lambda)*tdL13
(lambda)*h-tdL11(lambda)*(diff(tdL32(lambda), lambda))*tdL12
(lambda)*tdL23(lambda)*tdL13(lambda)*h+tdL11(lambda)*(diff
(tdL33(lambda), lambda))*tdL22(lambda)*tdL12(lambda)*tdL13
(lambda)*h+tdL11(lambda)*tdL22(lambda)*tdL32(lambda)*tdL12
(lambda)*tdL23(lambda)*tdL13(lambda)-tdL11(lambda)*tdL22
(lambda)*tdL32(lambda)*tdL13(lambda)*(diff(tdL13(lambda),
lambda))*h+tdL11(lambda)*tdL22(lambda)*(diff(tdL12(lambda),
lambda))*tdL12(lambda)*tdL23(lambda)*h-2*tdL11(lambda)*tdL22
(lambda)*(diff(tdL12(lambda), lambda))*tdL13(lambda)*tdL33
(lambda)*h+2*tdL11(lambda)*tdL22(lambda)*tdL12(lambda)*tdL33
(lambda)*(diff(tdL13(lambda), lambda))*h+3*tdL11(lambda)*tdL32
(lambda)*(diff(tdL12(lambda), lambda))*tdL23(lambda)*tdL13
(lambda)*h-tdL11(lambda)*tdL32(lambda)*tdL12(lambda)*tdL23
(lambda)*tdL13(lambda)*tdL33(lambda)-3*tdL11(lambda)*tdL32
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```
(lambda)*tdL12(lambda)*tdL23(lambda)*(diff(tdL13(lambda),
lambda))*h-tdL11(lambda)*tdL32(lambda)*tdL13(lambda)*tdL33
(lambda)*(diff(tdL13(lambda), lambda))*h+tdL11(lambda)*(diff
(tdL12(lambda), lambda))*tdL12(lambda)*tdL23(lambda)*tdL33
(lambda)*h+(diff(tdL13(lambda), lambda, lambda))*tdL11(lambda)*
tdL32(lambda)*tdL13(lambda)*h^2-(diff(tdL11(lambda), lambda,
lambda))*tdL22(lambda)*tdL12(lambda)*tdL13(lambda)*h^2+(diff
(tdL11(lambda), lambda))*(diff(tdL22(lambda), lambda))*tdL12
(lambda)*tdL13(lambda)*h^2-(diff(tdL11(lambda), lambda))*(diff
(tdL33(lambda), lambda))*tdL12(lambda)*tdL13(lambda)*h^2+(diff
(tdL11(lambda), lambda))*tdL22(lambda)^2*tdL12(lambda)*tdL13
(lambda)*h+(diff(tdL11(lambda), lambda))*tdL22(lambda)*tdL32
(lambda)*tdL13(lambda)^2*h+2*(diff(tdL11(lambda), lambda))*
tdL22(lambda)*(diff(tdL12(lambda), lambda))*tdL13(lambda)*h^2-
(diff(tdL11(lambda), lambda))*tdL22(lambda)*tdL12(lambda)^2*
tdL23(lambda)*h+(diff(tdL11(lambda), lambda))*tdL32(lambda)*
tdL13(lambda)^2*tdL33(lambda)*h+2*(diff(tdL11(lambda), lambda))
*tdL32(lambda)*tdL13(lambda)*(diff(tdL13(lambda), lambda))*h^2
-2*(diff(tdL11(lambda), lambda))*(diff(tdL12(lambda), lambda))*
tdL12(lambda)*tdL23(lambda)*h^2-(diff(tdL11(lambda), lambda))*
tdL12(lambda)^2*tdL23(lambda)*tdL33(lambda)*h-(diff(tdL11
(lambda), lambda))*tdL12(lambda)*tdL13(lambda)*tdL33(lambda)^2*
h-2*(diff(tdL11(lambda), lambda))*tdL12(lambda)*tdL33(lambda)*
(diff(tdL13(lambda), lambda))*h^2+tdL21(lambda)*(diff(tdL22
(lambda), lambda))*tdL12(lambda)^2*tdL13(lambda)*h+tdL21
(lambda)*(diff(tdL32(lambda), lambda))*tdL12(lambda)*tdL13
(lambda)^2*h-tdL21(lambda)*(diff(tdL33(lambda), lambda))*tdL12
(lambda)^2*tdL13(lambda)*h-tdL21(lambda)*tdL22(lambda)*tdL32
(lambda)*tdL12(lambda)*tdL13(lambda)^2+tdL21(lambda)*tdL22
(lambda)*tdL12(lambda)^2*tdL13(lambda)*tdL33(lambda)+tdL21
(lambda)*tdL22(lambda)*tdL12(lambda)^2*(diff(tdL13(lambda),
lambda))*h-3*tdL21(lambda)*tdL32(lambda)*(diff(tdL12(lambda),
lambda))*tdL13(lambda)^2*h+tdL21(lambda)*tdL32(lambda)*tdL12
(lambda)^2*tdL23(lambda)*tdL13(lambda)+2*tdL21(lambda)*tdL32
(lambda)*tdL12(lambda)*tdL13(lambda)^2*tdL33(lambda)+2*tdL21
(lambda)*(diff(tdL12(lambda), lambda))*tdL12(lambda)*(diff
(tdL13(lambda), lambda))*h^2-2*tdL21(lambda)*tdL12(lambda)^2*
tdL33(lambda)*(diff(tdL13(lambda), lambda))*h+tdL31(lambda)*
(diff(tdL22(lambda), lambda))*tdL12(lambda)*tdL13(lambda)^2*h-
tdL31(lambda)*(diff(tdL23(lambda), lambda))*tdL12(lambda)^2*
tdL13(lambda)*h-tdL31(lambda)*(diff(tdL33(lambda), lambda))*
tdL12(lambda)*tdL13(lambda)^2*h+2*tdL31(lambda)*tdL22(lambda)*
```

```
(diff(tdL12(lambda), lambda))*tdL13(lambda)^2*h-2*tdL31(lambda)
*tdL22(lambda)*tdL12(lambda)^2*tdL23(lambda)*tdL13(lambda)-
tdL31(lambda)*tdL22(lambda)*tdL12(lambda)*tdL13(lambda)^2*tdL33
(lambda)-tdL31(lambda)*tdL32(lambda)*tdL12(lambda)*tdL23
(lambda)*tdL13(lambda)^2-tdL31(lambda)*(diff(tdL12(lambda),
lambda))*tdL13(lambda)^2*tdL33(lambda)*h-2*tdL31(lambda)*(diff
(tdL12(lambda), lambda))*tdL13(lambda)*(diff(tdL13(lambda),
lambda))*h^2+tdL31(lambda)*tdL12(lambda)^2*tdL23(lambda)*tdL13
(lambda)*tdL33(lambda)+3*tdL31(lambda)*tdL12(lambda)^2*tdL23
(lambda)*(diff(tdL13(lambda), lambda))*h-tdL11(lambda)*(diff
(tdL22(lambda), lambda))*tdL12(lambda)^2*tdL23(lambda)*h-tdL11
(lambda)*(diff(tdL22(lambda), lambda))*tdL12(lambda)*(diff
(tdL13(lambda), lambda))*h^2+tdL11(lambda)*(diff(tdL23(lambda),
lambda))*tdL22(lambda)*tdL12(lambda)^2*h+tdL11(lambda)*(diff
(tdL23(lambda), lambda))*(diff(tdL12(lambda), lambda))*tdL12
(lambda)*h^2-tdL11(lambda)*(diff(tdL32(lambda), lambda))*tdL13
(lambda)^2*tdL33(lambda)*h-tdL11(lambda)*(diff(tdL32(lambda),
lambda))*tdL13(lambda)*(diff(tdL13(lambda), lambda))*h^2+tdL11
(lambda)*(diff(tdL33(lambda), lambda))*tdL32(lambda)*tdL13
(lambda)^2*h+tdL11(lambda)*(diff(tdL33(lambda), lambda))*(diff
(tdL12(lambda), lambda))*tdL13(lambda)*h^2-tdL11(lambda)*tdL22
(lambda)^2*tdL12(lambda)*tdL13(lambda)*tdL33(lambda)-tdL11
(lambda)*tdL22(lambda)^2*tdL12(lambda)*(diff(tdL13(lambda),
lambda))*h-tdL11(lambda)*tdL22(lambda)*tdL32(lambda)*tdL13
(lambda)^2*tdL33(lambda)-2*tdL11(lambda)*tdL22(lambda)*(diff
(tdL12(lambda), lambda))*(diff(tdL13(lambda), lambda))*h^2+
tdL11(lambda)*tdL22(lambda)*tdL12(lambda)^2*tdL23(lambda)*tdL33
(lambda)+tdL11(lambda)*tdL22(lambda)*tdL12(lambda)*tdL13
(lambda)*tdL33(lambda)^2+tdL11(lambda)*(diff(tdL12(lambda),
lambda))*tdL13(lambda)*tdL33(lambda)^2*h+2*tdL11(lambda)*(diff
(tdL12(lambda), lambda))*tdL33(lambda)*(diff(tdL13(lambda),
lambda))*h^2-(diff(tdL21(lambda), lambda))*tdL22(lambda)*tdL12
(lambda)^2*tdL13(lambda)*h-(diff(tdL21(lambda), lambda))*tdL32
(lambda)*tdL12(lambda)*tdL13(lambda)^2*h-(diff(tdL21(lambda),
lambda))*(diff(tdL12(lambda), lambda))*tdL12(lambda)*tdL13
(lambda)*h^2+(diff(tdL21(lambda), lambda))*tdL12(lambda)^2*
tdL13(lambda)*tdL33(lambda)*h-(diff(tdL31(lambda), lambda))*
tdL22(lambda)*tdL12(lambda)*tdL13(lambda)^2*h+(diff(tdL31
(lambda), lambda))*tdL12(lambda)^2*tdL23(lambda)*tdL13(lambda)*
h+(diff(tdL31(lambda), lambda))*tdL12(lambda)*tdL13(lambda)^2*
tdL33(lambda)*h+(diff(tdL31(lambda), lambda))*tdL12(lambda)*
tdL13(lambda)*(diff(tdL13(lambda), lambda))*h^2+(diff(tdL11
```

```
(lambda), lambda, lambda))*tdL12(lambda)*tdL13(lambda)*tdL33
(lambda)*h^2-(diff(tdL12(lambda), lambda, lambda))*tdL11
(lambda)*tdL12(lambda)*tdL23(lambda)*h^2-(diff(tdL12(lambda),
lambda, lambda))*tdL11(lambda)*tdL13(lambda)*tdL33(lambda)*h^2+
(diff(tdL13(lambda), lambda, lambda))*tdL11(lambda)*tdL22
(lambda)*tdL12(lambda)*h^2)/(tdL13(lambda)*tdL12(lambda)*tdL22
(lambda)+tdL13(lambda)^2*tdL32(lambda)+h*tdL13(lambda)*(diff
(tdL12(lambda), lambda))-tdL23(lambda)*tdL12(lambda)^2-tdL13
(lambda)*tdL33(lambda)*tdL12(lambda)-h*tdL12(lambda)*(diff
(tdL13(lambda), lambda))),lambda):
```

```
> simplify(ToCancel);
  simplify(G);
  factor(Determinant(G));
```

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(1.4)

$$\left[\left[1, 0, 0 \right], \right.$$

$$\left[tdL11(\lambda), tdL12(\lambda), tdL13(\lambda) \right],$$

$$\left[tdL12(\lambda)\, tdL21(\lambda) + tdL13(\lambda)\, tdL31(\lambda) + tdL11(\lambda)^2 + h\left(\frac{d}{d\lambda}\, tdL11(\lambda) \right), \right.$$

$$h\left(\frac{d}{d\lambda}\, tdL12(\lambda) \right) + (tdL11(\lambda) + tdL22(\lambda))\, tdL12(\lambda) + tdL13(\lambda)\, tdL32(\lambda),$$

$$\left. h\left(\frac{d}{d\lambda}\, tdL13(\lambda) \right) + (tdL11(\lambda) + tdL33(\lambda))\, tdL13(\lambda) + tdL12(\lambda)\, tdL23(\lambda) \right] \right]$$

$$-tdL13(\lambda)\, tdL12(\lambda)\, tdL22(\lambda) - tdL13(\lambda)^2\, tdL32(\lambda) - h\, tdL13(\lambda) \left(\frac{d}{d\lambda}\, tdL12(\lambda) \right)$$

$$+ tdL23(\lambda)\, tdL12(\lambda)^2 + tdL13(\lambda)\, tdL33(\lambda)\, tdL12(\lambda)$$

$$+ h\, tdL12(\lambda) \left(\frac{d}{d\lambda}\, tdL13(\lambda) \right)$$

Application to the case of \td{L}=L_1\lambda+L_0. We take L_1=diag(t12,t22, t32) but L_0 remains free. In particular, we keep a_{1,2}, a_{1,3} as free parameters that shall not be determined since they belongs to the action by global conjugation of GL_3.

```
> tdL11:=unapply(t12*lambda+a11,lambda):
  tdL12:=unapply(a12,lambda):
  tdL13:=unapply(a13,lambda):
  tdL21:=unapply(a21,lambda):
  tdL22:=unapply(t22*lambda+a22,lambda):
```

```
tdL23:=unapply(a23,lambda):
tdL31:=unapply(a31,lambda):
tdL32:=unapply(a32,lambda):
tdL33:=unapply(t32*lambda+a33,lambda):
simplify(tdL);
simplify(series(factor(Determinant(G)),lambda=0));
simplify(G);

tdLinfty1:=Matrix(3,3,0):
tdLinfty1[1,1]:=t12:
tdLinfty1[2,2]:=t22:
tdLinfty1[3,3]:=t32:

tdLinfty0:=Matrix(3,3,0):
tdLinfty0[1,1]:=a11:
tdLinfty0[1,2]:=a12:
tdLinfty0[1,3]:=a13:
tdLinfty0[2,1]:=a21:
tdLinfty0[2,2]:=a22:
tdLinfty0[2,3]:=a23:
tdLinfty0[3,1]:=a31:
tdLinfty0[3,2]:=a32:
tdLinfty0[3,3]:=a33:

simplify(tdL);
Trace(tdL);
```

$$
\begin{bmatrix}
\lambda\, t12 + a11 & a12 & a13 \\
a21 & \lambda\, t22 + a22 & a23 \\
a31 & a32 & \lambda\, t32 + a33
\end{bmatrix}
\tag{1.1.1}
$$

$$
a12^2\, a23 - a13\,(a22 - a33)\, a12 - a13^2\, a32 - a12\, a13\,(t22 - t32)\,\lambda
$$

$$
\Big[\big[\,1,\, 0,\, 0\,\big],
$$
$$
\big[\,\lambda\, t12 + a11,\, a12,\, a13\,\big],
$$
$$
\big[\,\lambda^2\, t12^2 + (2\, a11\, \lambda + h)\, t12 + a11^2 + a12\, a21 + a13\, a31,\, \big((t12 + t22)\,\lambda + a11
$$
$$
+ a22\big)\, a12 + a13\, a32,\, \big((t12 + t32)\,\lambda + a11 + a33\big)\, a13 + a12\, a23\,\big]\Big]
$$

$$
\begin{bmatrix}
\lambda\, t12 + a11 & a12 & a13 \\
a21 & \lambda\, t22 + a22 & a23 \\
a31 & a32 & \lambda\, t32 + a33
\end{bmatrix}
$$

$$\lambda\,t12 + \lambda\,t22 + \lambda\,t32 + a11 + a22 + a33$$

Definition of the apparent singularity q

```
> Qsing:=(a12^2*a23-a12*a13*a22+a12*a13*a33-a13^2*a32)/(a12*
  a13*(t22-t32));
```

$$Qsing := \frac{a12^2\,a23 - a12\,a13\,a22 + a12\,a13\,a33 - a13^2\,a32}{a12\,a13\,(t22 - t32)} \qquad \textbf{(1.1.2)}$$

```
> R31:=factor(residue(L31(lambda),lambda=Qsing)):
  R32:=factor(residue(L32(lambda),lambda=Qsing)):
  R33:=factor(residue(L33(lambda),lambda=Qsing));
```

$$R33 := h \qquad \textbf{(1.1.3)}$$

```
> L31bis:=(a11*a12*a13*a22*a33*t22-a11*a12*a13*a22*a33*t32-a11*
  a12*a13*a23*a32*t22+a11*a12*a13*a23*a32*t32-a12^2*a13*a21*
  a33*t22+a12^2*a13*a21*a33*t32+a12^2*a13*a23*a31*t22-a12^2*
  a13*a23*a31*t32+a12*a13^2*a21*a32*t22-a12*a13^2*a21*a32*t32-
  a12*a13^2*a22*a31*t22+a12*a13^2*a22*a31*t32+h*a12^2*a23*t12*
  t22-h*a12*a13*a22*t12*t22+h*a12*a13*a33*t12*t32-h*a13^2*a32*
  t12*t32)/(a12*a13*(t22-t32))+(a11*a22*t32+a11*a33*t22-a12*
  a21*t32-a13*a31*t22+a22*a33*t12-a23*a32*t12-h*t12*t22-h*t12*
  t32)*lambda+(a11*t22*t32+a22*t12*t32+a33*t12*t22)*lambda^2+
  t12*t22*t32*lambda^3+R31/(lambda-Qsing);
  L32bis:=-a11*a22-a11*a33+a12*a21+a13*a31-a22*a33+a23*a32+h*
  t12+(-a11*t22-a11*t32-a22*t12-a22*t32-a33*t12-a33*t22)*
  lambda+(-t12*t22-t12*t32-t22*t32)*lambda^2+R32/
  (lambda-Qsing);
  L33bis:=(t12+t22+t32)*lambda+a11+a22+a33+ h/(lambda-Qsing);

  simplify(L31(lambda)-L31bis);
  simplify(L32(lambda)-L32bis);
  simplify(L33(lambda)-L33bis);

  L:=simplify(L):

  L31:=unapply(L31bis,lambda):
  L32:=unapply(L32bis,lambda):
  L33:=unapply(L33bis,lambda):
```

$$
\begin{aligned}
L31bis := {} & \frac{1}{a12\,a13\,(t22 - t32)}\big(a11\,a12\,a13\,a22\,a33\,t22 - a11\,a12\,a13\,a22\,a33\,t32 \\
& - a11\,a12\,a13\,a23\,a32\,t22 + a11\,a12\,a13\,a23\,a32\,t32 - a12^2\,a13\,a21\,a33\,t22 \\
& + a12^2\,a13\,a21\,a33\,t32 + a12^2\,a13\,a23\,a31\,t22 - a12^2\,a13\,a23\,a31\,t32 \\
& + a12^2\,a23\,h\,t12\,t22 + a12\,a13^2\,a21\,a32\,t22 - a12\,a13^2\,a21\,a32\,t32 \\
& - a12\,a13^2\,a22\,a31\,t22 + a12\,a13^2\,a22\,a31\,t32 - a12\,a13\,a22\,h\,t12\,t22 \\
& + a12\,a13\,a33\,h\,t12\,t32 - a13^2\,a32\,h\,t12\,t32\big) + \big(a11\,a22\,t32 + a11\,a33\,t22 \\
& - a12\,a21\,t32 - a13\,a31\,t22 + a22\,a33\,t12 - a23\,a32\,t12 - h\,t12\,t22 - h\,t12\,t32\big)
\end{aligned}
$$

$$\lambda + (a11\ t22\ t32 + a22\ t12\ t32 + a33\ t12\ t22)\ \lambda^2 + t12\ t22\ t32\ \lambda^3$$
$$+ \big((a11\ a12^3\ a13\ a23\ t22^2 - a11\ a12^3\ a13\ a23\ t22\ t32 - a11\ a12^2\ a13^2\ a22\ t22\ t32 + a11\ a12^2\ a$$
$$+ a12^4\ a23^2\ t12\ t22 - a12^3\ a13^2\ a21\ t22^2 + 2\ a12^3\ a13^2\ a21\ t22\ t32$$
$$- a12^3\ a13^2\ a21\ t32^2 - a12^3\ a13\ a22\ a23\ t12\ t22 - a12^3\ a13\ a22\ a23\ t12\ t32$$
$$+ 2\ a12^3\ a13\ a23\ a33\ t12\ t22 - a12^2\ a13^3\ a31\ t22^2 + 2\ a12^2\ a13^3\ a31\ t22\ t32$$
$$- a12^2\ a13^3\ a31\ t32^2 + a12^2\ a13^2\ a22^2\ t12\ t32 - a12^2\ a13^2\ a22\ a33\ t12\ t22$$
$$- a12^2\ a13^2\ a22\ a33\ t12\ t32 - a12^2\ a13^2\ a23\ a32\ t12\ t22$$
$$- a12^2\ a13^2\ a23\ a32\ t12\ t32 + a12^2\ a13^2\ a33^2\ t12\ t22 - a12^2\ a13^2\ h\ t12\ t22^2$$
$$+ 2\ a12^2\ a13^2\ h\ t12\ t22\ t32 - a12^2\ a13^2\ h\ t12\ t32^2 + 2\ a12\ a13^3\ a22\ a32\ t12\ t32$$
$$- a12\ a13^3\ a32\ a33\ t12\ t22 - a12\ a13^3\ a32\ a33\ t12\ t32 + a13^4\ a32^2\ t12\ t32\big)\ h\Big) \Big/$$
$$\left(a12^2\ a13^2\ (t22 - t32)^2 \left(\lambda\right.\right.$$
$$\left.\left. - \frac{a12^2\ a23 - a12\ a13\ a22 + a12\ a13\ a33 - a13^2\ a32}{a12\ a13\ (t22 - t32)}\right)\right)$$

$L32bis := -a11\ a22 - a11\ a33 + a12\ a21 + a13\ a31 - a22\ a33 + a23\ a32 + h\ t12 + ($
$\quad -a11\ t22 - a11\ t32 - a22\ t12 - a22\ t32 - a33\ t12 - a33\ t22)\ \lambda + (-t12\ t22$
$\quad - t12\ t32 - t22\ t32)\ \lambda^2 - \big((a11\ a12\ a13\ t22 - a11\ a12\ a13\ t32 + a12^2\ a23\ t12$
$\quad + a12^2\ a23\ t22 - a12\ a13\ a22\ t12 - a12\ a13\ a22\ t32 + a12\ a13\ a33\ t12$
$\quad + a12\ a13\ a33\ t22 - a13^2\ a32\ t12 - a13^2\ a32\ t32)\ h\big) \Big/ \left(a12\ a13\ (t22 - t32)\left(\lambda\right.\right.$
$\quad \left.\left. - \frac{a12^2\ a23 - a12\ a13\ a22 + a12\ a13\ a33 - a13^2\ a32}{a12\ a13\ (t22 - t32)}\right)\right)$

$L33bis := (t12 + t22 + t32)\ \lambda + a11 + a22 + a33$
$\quad + \dfrac{h}{\lambda - \dfrac{a12^2\ a23 - a12\ a13\ a22 + a12\ a13\ a33 - a13^2\ a32}{a12\ a13\ (t22 - t32)}}$

$$0$$
$$0$$
$$0$$

▼ Verification of the gauge formula with the formula

```
> J33:=unapply(j33/(lambda-Qsing),lambda):
  J32:=unapply(j32/(lambda-Qsing),lambda):
  J31:=unapply(j31/(lambda-Qsing),lambda):
  j33:=1:

  J:=Matrix(3,3,0):
  J[1,1]:=1:
  J[2,2]:=1:
  J[3,1]:=J31(lambda):
  J[3,2]:=J32(lambda):
  J[3,3]:=J33(lambda):
```

```
  J:

  Jprime:=Matrix(3,3,0):
  for i from 1 to 3 do for j from 1 to 3 do Jprime[i,j]:=diff(J
  [i,j],lambda): od: od:
  Jprime:

  checkL:=simplify(Multiply(Multiply(J,L),J^(-1))+h*Multiply
  (Jprime,J^(-1))):
> j31 :=factor( (a11*a12^3*a13*a23*t22^2-a11*a12^3*a13*a23*t22*
  t32-a11*a12^2*a13^2*a22*t22*t32+a11*a12^2*a13^2*a22*t32^2+a11*
  a12^2*a13^2*a33*t22^2-a11*a12^2*a13^2*a33*t22*t32-a11*a12*
  a13^3*a32*t22*t32+a11*a12*a13^3*a32*t32^2+a12^4*a23^2*t12*t22-
  a12^3*a13^2*a21*t22^2+2*a12^3*a13^2*a21*t22*t32-a12^3*a13^2*
  a21*t32^2-a12^3*a13*a22*a23*t12*t22-a12^3*a13*a22*a23*t12*
  t32+2*a12^3*a13*a23*a33*t12*t22-a12^2*a13^3*a31*t22^2+2*a12^2*
  a13^3*a31*t22*t32-a12^2*a13^3*a31*t32^2+a12^2*a13^2*a22^2*t12*
  t32-a12^2*a13^2*a22*a33*t12*t22-a12^2*a13^2*a22*a33*t12*t32-
  a12^2*a13^2*a23*a32*t12*t22-a12^2*a13^2*a23*a32*t12*t32+a12^2*
  a13^2*a33^2*t12*t22+2*a12*a13^3*a22*a32*t12*t32-a12*a13^3*a32*
  a33*t12*t22-a12*a13^3*a32*a33*t12*t32+a13^4*a32^2*t12*t32)/
  (a12^2*a13^2*(t22-t32)^2))-h*t12;
  j32 :=factor( -(a11*a12*a13*t22-a11*a12*a13*t32+a12^2*a23*t12+
  a12^2*a23*t22-a12*a13*a22*t12-a12*a13*a22*t32+a12*a13*a33*t12+
  a12*a13*a33*t22-a13^2*a32*t12-a13^2*a32*t32)/(a12*a13*(t22-t32)
  ));

  factor(simplify(residue(checkL[3,1],lambda=Q)));
  factor(simplify(residue(checkL[3,2],lambda=Q)));
  factor(simplify(residue(checkL[3,3],lambda=Q)));

  J:=simplify(J):
  checkL:=simplify(checkL):
```

$$j31 := \frac{1}{a12^2\,a13^2\,(t22-t32)^2}\left(a11\,a12^3\,a13\,a23\,t22^2 - a11\,a12^3\,a13\,a23\,t22\,t32\right. \tag{2.1}$$
$$-a11\,a12^2\,a13^2\,a22\,t22\,t32 + a11\,a12^2\,a13^2\,a22\,t32^2 + a11\,a12^2\,a13^2\,a33\,t22^2$$
$$-a11\,a12^2\,a13^2\,a33\,t22\,t32 - a11\,a12\,a13^3\,a32\,t22\,t32 + a11\,a12\,a13^3\,a32\,t32^2$$
$$+a12^4\,a23^2\,t12\,t22 - a12^3\,a13^2\,a21\,t22^2 + 2\,a12^3\,a13^2\,a21\,t22\,t32$$
$$-a12^3\,a13^2\,a21\,t32^2 - a12^3\,a13\,a22\,a23\,t12\,t22 - a12^3\,a13\,a22\,a23\,t12\,t32$$
$$+2\,a12^3\,a13\,a23\,a33\,t12\,t22 - a12^2\,a13^3\,a31\,t22^2 + 2\,a12^2\,a13^3\,a31\,t22\,t32$$
$$-a12^2\,a13^3\,a31\,t32^2 + a12^2\,a13^2\,a22^2\,t12\,t32 - a12^2\,a13^2\,a22\,a33\,t12\,t22$$
$$-a12^2\,a13^2\,a22\,a33\,t12\,t32 - a12^2\,a13^2\,a23\,a32\,t12\,t22 - a12^2\,a13^2\,a23\,a32\,t12\,t32$$

$$+ a12^2\, a13^2\, a33^2\, t12\, t22 + 2\, a12\, a13^3\, a22\, a32\, t12\, t32 - a12\, a13^3\, a32\, a33\, t12\, t22$$
$$- a12\, a13^3\, a32\, a33\, t12\, t32 + a13^4\, a32^2\, t12\, t32) - h\, t12$$

$$j32 := -\frac{1}{a12\, a13\, (t22 - t32)} \big(a11\, a12\, a13\, t22 - a11\, a12\, a13\, t32 + a12^2\, a23\, t12$$
$$+ a12^2\, a23\, t22 - a12\, a13\, a22\, t12 - a12\, a13\, a22\, t32 + a12\, a13\, a33\, t12$$
$$+ a12\, a13\, a33\, t22 - a13^2\, a32\, t12 - a13^2\, a32\, t32\big)$$

$$0$$
$$0$$
$$0$$

> ```
series(simplify(series(factor(series(checkL[2,1]-h*t12,lambda=
infinity)),lambda=infinity)),lambda=infinity);
series(simplify(series(factor(series(checkL[2,2],lambda=
infinity)),lambda=infinity)),lambda=infinity);
series(simplify(series(factor(series(checkL[2,3],lambda=
infinity)),lambda=infinity)),lambda=infinity);

series(simplify(series(factor(series(checkL[3,1]-h*t12^2,
lambda=infinity)),lambda=infinity)),lambda=infinity);
series(simplify(series(factor(series(checkL[3,2],lambda=
infinity)),lambda=infinity)),lambda=infinity);
series(simplify(series(factor(series(checkL[3,3],lambda=
infinity)),lambda=infinity)),lambda=infinity);
```

$$\frac{1}{a12^2\, a13^2\, (t22 - t32)^2} \Big( -a12^4\, a23^2\, t12\, t22 - \big( -a21\, (t22 - t32)^2\, a13 + a23\, (t22^2\, a11 \tag{2.2}$$
$$+ (-t32\, a11 - t12\, (-2\, a33 + a22))\, t22 - t12\, t32\, a22)\big)\, a13\, a12^3 - \big( -a31\, (t22$$
$$- t32)^2\, a13 + a11\, t22^2\, a33 + \big( -a11\, (a33 + a22)\, t32 - t12\, (a22\, a33 + a23\, a32$$
$$- a33^2)\big)\, t22 + \big(t32\, a11\, a22 + t12\, (a22^2 - a22\, a33 - a23\, a32)\big)\, t32\big)\, a13^2\, a12^2$$
$$+ a32\, a13^3 \left( (a11\, t32 + a33\, t12)\, t22 - \left( t32\, a11 + 2\, \left( a22 - \frac{1}{2}\, a33 \right)\, t12 \right)\, t32 \right)\, a12$$
$$- a13^4\, a32^2\, t12\, t32 \Big)$$

$$\frac{1}{a12\, a13\, (-t22 + t32)} \big( -a23\, (t12 + t22)\, a12^2 + (t12\, (a22 - a33) + (-a11 - a33)\, t22$$
$$+ t32\, (a11 + a22))\, a13\, a12 + a13^2\, a32\, (t12 + t32) \big)$$

$$\lambda + \frac{-a12^2\, a23 + a13\, (a22 - a33)\, a12 + a13^2\, a32}{a12\, a13\, (t22 - t32)}$$

$$-\frac{(-t22^3\, t32 + 2\, t22^2\, t32^2 - t22\, t32^3)\, t12\, \lambda^2}{(t22 - t32)^2} + \frac{1}{a12^2\, a13^2\, (t22 - t32)^2} \big( (-a13\, (-t22\, t32$$
$$+ t32^2)\, t22\, t12\, a23\, a12^3 - \big( (a22\, t22\, t32^2 - a22\, t32^3 - a33\, t22^3 + a33\, t22^2\, t32)\, t12$$
$$- t22\, t32\, a11\, (t22 - t32)^2 \big)\, a13^2\, a12^2 + a32\, a13^3\, t32\, (-t22^2 + t22\, t32)\, t12\, a12 \big)\, \lambda \big)$$

$$+ \frac{1}{a12^2\, a13^2\, (t22 - t32)^2} \Big( -t12\, t22\, a23^2\, (t12 + t22)\, a12^4 - \big( -a21\, (t22$$

$$- t32)^2\, (t12 + t22)\, a13 + a23\, \big( ((-a22 + 2\, a33)\, t22 - a22\, t32)\, t12^2 + t22\, ((a11$$

$$- a22 + a33)\, t22 - t32\, (a11 + a22 - a33))\, t12 + t22^2\, a11\, (t22 - t32) \big) \big)\, a13\, a12^3$$

$$- \big( -a31\, (t22 - t32)^2\, (t12 + t32)\, a13 + \big( (-a22\, a33 - a23\, a32 + a33^2)\, t22$$

$$+ t32\, (a22^2 - a22\, a33 - a23\, a32) \big)\, t12^2 + (a33\, (a11 - a22)\, t22^2 - (a11\, (a33$$

$$+ a22) - a22^2 + 2\, a23\, a32 - a33^2)\, t32\, t22 + a22\, t32^2\, (a11 - a33))\, t12$$

$$- t22\, t32\, a11\, (t22 - t32)\, (a22 - a33) \big)\, a13^2\, a12^2 + a32\, \Big( \Big( a33\, t22 - 2\, \Big( a22$$

$$- \frac{1}{2}\, a33 \Big)\, t32 \Big)\, t12^2 + ((a11 - a22 + a33)\, t22 - t32\, (a11 + a22 - a33))\, t32\, t12$$

$$+ t32^2\, a11\, (t22 - t32) \Big)\, a13^3\, a12 - t12\, t32\, a13^4\, a32^2\, (t12 + t32) \Big)$$

$$\frac{((t22^2 - t32^2)\, t12 - t32\, t22\, (-t22 + t32))\, \lambda}{-t22 + t32} + \frac{1}{a12\, a13\, (-t22 + t32)} \big( -a23\, (t12^2$$

$$+ t12\, t22 + t22^2)\, a12^2 + ((a22 - a33)\, t12^2 + ((-a11 + a22)\, t22 + t32\, (a11$$

$$- a33))\, t12 - t32\, (a33 - a22)\, t22)\, a13\, a12 + a13^2\, a32\, (t12^2 + t12\, t32 + t32^2) \big)$$

$$\frac{((t22 - t32)\, t12 + t22^2 - t32^2)\, \lambda}{t22 - t32} + \frac{1}{a12\, a13\, (t22 - t32)} \big( -a23\, (t12 + t22)\, a12^2$$

$$+ a13\, (t12\, (a22 - a33) + a22\, t22 - a33\, t32)\, a12 + a13^2\, a32\, (t12 + t32) \big)$$

Computation of the additional gauge matrix to normalize properly at infinity

```
> Gnorm:=Matrix(3,3,0):
  Gnorm[1,1]:=Gnorm111*lambda+Gnorm11:
  Gnorm[1,2]:=Gnorm112*lambda+Gnorm12:
  Gnorm[1,3]:=Gnorm113*lambda+Gnorm13:
  Gnorm[2,1]:=Gnorm121*lambda+Gnorm21:
  Gnorm[2,2]:=Gnorm122*lambda+Gnorm22:
  Gnorm[2,3]:=Gnorm123*lambda+Gnorm23:
  Gnorm[3,1]:=Gnorm131*lambda+Gnorm31:
  Gnorm[3,2]:=Gnorm132*lambda+Gnorm32:
  Gnorm[3,3]:=Gnorm133*lambda+Gnorm33:
  Gnorm;

  Gnormprime:=Matrix(3,3,0):
  for i from 1 to 3 do for j from 1 to 3 do Gnormprime[i,j]:=diff
  (Gnorm[i,j],lambda): od: od:
  Gnormprime;
```

$$
\begin{bmatrix}
\lambda\, Gnorm111 + Gnorm11 & \lambda\, Gnorm112 + Gnorm12 & \lambda\, Gnorm113 + Gnorm13 \\
\lambda\, Gnorm121 + Gnorm21 & \lambda\, Gnorm122 + Gnorm22 & \lambda\, Gnorm123 + Gnorm23 \\
\lambda\, Gnorm131 + Gnorm31 & \lambda\, Gnorm132 + Gnorm32 & \lambda\, Gnorm133 + Gnorm33
\end{bmatrix}
\qquad (2.3)
$$

$$
\begin{bmatrix}
Gnorm111 & Gnorm112 & Gnorm113 \\
Gnorm121 & Gnorm122 & Gnorm123 \\
Gnorm131 & Gnorm132 & Gnorm133
\end{bmatrix}
$$

```
> TdLtheo:=Matrix(3,3,0):
  TdLtheo[1,1]:=t12*lambda+a11:
  TdLtheo[1,2]:=a12:
  TdLtheo[1,3]:=a13:
  TdLtheo[2,1]:=a21:
  TdLtheo[2,2]:=t22*lambda+a22:
  TdLtheo[2,3]:=a23:
  TdLtheo[3,1]:=a31:
  TdLtheo[3,2]:=a32:
  TdLtheo[3,3]:=t32*lambda+a33:
  TdLtheo;

  EquationToCancel:=simplify(Multiply(TdLtheo,Gnorm)-Multiply
  (Gnorm,checkL)-h*Gnormprime):
```

$$
\begin{bmatrix}
\lambda\, t12 + a11 & a12 & a13 \\
a21 & \lambda\, t22 + a22 & a23 \\
a31 & a32 & \lambda\, t32 + a33
\end{bmatrix}
\qquad (2.4)
$$

```
> Gnorm113:=0:
  factor(-residue(EquationToCancel[1,1]/lambda^4,lambda=infinity)
  );
  factor(-residue(EquationToCancel[1,2]/lambda^4,lambda=infinity)
  );
  factor(-residue(EquationToCancel[1,3]/lambda^4,lambda=infinity)
  );
  Gnorm123:=0:
  factor(-residue(EquationToCancel[2,1]/lambda^4,lambda=infinity)
  );
  factor(-residue(EquationToCancel[2,2]/lambda^4,lambda=infinity)
  );
  factor(-residue(EquationToCancel[2,3]/lambda^4,lambda=infinity)
  );
  Gnorm133:=0:
```

```
factor(-residue(EquationToCancel[3,1]/lambda^4,lambda=infinity)
);
factor(-residue(EquationToCancel[3,2]/lambda^4,lambda=infinity)
);
factor(-residue(EquationToCancel[3,3]/lambda^4,lambda=infinity)
);
```

$$0 \tag{2.5}$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$

```
> Gnorm112:=0:
  Gnorm111:=Gnorm13*t32*t22;
  factor(-residue(EquationToCancel[1,1]/lambda^3,lambda=infinity)
  );
  factor(-residue(EquationToCancel[1,2]/lambda^3,lambda=infinity)
  );
  factor(-residue(EquationToCancel[1,3]/lambda^3,lambda=infinity)
  );
  Gnorm122:=0:
  Gnorm121:=Gnorm23*t32*t12;
  factor(-residue(EquationToCancel[2,1]/lambda^3,lambda=infinity)
  );
  factor(-residue(EquationToCancel[2,2]/lambda^3,lambda=infinity)
  );
  factor(-residue(EquationToCancel[2,3]/lambda^3,lambda=infinity)
  );
  Gnorm132:=0:
  Gnorm131:=Gnorm33*t22*t12;
  factor(-residue(EquationToCancel[3,1]/lambda^3,lambda=infinity)
  );
  factor(-residue(EquationToCancel[3,2]/lambda^3,lambda=infinity)
  );
  factor(-residue(EquationToCancel[3,3]/lambda^3,lambda=infinity)
  );

  Gnorm:=simplify(Gnorm);
```

$$Gnorm111 := Gnorm13 \; t32 \; t22 \tag{2.6}$$
$$0$$
$$0$$
$$0$$

$$Gnorm121 := Gnorm23\ t12\ t32$$
$$0$$
$$0$$
$$0$$
$$Gnorm131 := Gnorm33\ t12\ t22$$
$$0$$
$$0$$
$$0$$

$$\begin{bmatrix} Gnorm13\ \lambda\ t22\ t32 + Gnorm11 & Gnorm12 & Gnorm13 \\ Gnorm23\ \lambda\ t12\ t32 + Gnorm21 & Gnorm22 & Gnorm23 \\ Gnorm33\ \lambda\ t12\ t22 + Gnorm31 & Gnorm32 & Gnorm33 \end{bmatrix}$$

```
> Gnorm12:=-Gnorm13*(t22+t32);
  Gnorm13:=(Gnorm23*a12*t32+Gnorm33*a13*t22+Gnorm11)*a13*a12*(t22
  -t32)/(a12^2*a23*t22*t32-a12*a13*a22*t32^2+a12*a13*a33*t22^2-
  a13^2*a32*t22*t32);
  factor(-residue(EquationToCancel[1,1]/lambda^2,lambda=infinity)
  );
  factor(-residue(EquationToCancel[1,2]/lambda^2,lambda=infinity)
  );
  factor(-residue(EquationToCancel[1,3]/lambda^2,lambda=infinity)
  );
```

$$Gnorm12 := -Gnorm13\ (t22 + t32) \tag{2.7}$$

$$Gnorm13 := \frac{(Gnorm23\ a12\ t32 + Gnorm33\ a13\ t22 + Gnorm11)\ a13\ a12\ (t22 - t32)}{a12^2\ a23\ t22\ t32 - a12\ a13\ a22\ t32^2 + a12\ a13\ a33\ t22^2 - a13^2\ a32\ t22\ t32}$$
$$0$$
$$0$$
$$0$$

```
> Gnorm22:=-Gnorm23*(t12+t32);
  Gnorm23:=(Gnorm33*a12^2*a23^2*t12*t22*t32+Gnorm33*a12*a13^2*
  a21*t22^2*t32-Gnorm33*a12*a13^2*a21*t22*t32^2-Gnorm33*a12*a13*
  a22*a23*t12*t32^2+Gnorm33*a12*a13*a23*a33*t12*t22^2-Gnorm33*
  a13^2*a23*a32*t12*t22*t32+Gnorm11*a12*a13*a21*t22*t32-Gnorm11*
  a12*a13*a21*t32^2+Gnorm21*a12^2*a23*t22*t32-Gnorm21*a12*a13*
  a22*t32^2+Gnorm21*a12*a13*a33*t22^2-Gnorm21*a13^2*a32*t22*t32)*
  a12*a13*(t22-t32)/(a11*a12^3*a13*a23*t22^2*t32^2-a11*a12^3*a13*
  a23*t22*t32^3-a11*a12^2*a13^2*a22*t22*t32^3+a11*a12^2*a13^2*
  a22*t32^4+a11*a12^2*a13^2*a33*t22^3*t32-a11*a12^2*a13^2*a33*
  t22^2*t32^2-a11*a12*a13^3*a32*t22^2*t32^2+a11*a12*a13^3*a32*
  t22*t32^3+a12^4*a23^2*t12*t22*t32^2-a12^3*a13^2*a21*t22^2*
  t32^2+2*a12^3*a13^2*a21*t22*t32^3-a12^3*a13^2*a21*t32^4-a12^3*
  a13*a22*a23*t12*t22*t32^2-a12^3*a13*a22*a23*t12*t32^3+2*a12^3*
  a13*a23*a33*t12*t22^2*t32+a12^2*a13^2*a22^2*t12*t32^3-a12^2*
  a13^2*a22*a33*t12*t22^2*t32-a12^2*a13^2*a22*a33*t12*t22*t32^2
```

```
    -2*a12^2*a13^2*a23*a32*t12*t22*t32^2+a12^2*a13^2*a33^2*t12*
    t22^3+a12*a13^3*a22*a32*t12*t22*t32^2+a12*a13^3*a22*a32*t12*
    t32^3-2*a12*a13^3*a32*a33*t12*t22^2*t32+a13^4*a32^2*t12*t22*
    t32^2);
    factor(-residue(EquationToCancel[2,1]/lambda^2,lambda=infinity)
    );
    factor(-residue(EquationToCancel[2,2]/lambda^2,lambda=infinity)
    );
    factor(-residue(EquationToCancel[2,3]/lambda^2,lambda=infinity)
    );
```

$$Gnorm22 := -Gnorm23 \, (t12 + t32) \tag{2.8}$$

$$Gnorm23 := \Big(\big(Gnorm33 \, a12^2 \, a23^2 \, t12 \, t22 \, t32 + Gnorm33 \, a12 \, a13^2 \, a21 \, t22^2 \, t32$$
$$- Gnorm33 \, a12 \, a13^2 \, a21 \, t22 \, t32^2 - Gnorm33 \, a12 \, a13 \, a22 \, a23 \, t12 \, t32^2$$
$$+ Gnorm33 \, a12 \, a13 \, a23 \, a33 \, t12 \, t22^2 - Gnorm33 \, a13^2 \, a23 \, a32 \, t12 \, t22 \, t32$$
$$+ Gnorm11 \, a12 \, a13 \, a21 \, t22 \, t32 - Gnorm11 \, a12 \, a13 \, a21 \, t32^2$$
$$+ Gnorm21 \, a12^2 \, a23 \, t22 \, t32 - Gnorm21 \, a12 \, a13 \, a22 \, t32^2$$
$$+ Gnorm21 \, a12 \, a13 \, a33 \, t22^2 - Gnorm21 \, a13^2 \, a32 \, t22 \, t32 \big) \, a12 \, a13 \, (t22 - t32) \big) \Big/$$
$$\big(a11 \, a12^3 \, a13 \, a23 \, t22^2 \, t32^2 - a11 \, a12^3 \, a13 \, a23 \, t22 \, t32^3 - a11 \, a12^2 \, a13^2 \, a22 \, t22 \, t32^3$$
$$+ a11 \, a12^2 \, a13^2 \, a22 \, t32^4 + a11 \, a12^2 \, a13^2 \, a33 \, t22^3 \, t32 - a11 \, a12^2 \, a13^2 \, a33 \, t22^2 \, t32^2$$
$$- a11 \, a12 \, a13^3 \, a32 \, t22^2 \, t32^2 + a11 \, a12 \, a13^3 \, a32 \, t22 \, t32^3 + a12^4 \, a23^2 \, t12 \, t22 \, t32^2$$
$$- a12^3 \, a13^2 \, a21 \, t22^2 \, t32^2 + 2 \, a12^3 \, a13^2 \, a21 \, t22 \, t32^3 - a12^3 \, a13^2 \, a21 \, t32^4$$
$$- a12^3 \, a13 \, a22 \, a23 \, t12 \, t22 \, t32^2 - a12^3 \, a13 \, a22 \, a23 \, t12 \, t32^3$$
$$+ 2 \, a12^3 \, a13 \, a23 \, a33 \, t12 \, t22^2 \, t32 + a12^2 \, a13^2 \, a22^2 \, t12 \, t32^3$$
$$- a12^2 \, a13^2 \, a22 \, a33 \, t12 \, t22^2 \, t32 - a12^2 \, a13^2 \, a22 \, a33 \, t12 \, t22 \, t32^2$$
$$- 2 \, a12^2 \, a13^2 \, a23 \, a32 \, t12 \, t22 \, t32^2 + a12^2 \, a13^2 \, a33^2 \, t12 \, t22^3$$
$$+ a12 \, a13^3 \, a22 \, a32 \, t12 \, t22 \, t32^2 + a12 \, a13^3 \, a22 \, a32 \, t12 \, t32^3$$
$$- 2 \, a12 \, a13^3 \, a32 \, a33 \, t12 \, t22^2 \, t32 + a13^4 \, a32^2 \, t12 \, t22 \, t32^2 \big)$$

$$0$$
$$0$$
$$0$$

```
> Gnorm32:=-Gnorm33*(t12+t22);
  Gnorm33:=factor(solve(factor(-residue(EquationToCancel[3,1]
  /lambda^2,lambda=infinity)),Gnorm33)):
  factor(-residue(EquationToCancel[3,1]/lambda^2,lambda=infinity)
  );
  factor(-residue(EquationToCancel[3,2]/lambda^2,lambda=infinity)
  );
  factor(-residue(EquationToCancel[3,3]/lambda^2,lambda=infinity)
  );
  Gnorm:=simplify(Gnorm):
```

$$Gnorm32 := -Gnorm33 \, (t12 + t22) \tag{2.9}$$
$$0$$
$$0$$
$$0$$

```
> Gnorm31:=factor(solve(factor(-residue(EquationToCancel[3,3]
  /lambda^1,lambda=infinity)),Gnorm31)):
  factor(-residue(EquationToCancel[3,3]/lambda^1,lambda=infinity)
  );
  factor(-residue(EquationToCancel[3,2]/lambda^1,lambda=infinity)
  );
```

$$0 \tag{2.10}$$
$$0$$

```
> Gnorm11:=Gnorm21*a12^2*a13*(t22-t32)^2/(a11*a12*a13*t22*t32-
  a11*a12*a13*t32^2+a12^2*a23*t12*t22-a12*a13*a22*t12*t32+a12*
  a13*a33*t12*t22-a13^2*a32*t12*t32);
```

$$Gnorm11 := \left(Gnorm21 \; a12^2 \; a13 \; (t22 - t32)^2 \right) \Big/ \left(a11 \; a12 \; a13 \; t22 \; t32 - a11 \; a12 \; a13 \; t32^2 \right. \tag{2.11}$$
$$\left. + a12^2 \; a23 \; t12 \; t22 - a12 \; a13 \; a22 \; t12 \; t32 + a12 \; a13 \; a33 \; t12 \; t22 - a13^2 \; a32 \; t12 \; t32 \right)$$

```
> factor(-residue(EquationToCancel[3,1]/lambda^1,lambda=infinity)
  );
  factor(-residue(EquationToCancel[2,1]/lambda^1,lambda=infinity)
  );
  factor(-residue(EquationToCancel[2,2]/lambda^1,lambda=infinity)
  );
  factor(-residue(EquationToCancel[2,3]/lambda^1,lambda=infinity)
  );
  factor(-residue(EquationToCancel[1,1]/lambda^1,lambda=infinity)
  );
  factor(-residue(EquationToCancel[1,2]/lambda^1,lambda=infinity)
  );
  factor(-residue(EquationToCancel[1,3]/lambda^1,lambda=infinity)
  );
```

$$0 \tag{2.12}$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$

```
> Gnorm11:=factor(Gnorm11);
  Gnorm12:=factor(Gnorm12);
  Gnorm13:=factor(Gnorm13);
  Gnorm21:=factor(Gnorm21);
  Gnorm22:=factor(Gnorm22);
  Gnorm23:=factor(Gnorm23);
  Gnorm31:=factor(Gnorm31);
  Gnorm32:=factor(Gnorm32);
  Gnorm33:=factor(Gnorm33);
```

```
    Gnorm21:=(a11*a12*a13*t22*t32-a11*a12*a13*t32^2+a12^2*a23*t12*
    t22-a12*a13*a22*t12*t32+a12*a13*a33*t12*t22-a13^2*a32*t12*t32)/
    (a12^2*(t22-t32)^2*a13):
    Gnorm:=simplify(Gnorm);
```

$$Gnorm11 := \left(Gnorm21\ a12^2\ a13\ (t22 - t32)^2\right) \Big/ \left(a11\ a12\ a13\ t22\ t32 - a11\ a12\ a13\ t32^2 \right. \qquad \textbf{(2.13)}$$
$$\left. + a12^2\ a23\ t12\ t22 - a12\ a13\ a22\ t12\ t32 + a12\ a13\ a33\ t12\ t22 - a13^2\ a32\ t12\ t32\right)$$
$$Gnorm12 := 0$$
$$Gnorm13 := 0$$
$$Gnorm21 := Gnorm21$$
$$Gnorm22 := -\left((t12 + t32)\ (t22 - t32)\ a12\ a13\ Gnorm21\right) \Big/ \left(a11\ a12\ a13\ t22\ t32 \right.$$
$$\left. - a11\ a12\ a13\ t32^2 + a12^2\ a23\ t12\ t22 - a12\ a13\ a22\ t12\ t32 + a12\ a13\ a33\ t12\ t22 \right.$$
$$\left. - a13^2\ a32\ t12\ t32\right)$$
$$Gnorm23 := \left((t22 - t32)\ a12\ a13\ Gnorm21\right) \Big/ \left(a11\ a12\ a13\ t22\ t32 - a11\ a12\ a13\ t32^2 \right.$$
$$\left. + a12^2\ a23\ t12\ t22 - a12\ a13\ a22\ t12\ t32 + a12\ a13\ a33\ t12\ t22 - a13^2\ a32\ t12\ t32\right)$$
$$Gnorm31 := -\left(\left(a11\ a12\ a13\ t22^2 - a11\ a12\ a13\ t22\ t32 + a12^2\ a23\ t12\ t22 \right.\right.$$
$$\left.\left. - a12\ a13\ a22\ t12\ t32 + a12\ a13\ a33\ t12\ t22 - a13^2\ a32\ t12\ t32\right)\ Gnorm21\ a12\right) \Big/$$
$$\left(\left(a11\ a12\ a13\ t22\ t32 - a11\ a12\ a13\ t32^2 + a12^2\ a23\ t12\ t22 - a12\ a13\ a22\ t12\ t32 \right.\right.$$
$$\left.\left. + a12\ a13\ a33\ t12\ t22 - a13^2\ a32\ t12\ t32\right)\ a13\right)$$
$$Gnorm32 := \left((t12 + t22)\ (t22 - t32)\ a12^2\ Gnorm21\right) \Big/ \left(a11\ a12\ a13\ t22\ t32 \right.$$
$$\left. - a11\ a12\ a13\ t32^2 + a12^2\ a23\ t12\ t22 - a12\ a13\ a22\ t12\ t32 + a12\ a13\ a33\ t12\ t22 \right.$$
$$\left. - a13^2\ a32\ t12\ t32\right)$$
$$Gnorm33 := -\left((t22 - t32)\ a12^2\ Gnorm21\right) \Big/ \left(a11\ a12\ a13\ t22\ t32 - a11\ a12\ a13\ t32^2 \right.$$
$$\left. + a12^2\ a23\ t12\ t22 - a12\ a13\ a22\ t12\ t32 + a12\ a13\ a33\ t12\ t22 - a13^2\ a32\ t12\ t32\right)$$

$$\left[\left[1, 0, 0\right],\right.$$
$$\left[\frac{1}{a13\ a12^2\ (-t22 + t32)^2}\left(a12^2\ a23\ t12\ t22 - \left(\left(\lambda\ t12 + a11\right)\ t32^2 + \left(\left(-\lambda\ t22 \right.\right.\right.\right.\right.$$
$$\left.\left.\left.\left. + a22\right)\ t12 - a11\ t22\right)\ t32 - a33\ t12\ t22\right)\ a13\ a12 - a13^2\ a32\ t12\ t32\right),$$
$$\frac{-t12 - t32}{a12\ (t22 - t32)},\ \frac{1}{a12\ (t22 - t32)}\right],$$
$$\left[\frac{1}{a12\ (-t22 + t32)^2\ a13^2}\left(-a12^2\ a23\ t12\ t22 + a13\left(\left(-\lambda\ t12 - a11\right)\ t22^2 \right.\right.\right.$$
$$\left.\left.\left. + \left(\left(\lambda\ t32 - a33\right)\ t12 + t32\ a11\right)\ t22 + t12\ t32\ a22\right)\ a12 + a13^2\ a32\ t12\ t32\right),$$
$$\left.\frac{t12 + t22}{a13\ (t22 - t32)},\ -\frac{1}{a13\ (t22 - t32)}\right]\right]$$

We verify that G is indeed equal to G^{-1} J^{-1} and give the expression of Gnorm

```
> Gnormalisationinfini:=simplify(Multiply(G^(-1),J^(-1))):
  dGnormalisationinfinidlambda:=Matrix(3,3,0):
  for i from 1 to 3 do for j from 1 to 3 do
  dGnormalisationinfinidlambda[i,j]:=diff(Gnormalisationinfini[i,
```

```
j],lambda): od: od:
simplify(Gnorm-Gnormalisationinfini);
factor(-(residue(Gnorm[1,1]/lambda^2,lambda=infinity) ));
factor(-(residue(Gnorm[1,1]/lambda,lambda=infinity) ));

factor(-(residue(Gnorm[2,1]/lambda^2,lambda=infinity) ));
factor(-(residue(Gnorm[2,1]/lambda,lambda=infinity) ));

factor(-(residue(Gnorm[2,2]/lambda^2,lambda=infinity) ));
factor(-(residue(Gnorm[2,2]/lambda,lambda=infinity) ));

factor(-(residue(Gnorm[2,3]/lambda^2,lambda=infinity) ));
factor(-(residue(Gnorm[2,3]/lambda,lambda=infinity) ));

factor(-(residue(Gnorm[3,1]/lambda^2,lambda=infinity) ));
factor(-(residue(Gnorm[3,1]/lambda,lambda=infinity) ));

factor(-(residue(Gnorm[3,2]/lambda^2,lambda=infinity) ));
factor(-(residue(Gnorm[3,2]/lambda,lambda=infinity) ));

factor(-(residue(Gnorm[3,3]/lambda^2,lambda=infinity) ));
factor(-(residue(Gnorm[3,3]/lambda,lambda=infinity) ));

factor(Determinant(Gnorm));
```

$$
\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\tag{2.14}
$$

$$0$$

$$1$$

$$\frac{t12\,t32}{a12\,(t22-t32)}$$

$$\frac{1}{a12^2\,(t22-t32)^2\,a13}\left(a11\,a12\,a13\,t22\,t32 - a11\,a12\,a13\,t32^2 + a12^2\,a23\,t12\,t22 \right.$$
$$\left. - a12\,a13\,a22\,t12\,t32 + a12\,a13\,a33\,t12\,t22 - a13^2\,a32\,t12\,t32\right)$$

$$0$$

$$-\frac{t12+t32}{a12\,(t22-t32)}$$

$$0$$

$$\frac{1}{a12\,(t22-t32)}$$

$$-\frac{t12\,t22}{a13\,(t22-t32)}$$

$$-\frac{1}{a12\,(t22-t32)^2\,a13^2}\big(a11\,a12\,a13\,t22^2 - a11\,a12\,a13\,t22\,t32 + a12^2\,a23\,t12\,t22$$
$$-\,a12\,a13\,a22\,t12\,t32 + a12\,a13\,a33\,t12\,t22 - a13^2\,a32\,t12\,t32\big)$$
$$0$$
$$\frac{t12+t22}{a13\,(t22-t32)}$$
$$0$$
$$-\frac{1}{a13\,(t22-t32)}$$
$$-\frac{1}{a12\,a13\,(t22-t32)}$$

Verification that TdLtheo is really the normalized version of \check{L}

```
> simplify(TdLtheo-Multiply(Multiply(Gnorm,checkL),Gnorm^(-1))-h*
  Multiply(Gnormprime,Gnorm^(-1)));
```

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(2.15)

Expression of the Lax matrices \td{L}, L and gauge matrices using irregular times after computing the local diagonalization of the singular part at infinity.

```
> Ginfinity:=Matrix(3,3,0):
  Ginfinity[1,1]:=1+g111*lambda^(-1)+ g112*lambda^(-2)+E11*
  lambda^(-3):
  Ginfinity[1,2]:=  g121*lambda^(-1)+ g122*lambda^(-2)+E12*
  lambda^(-3):
  Ginfinity[1,3]:=  g131*lambda^(-1)+ g132*lambda^(-2)+E13*
  lambda^(-3):
  Ginfinity[2,1]:=  g211*lambda^(-1)+ g212*lambda^(-2)+E21*
  lambda^(-3):
  Ginfinity[2,2]:=1+g221*lambda^(-1)+ g222*lambda^(-2)+E22*
  lambda^(-3):
  Ginfinity[2,3]:=  g231*lambda^(-1)+ g231*lambda^(-2)+E23*
  lambda^(-3):
  Ginfinity[3,1]:=  g311*lambda^(-1)+ g312*lambda^(-2)+E31*
  lambda^(-3):
  Ginfinity[3,2]:=  g321*lambda^(-1)+ g322*lambda^(-2)+E32*
  lambda^(-3):
  Ginfinity[3,3]:=1+g331*lambda^(-1)+ g332*lambda^(-2)+E33*
  lambda^(-3):
  Ginfinity;
```

```
Ginfinityprime:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do Ginfinityprime[i,j]:=
diff(Ginfinity[i,j],lambda): od: od:

Linfinity:=Matrix(3,3,0):
Linfinity[1,1]:=t12*lambda+t11+t10/lambda+u11/lambda^2+
v11/lambda^3:
Linfinity[1,2]:=u12/lambda^2+v12/lambda^3:
Linfinity[1,3]:=u13/lambda^2+v13/lambda^3:
Linfinity[2,1]:=u21/lambda^2+v21/lambda^3:
Linfinity[2,2]:=t22*lambda+t21+t20/lambda+u22/lambda^2+
v22/lambda^3:
Linfinity[2,3]:=u23/lambda^2+v23/lambda^3:
Linfinity[3,1]:=u31/lambda^2+v31/lambda^3:
Linfinity[3,2]:=u32/lambda^2+v32/lambda^3:
Linfinity[3,3]:=t32*lambda+t31+t30/lambda+
u33/lambda^2+v33/lambda^3:
Linfinity;
```

$$\left[\left[1+\frac{g111}{\lambda}+\frac{g112}{\lambda^2}+\frac{E11}{\lambda^3},\ \frac{g121}{\lambda}+\frac{g122}{\lambda^2}+\frac{E12}{\lambda^3},\ \frac{g131}{\lambda}+\frac{g132}{\lambda^2}+\frac{E13}{\lambda^3}\right],\right. \tag{3.1}$$

$$\left[\frac{g211}{\lambda}+\frac{g212}{\lambda^2}+\frac{E21}{\lambda^3},\ 1+\frac{g221}{\lambda}+\frac{g222}{\lambda^2}+\frac{E22}{\lambda^3},\ \frac{g231}{\lambda}+\frac{g231}{\lambda^2}+\frac{E23}{\lambda^3}\right],$$

$$\left.\left[\frac{g311}{\lambda}+\frac{g312}{\lambda^2}+\frac{E31}{\lambda^3},\ \frac{g321}{\lambda}+\frac{g322}{\lambda^2}+\frac{E32}{\lambda^3},\ 1+\frac{g331}{\lambda}+\frac{g332}{\lambda^2}+\frac{E33}{\lambda^3}\right]\right]$$

$$\left[\left[\lambda\,t12+t11+\frac{t10}{\lambda}+\frac{u11}{\lambda^2}+\frac{v11}{\lambda^3},\ \frac{u12}{\lambda^2}+\frac{v12}{\lambda^3},\ \frac{u13}{\lambda^2}+\frac{v13}{\lambda^3}\right],\right.$$

$$\left[\frac{u21}{\lambda^2}+\frac{v21}{\lambda^3},\ \lambda\,t22+t21+\frac{t20}{\lambda}+\frac{u22}{\lambda^2}+\frac{v22}{\lambda^3},\ \frac{u23}{\lambda^2}+\frac{v23}{\lambda^3}\right],$$

$$\left.\left[\frac{u31}{\lambda^2}+\frac{v31}{\lambda^3},\ \frac{u32}{\lambda^2}+\frac{v32}{\lambda^3},\ \lambda\,t32+t31+\frac{t30}{\lambda}+\frac{u33}{\lambda^2}+\frac{v33}{\lambda^3}\right]\right]$$

```
> tdL:=simplify(tdL):
ToCancel2:=simplify(simplify(tdL)-(Multiply(Multiply(Ginfinity,
Linfinity),Ginfinity^(-1))+h*Multiply(Ginfinityprime,Ginfinity^
(-1)))):
> g121:=-a12/(t12-t22);
residue(ToCancel2[1,2]/lambda^2,lambda=infinity);
residue(ToCancel2[1,2]/lambda,lambda=infinity);
g131:=-a13/(t12-t32);
residue(ToCancel2[1,3]/lambda^2,lambda=infinity);
residue(ToCancel2[1,3]/lambda,lambda=infinity);
g211:=-a21/(t22-t12);
```

```
residue(ToCancel2[2,1]/lambda^2,lambda=infinity);
residue(ToCancel2[2,1]/lambda,lambda=infinity);
a22:=t21;
residue(ToCancel2[2,2]/lambda^2,lambda=infinity);
residue(ToCancel2[2,2]/lambda,lambda=infinity);
g231:=-a23/(t22-t32);
residue(ToCancel2[2,3]/lambda^2,lambda=infinity);
residue(ToCancel2[2,3]/lambda,lambda=infinity);
g311:=-a31/(t32-t12);
residue(ToCancel2[3,1]/lambda^2,lambda=infinity);
residue(ToCancel2[3,1]/lambda,lambda=infinity);
g321:=a32/(t22-t32);
residue(ToCancel2[3,2]/lambda^2,lambda=infinity);
simplify(residue(ToCancel2[3,2]/lambda,lambda=infinity));
a33:=t31;
residue(ToCancel2[3,3]/lambda^2,lambda=infinity);
simplify(residue(ToCancel2[3,3]/lambda,lambda=infinity));
a11:=t11;
residue(ToCancel2[1,1]/lambda,lambda=infinity);
```

$$g121 := -\frac{a12}{t12-t22}$$ (3.2)

$$0$$

$$0$$

$$g131 := -\frac{a13}{t12-t32}$$

$$0$$

$$0$$

$$g211 := -\frac{a21}{-t12+t22}$$

$$0$$

$$0$$

$$a22 := t21$$

$$0$$

$$0$$

$$g231 := -\frac{a23}{t22-t32}$$

$$0$$

$$0$$

$$g311 := -\frac{a31}{-t12+t32}$$

$$0$$

$$0$$

$$g321 := \frac{a32}{t22-t32}$$

$$0$$

$$0$$

$$a33 := t31$$

$$
\begin{array}{c}
0 \\
0 \\
a11 := t11 \\
0
\end{array}
$$

```
> NonTrivialRelation:=residue(ToCancel2[1,1],lambda=infinity);
  tdL:=simplify(tdL);

  g212:=(a21*t12^2*g111-a21*t12*t22*g111-a21*t12*t32*g111+a21*
  t22*t32*g111-a21*t12*t11+a21*t12*t21+a21*t32*t11-a21*t32*t21+
  a23*a31*t12-a23*a31*t22)/(t12-t22)^2/(t12-t32);
  simplify(residue(ToCancel2[2,1],lambda=infinity));

  NonTrivialRelation2:=simplify(residue(ToCancel2[2,2],lambda=
  infinity));

  g331:=(a13*a21*t22-a13*a21*t32+a23*t12*t22-a23*t12*t32+a23*t12*
  t21-a23*t12*t31-a23*t22*t32+a23*t32^2-a23*t32*t21+a23*t32*t31)/
  ((t12-t32)*(t22-t32)*a23);
  simplify(residue(ToCancel2[2,3],lambda=infinity));

  g312:=(a31*t12^2*g111-a31*t12*t22*g111-a31*t12*t32*g111+a31*
  t22*t32*g111+a21*a32*t12-a21*a32*t32-a31*t12*t11+a31*t12*t31+
  a31*t22*t11-a31*t22*t31)/(t12-t32)^2/(t12-t22);
  simplify(residue(ToCancel2[2,3],lambda=infinity));

  g322:=-(-a32*t12*t22*g221+a32*t12*t32*g221+a32*t22^2*g221-a32*
  t22*t32*g221+a12*a31*t22-a12*a31*t32+a32*t12*t21-a32*t12*t31-
  a32*t22*t21+a32*t22*t31)/(t22-t32)^2/(t12-t22);
  simplify(residue(ToCancel2[3,2],lambda=infinity));

  NonTrivialRelation3:=simplify(residue(ToCancel2[3,3],lambda=
  infinity));
```

$$
NonTrivialRelation := \frac{a21\,a12}{-t12 + t22} + \frac{a31\,a13}{-t12 + t32} + t10 \tag{3.3}
$$

$$
\begin{bmatrix}
\lambda\,t12 + t11 & a12 & a13 \\
a21 & \lambda\,t22 + t21 & a23 \\
a31 & a32 & \lambda\,t32 + t31
\end{bmatrix}
$$

$$
\begin{aligned}
g212 := {}& \frac{1}{(t12 - t22)^2\,(t12 - t32)} \big(a21\,g111\,t12^2 - a21\,g111\,t12\,t22 - a21\,g111\,t12\,t32 \\
& + a21\,g111\,t22\,t32 - a21\,t11\,t12 + a21\,t11\,t32 + a21\,t12\,t21 - a21\,t21\,t32 \\
& + a23\,a31\,t12 - a23\,a31\,t22 \big)
\end{aligned}
$$

$$
0
$$

$$NonTrivialRelation2 := \frac{1}{(t12 - t22)(t22 - t32)}\left(-t20\,t22^2 + (a12\,a21 + a23\,a32\right.$$
$$\left. + t12\,t20 + t20\,t32)\,t22 + (-a23\,a32 - t20\,t32)\,t12 - a12\,a21\,t32\right)$$

$$g331 := \frac{1}{(t12 - t32)(t22 - t32)\,a23}\left(a13\,a21\,t22 - a13\,a21\,t32 + a23\,t12\,t21\right.$$
$$+ a23\,t12\,t22 - a23\,t12\,t31 - a23\,t12\,t32 - a23\,t21\,t32 - a23\,t22\,t32 + a23\,t31\,t32$$
$$\left. + a23\,t32^2\right)$$

$$0$$

$$g312 := \frac{1}{(t12 - t32)^2(t12 - t22)}\left(a31\,g111\,t12^2 - a31\,g111\,t12\,t22 - a31\,g111\,t12\,t32\right.$$
$$+ a31\,g111\,t22\,t32 + a21\,a32\,t12 - a21\,a32\,t32 - a31\,t11\,t12 + a31\,t11\,t22$$
$$\left. + a31\,t12\,t31 - a31\,t22\,t31\right)$$

$$0$$

$$g322 := -\frac{1}{(t22 - t32)^2(t12 - t22)}\left(-a32\,g221\,t12\,t22 + a32\,g221\,t12\,t32\right.$$
$$+ a32\,g221\,t22^2 - a32\,g221\,t22\,t32 + a12\,a31\,t22 - a12\,a31\,t32 + a32\,t12\,t21$$
$$\left. - a32\,t12\,t31 - a32\,t21\,t22 + a32\,t22\,t31\right)$$

$$0$$

$$NonTrivialRelation3 := \frac{1}{(t12 - t32)(t22 - t32)}\left(t30\,t32^2 + (-a13\,a31 - a23\,a32\right.$$
$$\left. - t12\,t30 - t22\,t30)\,t32 + (a23\,a32 + t22\,t30)\,t12 + a13\,a31\,t22\right)$$

> **`NonTrivialRelation:=simplify(NonTrivialRelation);`**
  `NonTrivialRelation2;`
  `NonTrivialRelation3;`

  `t10:=-t20-t30;`
  `a21 := (a23*a32*t12-a23*a32*t22-t12*t22*t20+t12*t32*t20+t22^2*`
  `t20-t22*t32*t20)/(a12*(t22-t32));`
  `a31 := -(a23*a32*t12-a23*a32*t32+t12*t22*t30-t12*t32*t30-t22*`
  `t32*t30+t32^2*t30)/(a13*(t22-t32));`
  `simplify(NonTrivialRelation);`
  `simplify(NonTrivialRelation2);`
  `simplify(NonTrivialRelation3);`
  `tdL:=simplify(tdL);`

$$NonTrivialRelation := \frac{1}{(t12 - t22)(t12 - t32)}\left(t10\,t12^2 + ((-t22 - t32)\,t10 - a12\,a21\right. \qquad \textbf{(3.4)}$$
$$\left. - a13\,a31)\,t12 + a12\,a21\,t32 + a13\,a31\,t22 + t10\,t22\,t32\right)$$

$$\frac{1}{(t12 - t22)(t22 - t32)}\left(-t20\,t22^2 + (a12\,a21 + a23\,a32 + t12\,t20 + t20\,t32)\,t22 + (\right.$$
$$\left. -a23\,a32 - t20\,t32)\,t12 - a12\,a21\,t32\right)$$

$$\frac{1}{(t12 - t32)(t22 - t32)}\left(t30\,t32^2 + (-a13\,a31 - a23\,a32 - t12\,t30 - t22\,t30)\,t32\right.$$
$$\left. + (a23\,a32 + t22\,t30)\,t12 + a13\,a31\,t22\right)$$

$$t10 := -t20 - t30$$

$$a21 := \frac{a23\,a32\,t12 - a23\,a32\,t22 - t12\,t20\,t22 + t12\,t20\,t32 + t20\,t22^2 - t20\,t22\,t32}{a12\,(t22 - t32)}$$

$$a31 := -\frac{a23\,a32\,t12 - a23\,a32\,t32 + t12\,t22\,t30 - t12\,t30\,t32 - t22\,t30\,t32 + t30\,t32^2}{a13\,(t22 - t32)}$$

$$0$$
$$0$$
$$0$$

$$\begin{bmatrix} \lambda\,t12 + t11 & a12 & a13 \\ \dfrac{(t12 - t22)\,((-t22 + t32)\,t20 + a23\,a32)}{a12\,(t22 - t32)} & \lambda\,t22 + t21 & a23 \\ -\dfrac{(t12 - t32)\,(t30\,(t22 - t32) + a23\,a32)}{a13\,(t22 - t32)} & a32 & \lambda\,t32 + t31 \end{bmatrix}$$

Definition of the Darboux coordinates to end the expression of the Lax matrices and gauge matrices.
The coordinate q was introduced earlier in the oper gauge transform as the apparent singularity

```
> Conditionp:=unapply(factor(Determinant(Psing*IdentityMatrix(3)-
  tdL)),lambda):
> Psing:=(a12^2*a23*t32+a12*a13*t22*t31-a12*a13*t32*t21-a13^2*
  a32*t22)/(a12*(t22-t32)*a13);
  factor(Conditionp(Qsing));
  Qsing=q;
  Psing=p;
  solve({Qsing-q,Psing-p},{a23,a32});

  a23 := -a13*(-q*t22+p-t21)/a12;
  a32 := -a12*(-q*t32+p-t31)/a13;
  simplify(Qsing-q);
  simplify(Psing-p);
```

$$Psing := \frac{a12^2\,a23\,t32 - a12\,a13\,t21\,t32 + a12\,a13\,t22\,t31 - a13^2\,a32\,t22}{a12\,a13\,(t22 - t32)}$$ (3.5)

$$0$$

$$\frac{a12^2\,a23 - a12\,a13\,t21 + a12\,a13\,t31 - a13^2\,a32}{a12\,a13\,(t22 - t32)} = q$$

$$\frac{a12^2\,a23\,t32 - a12\,a13\,t21\,t32 + a12\,a13\,t22\,t31 - a13^2\,a32\,t22}{a12\,a13\,(t22 - t32)} = p$$

$$\left\{ a23 = -\frac{a13\,(-q\,t22 + p - t21)}{a12}, \; a32 = -\frac{a12\,(-q\,t32 + p - t31)}{a13} \right\}$$

$$a23 := -\frac{a13\,(-q\,t22 + p - t21)}{a12}$$

$$a32 := -\frac{a12\,(-q\,t32 + p - t31)}{a13}$$

$$0$$
$$0$$

```
> g132:=solve(simplify(residue(ToCancel2[1,3]/lambda^0,lambda=
  infinity)),g132);
  g221:=solve(simplify(residue(ToCancel2[1,2]/lambda^0,lambda=
  infinity)),g221);

  simplify(residue(ToCancel2[1,1],lambda=infinity));
  simplify(residue(ToCancel2[1,2],lambda=infinity));
  simplify(residue(ToCancel2[1,3],lambda=infinity));
  simplify(residue(ToCancel2[2,1],lambda=infinity));
  simplify(residue(ToCancel2[2,2],lambda=infinity));
  simplify(residue(ToCancel2[2,3],lambda=infinity));
  simplify(residue(ToCancel2[3,1],lambda=infinity));
  simplify(residue(ToCancel2[3,2],lambda=infinity));
  simplify(residue(ToCancel2[3,3],lambda=infinity));
```

$$g132 := -\frac{1}{(t12-t32)^2\,(-q\,t22+p-t21)}\Big(\big(q^2\,t12\,t22 - p\,q\,t12 - p\,q\,t22 + q\,t11\,t22 \tag{3.6}$$
$$+\,q\,t12\,t21 - q\,t12\,t22 + q\,t22\,t32 + p^2 - p\,t11 + p\,t12 - p\,t21 - p\,t32 + t11\,t21$$
$$+\,t12\,t20 - t12\,t21 - t20\,t22 + t21\,t32\big)\,a13\Big)$$

$$g221 := \frac{1}{(t12-t22)\,(t22-t32)\,a12}\big(-a12\,q\,t12\,t32 + a12\,q\,t22\,t32 - g122\,t12^2\,t22$$
$$+\,g122\,t12^2\,t32 + 2\,g122\,t12\,t22^2 - 2\,g122\,t12\,t22\,t32 - g122\,t22^3 + g122\,t22^2\,t32$$
$$+\,a12\,p\,t12 - a12\,p\,t22 + a12\,t11\,t22 - a12\,t11\,t32 - a12\,t12\,t31 - a12\,t21\,t22$$
$$+\,a12\,t21\,t32 + a12\,t22\,t31\big)$$

$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$

```
> u11:=solve( simplify(residue(ToCancel2[1,1]*lambda,lambda=
  infinity)),u11):
  simplify(residue(ToCancel2[1,1]*lambda,lambda=infinity));
  u22:=solve( simplify(residue(ToCancel2[2,2]*lambda,lambda=
  infinity)),u22):
  simplify(residue(ToCancel2[2,2]*lambda,lambda=infinity));
  u33:=solve( simplify(residue(ToCancel2[3,3]*lambda,lambda=
  infinity)),u33):
  simplify(residue(ToCancel2[3,3]*lambda,lambda=infinity));
```

$$0 \tag{3.7}$$
$$0$$
$$0$$

Summary of the formulas for Lax matrices and gauge matrices

```
> tdL:=simplify(tdL):
  tdL[1,1];
  tdL[1,2];
  tdL[1,3];
  tdL[2,1];
  tdL21bis:=(t12-t22)/a12/(t22-t32)*(p^2-(q*(t22+t32)+t21+t31)*p+
  t32*t22*q^2+(t22*t31+t32*t21)*q+(t32-t22)*t20+t21*t31 );
  factor(series(factor(tdL[2,1]-tdL21bis),q=0));
  tdL[2,2];
  tdL[2,3];
  tdL[3,1];
  tdL31bis:=(t12-t32)*(p^2-(q*(t22+t32)+t21+t31)*p+t22*t32*q^2+
  (t22*t31+t32*t21)*q+((t22-t32)*t30+t21*t31) )/a13/(t32-t22);
  factor(series(factor(tdL[3,1]-tdL31bis),q=0));
  tdL[3,2];
  tdL[3,3];
  tdL;
```

$$\frac{\lambda\,t12 + t11}{a12}$$ (3.8)

$$a13$$

$$\frac{1}{a12\,(t22-t32)}\Big((t12-t22)\big((q^2\,t32 - p\,q + q\,t31 - t20)\,t22 + (-p\,q + q\,t21 + t20)\,t32 + (p-t31)\,(p-t21)\big)\Big)$$

$$tdL21bis := \frac{1}{a12\,(t22-t32)}\Big((t12-t22)\big(p^2 - (q\,(t22+t32) + t21 + t31)\,p + q^2\,t22\,t32 + (t21\,t32 + t22\,t31)\,q + (-t22+t32)\,t20 + t21\,t31\big)\Big)$$

$$0$$

$$\lambda\,t22 + t21$$

$$-\frac{a13\,(-q\,t22 + p - t21)}{a12}$$

$$-\frac{1}{a13\,(t22-t32)}\Big((t12-t32)\big((q^2\,t22 - p\,q + q\,t21 - t30)\,t32 + (-p\,q + q\,t31 + t30)\,t22 + (p-t31)\,(p-t21)\big)\Big)$$

$$tdL31bis := \frac{1}{a13\,(-t22+t32)}\Big((t12-t32)\big(p^2 - (q\,(t22+t32) + t21 + t31)\,p + q^2\,t22\,t32 + (t21\,t32 + t22\,t31)\,q + t30\,(t22-t32) + t21\,t31\big)\Big)$$

$$0$$

$$-\frac{a12\,(-q\,t32 + p - t31)}{a13}$$

$$\lambda\,t32 + t31$$

$$\left[\left[\lambda\,t12 + t11,\ a12,\ a13\right],\right.$$

$$\left[\frac{1}{a12\,(t22-t32)}\Big((t12-t22)\big((q^2\,t32 - p\,q + q\,t31 - t20)\,t22 + (-p\,q\right.$$

$$+ q\,t21 + t20)\,t32 + (p - t31)\,(p - t21))), \; \lambda\,t22 + t21, \; -\frac{a13\,(-q\,t22 + p - t21)}{a12}$$

$$\Big],$$

$$\left[-\frac{1}{a13\,(t22 - t32)}\left((t12 - t32)\left((q^2\,t22 - p\,q + q\,t21 - t30)\,t32 + (-p\,q\right.\right.\right.$$

$$\left.\left.\left. + q\,t31 + t30)\,t22 + (p - t31)\,(p - t21))\right), \; -\frac{a12\,(-q\,t32 + p - t31)}{a13}, \; \lambda\,t32 + t31\right.$$

$$\Big]\Big]$$

```
> L33ter:=h/(lambda-q)+(t12+t22+t32)*lambda+t11+t21+t31;
  simplify(L33bis-L33ter);

  L32ter:=h*(-(t12+t22+t32)*q+p-t11-t21-t31)/(lambda-q)-(t12*t22+
  t12*t32+t22*t32)*lambda^2-(t12*(t21+t31)+t22*(t11+t31)+t32*
  (t11+t21))*lambda+ t12*t10+t22*t20+t32*t30-t11*t21-t11*t31-t21*
  t31 +h*t12 ;
  factor(series(simplify(L32bis-L32ter),lambda=0));

  L31ter:=h*( p^2-(t11+t21+t31+q*(t12+t22+t32))*p+ (t12*t22+t12*
  t32+t22*t32)*q^2+(t12*(t21+t31)+t22*(t11+t31)+t32*(t11+t21))*q
  -t12*t10 -t22*t20-t32*t30+t11*t31+t11*t21+t21*t31 -h*t12 )/
  (lambda-q)
  +t12*t22*t32*lambda^3+(t22*t12*t31+t12*t32*t21+t22*t32*t11)*
  lambda^2
  +((t30-h)*t22*t12+(t20-h)*t32*t12+t10*t22*t32 +t12*t21*
  t31+t22*t11*t31+t32*t11*t21)*lambda
  +p^3-((t12+t22+t32)*q+t21+t31+t11)*p^2
  +(q^2*(t12*t22+t12*t32+t22*t32)+ (t12*(t21+t31)+t22*(t11+t31)+
  t32*(t11+t21))*q-(t10*t12+t20*t22+t30*t32)+ t11*t21+t11*t31+
  t21*t31 -h*t12 )*p
  -q^3*t12*t22*t32-(t22*t32*t11+t12*t32*t21+t12*t22*t31)*q^2
  -(t21*t31*t12+t11*t31*t22+t11*t21*t32 +t10*t22*t32+ t20*t12*
  t32+t30*t12*t22 -h*(t22+t32)*t12)*q;
  series(simplify(series(simplify(L31bis-L31ter),q=0)),q=0);
```

$$L33ter := \frac{h}{\lambda - q} + (t12 + t22 + t32)\,\lambda + t11 + t21 + t31 \tag{3.9}$$

$$0$$

$$L32ter := \frac{h\,(-(t12 + t22 + t32)\,q + p - t11 - t21 - t31)}{\lambda - q} - (t12\,t22 + t12\,t32$$

$$+ t22\,t32)\,\lambda^2 - (t12\,(t21 + t31) + t22\,(t11 + t31) + t32\,(t11 + t21))\,\lambda + t12\,(-t20$$

$$- t30) + t20\,t22 + t30\,t32 - t11\,t21 - t11\,t31 - t21\,t31 + h\,t12$$

$$0$$

$$L31ter := \frac{1}{\lambda - q}\left(h\left(p^2 - (t11 + t21 + t31 + (t12 + t22 + t32)\,q)\,p + (t12\,t22 + t12\,t32\right.\right.$$
$$+ t22\,t32)\,q^2 + (t12\,(t21 + t31) + t22\,(t11 + t31) + t32\,(t11 + t21))\,q - t12\,(-t20$$
$$- t30) - t20\,t22 - t30\,t32 + t11\,t31 + t11\,t21 + t21\,t31 - h\,t12\big)\big) + t12\,t22\,t32\,\lambda^3$$
$$+ (t11\,t22\,t32 + t12\,t21\,t32 + t12\,t22\,t31)\,\lambda^2 + ((-h + t30)\,t22\,t12 + (-h$$
$$+ t20)\,t32\,t12 + (-t20 - t30)\,t22\,t32 + t12\,t21\,t31 + t22\,t11\,t31 + t32\,t11\,t21)\,\lambda + p^3$$
$$- (t11 + t21 + t31 + (t12 + t22 + t32)\,q)\,p^2 + \big((t12\,t22 + t12\,t32 + t22\,t32)\,q^2$$
$$+ (t12\,(t21 + t31) + t22\,(t11 + t31) + t32\,(t11 + t21))\,q - t12\,(-t20 - t30)$$
$$- t20\,t22 - t30\,t32 + t11\,t21 + t11\,t31 + t21\,t31 - h\,t12\big)\,p - q^3\,t12\,t22\,t32$$
$$- (t11\,t22\,t32 + t12\,t21\,t32 + t12\,t22\,t31)\,q^2 - (t12\,t21\,t31 + t22\,t11\,t31 + t32\,t11\,t21$$
$$+ (-t20 - t30)\,t22\,t32 + t12\,t20\,t32 + t12\,t22\,t30 - h\,(t22 + t32)\,t12)\,q$$
$$0$$

```
> G:=simplify(G):
  simplify(G[1,1]);
  simplify(G[1,2]);
  simplify(G[1,3]);
  simplify(G[2,1]);
  simplify(G[2,2]);
  simplify(G[2,3]);
  simplify(G[3,3]);
  simplify(G[3,2]);
  simplify(G[3,1]):
  G31bis:=(t12*lambda+t11)^2-p^2+(q*(t22+t32)+t21+t31)*p- t22*
  t32*q^2-(t22*t31+t32*t21)*q- t21*t31+t10*t12+t20*t22+t30*t32+h*
  t12;
  G33bis:=a13*( (t12+t32)*lambda-p+t22*q+t11+t21+t31);
  simplify(G[3,3]-G33bis);
  G32bis:=a12*( (t12+t22)*lambda-p+t32*q+t11+t21+t31);
  simplify(G[3,2]-G32bis);
  series(simplify(G[3,1]-G31bis),q=0);
```

$$\tag{3.10}$$
$$1$$
$$0$$
$$0$$
$$\lambda\,t12 + t11$$
$$a12$$
$$a13$$
$$-a13\,((-t12 - t32)\,\lambda - q\,t22 + p - t11 - t21 - t31)$$
$$-((-t12 - t22)\,\lambda - q\,t32 + p - t11 - t21 - t31)\,a12$$

$$G31bis := (\lambda\,t12 + t11)^2 - p^2 + (q\,(t22 + t32) + t21 + t31)\,p - q^2\,t22\,t32 - (t21\,t32$$
$$+ t22\,t31)\,q - t21\,t31 + t12\,(-t20 - t30) + t20\,t22 + t30\,t32 + h\,t12$$
$$G33bis := a13\,((t12 + t32)\,\lambda - p + q\,t22 + t11 + t21 + t31)$$
$$0$$
$$G32bis := a12\,((t12 + t22)\,\lambda - p + q\,t32 + t11 + t21 + t31)$$

$$\begin{matrix} 0 \\ 0 \end{matrix}$$

```
> Gnorm:=simplify(Gnorm);
  J:=simplify(J);
  simplify(Gnorm-Multiply(G^(-1),J^(-1)));
  simplify(G^(-1)-Multiply(Gnorm,J));
  J[3,3];
  J[3,2];
  J[3,1];
  J31bis:=1/(lambda-q)*(p^2-p*((t12+t22+t32)*q+t31+t11+t21) +
  (t12*t22+t12*t32+t22*t32)*q^2 + (t12*(t21+t31)+ t22*(t11+t31)+
  t32*(t21+t11))*q -t12*t10-t22*t20-t32*t30+t11*t21+t11*t31+t21*
  t31-h*t12):
  series(simplify((lambda-q)*(J[3,1]-J31bis)),q=0);
```

$$\left[\left[1,0,0\right],\right.$$ (3.11)

$$\left[\frac{\left(\left(\lambda+q\right)t32+q\,t22-p+t21+t31\right)t12+t11\,t32}{a12\,(t22-t32)},\frac{-t12-t32}{a12\,(t22-t32)},\right.$$

$$\left.\frac{1}{a12\,(t22-t32)}\right],$$

$$\left[\frac{\left(\left(-\lambda-q\right)t22-q\,t32+p-t21-t31\right)t12-t11\,t22}{a13\,(t22-t32)},\frac{t12+t22}{a13\,(t22-t32)},\right.$$

$$\left.\left.-\frac{1}{a13\,(t22-t32)}\right]\right]$$

$$\left[\left[1,0,0\right],\right.$$

$$\left[0,1,0\right],$$

$$\left[\frac{1}{\lambda-q}\left(\left(\left(t22+t32\right)t12+t22\,t32\right)q^2+\left(\left(t31-p+t21\right)t12+\left(-t22-t32\right)p\right.\right.\right.$$

$$\left.+t22\left(t11+t31\right)+t32\left(t11+t21\right)\right)q+\left(-h+t20+t30\right)t12+p^2+\left(-t31-t11\right.$$

$$\left.\left.-t21\right)p-t20\,t22-t30\,t32+t11\left(t21+t31\right)+t21\,t31\right),$$

$$\left.\left.\frac{\left(t12+t22+t32\right)q-p+t11+t21+t31}{-\lambda+q},\frac{1}{\lambda-q}\right]\right]$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\frac{1}{\lambda - q}$$

$$\frac{(t12 + t22 + t32)\, q - p + t11 + t21 + t31}{-\lambda + q}$$

$$\frac{1}{\lambda - q}\Big(\big(((t22 + t32)\, t12 + t22\, t32)\, q^2 + ((t31 - p + t21)\, t12 + (-t22 - t32)\, p$$
$$+ t22\,(t11 + t31) + t32\,(t11 + t21))\, q + (-h + t20 + t30)\, t12 + p^2 + (-t31 - t11$$
$$- t21)\, p - t20\, t22 - t30\, t32 + t11\,(t21 + t31) + t21\, t31 \Big)$$

$$0$$

Definition of P_1, P_2 and P_3 and math with the theoretical formulas for L

```
> P1:=unapply( (t12+t22+t32)*lambda+t11+t21+t31,lambda);
  P2:=unapply( (t12*t22+t12*t32+t22*t32)*lambda^2+((t21+t31)*t12+
  (t11+t31)*t22+t32*(t11+t21))*lambda-t10*t12-t20*t22-t30*t32+
  t21*t11 +t31*t11+t21*t31,lambda);
  P3:=unapply(t12*t22*t32*lambda^3+(t12*t22*t31+t12*t32*t21+t22*
  t32*t11)*lambda^2+(t12*t22*t30+t12*t32*t20+t22*t32*t10  +t12*
  t21*t31+t22*t11*t31+t32*t11*t21)*lambda,lambda);
  L:=simplify(L):
  simplify(L[3,3]-h/(lambda-q)-P1(lambda));
  simplify(L[3,2]-(h*(p-P1(q))/(lambda-q)-P2(lambda)+h*t12));
  series( simplify(L[3,1]-(h*(p^2-P1(q)*p+P2(q)-h*t12)/(lambda-q)
  +p^3-P1(q)*p^2+(P2(q)-h*t12)*p-P3(q)+P3(lambda)-h*(t22+t32)*
  t12*(lambda-q) )),lambda=0);
  simplify(J31bis-(p^2-P1(q)*p+P2(q)-h*t12)/(lambda-q));
```

$$P1 := \lambda \rightarrow (t12 + t22 + t32)\,\lambda + t11 + t21 + t31 \tag{3.12}$$

$$P2 := \lambda \rightarrow (t12\, t22 + t12\, t32 + t22\, t32)\,\lambda^2 + (t12\,(t21 + t31) + t22\,(t11 + t31) + t32\,(t11 + t21))\,\lambda - t12\,(-t20 - t30) - t20\, t22 - t30\, t32 + t11\, t21 + t11\, t31 + t21\, t31$$

$$P3 := \lambda \rightarrow t12\, t22\, t32\, \lambda^3 + (t11\, t22\, t32 + t12\, t21\, t32 + t12\, t22\, t31)\,\lambda^2 + ((-t20 - t30)\, t22\, t32 + t32\, t11\, t21 + t22\, t11\, t31 + t12\, t20\, t32 + t12\, t21\, t31 + t12\, t22\, t30)\,\lambda$$

$$0$$
$$0$$
$$0$$
$$0$$

```
> Gnormalized:=Matrix(3,3,0):
  Gnormalized[1,1]:=1:
  Gnormalized[2,2]:=1:
  Gnormalized[3,3]:=1:
  Gnormalized[3,1]:=(t12-t32)/a13*lambda:
  Gnormalized;
```

```
dGnormalizeddlambda:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do dGnormalizeddlambda
[i,j]:=diff(Gnormalized[i,j],lambda): od: od:
LGnormalized:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do LGnormalized[i,j]:=
simplify( h*diff(Gnormalized[i,j],t12)*dott12+h*diff
(Gnormalized[i,j],t22)*dott22+h*diff(Gnormalized[i,j],t32)*
dott32+h*diff(Gnormalized[i,j],t11)*dott11+h*diff(Gnormalized
[i,j],t21)*dott21+h*diff(Gnormalized[i,j],t31)*dott31+h*diff
(Gnormalized[i,j],a12)*dota12+h*diff(Gnormalized[i,j],a13)*
dota13):
od: od:
Lnew:=simplify(Multiply(Multiply(Gnormalized,tdL),Gnormalized^
(-1))+h*Multiply(dGnormalizeddlambda,Gnormalized^(-1))):

Multiply(dGnormalizeddlambda,Gnormalized^(-1));
Multiply(Gnormalized^(-1),dGnormalizeddlambda);
simplify(tdL[1,2]*tdL[2,3]-tdL[2,2]*tdL[1,3]);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \dfrac{(t12 - t32)\,\lambda}{a13} & 0 & 1 \end{bmatrix}$$

(3.13)

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \dfrac{t12 - t32}{a13} & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \dfrac{t12 - t32}{a13} & 0 & 0 \end{bmatrix}$$

$$-((\lambda - q)\,t22 + p)\,a13$$

```
> GeneralSpectralCurve:=Determinant(y*IdentityMatrix(3)-tdL);
```

$GeneralSpectralCurve := -\lambda^3\, t12\, t22\, t32 + q^3\, t12\, t22\, t32 - \lambda^2\, t11\, t22\, t32 - \lambda^2\, t12\, t21\, t32$ (3.14)
$\quad - \lambda^2\, t12\, t22\, t31 + \lambda^2\, t12\, t22\, y + \lambda^2\, t12\, t32\, y + \lambda^2\, t22\, t32\, y - p\, q^2\, t12\, t22 - p\, q^2\, t12\, t32$
$\quad - p\, q^2\, t22\, t32 + q^2\, t11\, t22\, t32 + q^2\, t12\, t21\, t32 + q^2\, t12\, t22\, t31 - \lambda\, t11\, t21\, t32$
$\quad - \lambda\, t11\, t22\, t31 + \lambda\, t11\, t22\, y + \lambda\, t11\, t32\, y - \lambda\, t12\, t20\, t32 - \lambda\, t12\, t21\, t31 + \lambda\, t12\, t21\, y$
$\quad - \lambda\, t12\, t22\, t30 + \lambda\, t12\, t31\, y - \lambda\, t12\, y^2 + \lambda\, t20\, t22\, t32 + \lambda\, t21\, t32\, y + \lambda\, t22\, t30\, t32$
$\quad + \lambda\, t22\, t31\, y - \lambda\, t22\, y^2 - \lambda\, t32\, y^2 + p^2\, q\, t12 + p^2\, q\, t22 + p^2\, q\, t32 - p\, q\, t11\, t22$
$\quad - p\, q\, t11\, t32 - p\, q\, t12\, t21 - p\, q\, t12\, t31 - p\, q\, t21\, t32 - p\, q\, t22\, t31 + q\, t11\, t21\, t32$
$\quad + q\, t11\, t22\, t31 + q\, t12\, t20\, t32 + q\, t12\, t21\, t31 + q\, t12\, t22\, t30 - q\, t20\, t22\, t32$

$$-q\,t22\,t30\,t32 - p^3 + p^2\,t11 + p^2\,t21 + p^2\,t31 - p\,t11\,t21 - p\,t11\,t31 - p\,t12\,t20$$
$$- p\,t12\,t30 + p\,t20\,t22 - p\,t21\,t31 + p\,t30\,t32 + t11\,t21\,y + t11\,t31\,y - t11\,y^2$$
$$+ t12\,t20\,y + t12\,t30\,y - t20\,t22\,y + t21\,t31\,y - t21\,y^2 - t30\,t32\,y - t31\,y^2 + y^3$$

Jimbo-Miwa-Ueno tau function

```
> ExpLambdaInfinity:=Matrix(3,3,0):
  ExpLambdaInfinity[1,1]:=exp((t11*lambda+t12/2*lambda^2)/h):
  ExpLambdaInfinity[2,2]:=exp((t21*lambda+t22/2*lambda^2)/h):
  ExpLambdaInfinity[3,3]:=exp((t31*lambda+t32/2*lambda^2)/h):
  ExpLambdaInfinity;

  LambdaInfinity:=Matrix(3,3,0):
  LambdaInfinity[1,1]:=t11*lambda+t12/2*lambda^2:
  LambdaInfinity[2,2]:=t21*lambda+t22/2*lambda^2:
  LambdaInfinity[3,3]:=t31*lambda+t32/2*lambda^2:
  LambdaInfinity:

  dLambdaInfinitydlambda:=Matrix(3,3,0):
  for i from 1 to 3 do for j from 1 to 3 do
  dLambdaInfinitydlambda[i,j]:=diff(LambdaInfinity[i,j],lambda):
  od: od:
  dLambdaInfinitydlambda:

  ExpMonodromies:=Matrix(3,3,0):
  ExpMonodromies[1,1]:=exp(t10*ln(lambda)/h):
  ExpMonodromies[2,2]:=exp(t20*ln(lambda)/h):
  ExpMonodromies[3,3]:=exp(t30*ln(lambda)/h):
  ExpMonodromies;

  Monodromies:=Matrix(3,3,0):
  Monodromies[1,1]:=t10/lambda:
  Monodromies[2,2]:=t20/lambda:
  Monodromies[3,3]:=t30/lambda:
  Monodromies:

  F1:=Matrix(3,3,0):
  F2:=Matrix(3,3,0):
  F3:=Matrix(3,3,0):
  F4:=Matrix(3,3,0):
  FF5:=Matrix(3,3,0):
  for i from 1 to 3 do for j from 1 to 3 do F1[i,j]:=f1[i,j]:
  F2[i,j]:=f2[i,j]:
  F3[i,j]:=f3[i,j]:
```

```
F4[i,j]:=f4[i,j]:
FF5[i,j]:=ff5[i,j]:
od: od:


HatPsiReg:=IdentityMatrix(3)+F1/lambda+F2/lambda^2+F3/lambda^3+
F4/lambda^4+ FF5/lambda^5:
tdPsi:=Multiply(Multiply
(HatPsiReg,ExpLambdaInfinity),ExpMonodromies):

dHatPsiRegdlambda:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do dHatPsiRegdlambda[i,
j]:=diff(HatPsiReg[i,j],lambda): od: od:

dtdPsidlambda:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do dtdPsidlambda[i,j]:=
diff(tdPsi[i,j],lambda): od: od:

dLambdaInfinitydt:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do dLambdaInfinitydt[i,
j]:= simplify(diff(LambdaInfinity[i,j],t11)*dt11+diff
(LambdaInfinity[i,j],t21)*dt21+diff(LambdaInfinity[i,j],t31)*
dt31+diff(LambdaInfinity[i,j],t12)*dt12+diff(LambdaInfinity[i,
j],t22)*dt22+diff(LambdaInfinity[i,j],t32)*dt32): od: od:
dLambdaInfinitydt:

omegaJMU:=-h*residue(Trace(Multiply(Multiply(HatPsiReg^
(-1),dHatPsiRegdlambda), dLambdaInfinitydt)),lambda=infinity);
```

$$
\begin{bmatrix}
e^{\dfrac{\lambda t11 + \frac{1}{2} t12\lambda^2}{h}} & 0 & 0 \\[2em]
0 & e^{\dfrac{t21\lambda + \frac{1}{2} t22\lambda^2}{h}} & 0 \\[2em]
0 & 0 & e^{\dfrac{t31\lambda + \frac{1}{2}\lambda^2 t32}{h}}
\end{bmatrix}
\tag{3.15}
$$

$$
\begin{bmatrix}
e^{\frac{(-t20 - t30)\ln(\lambda)}{h}} & 0 & 0 \\
0 & e^{\frac{t20\ln(\lambda)}{h}} & 0 \\
0 & 0 & e^{\frac{t30\ln(\lambda)}{h}}
\end{bmatrix}
$$

$$
\begin{aligned}
omegaJMU := {}& -h\left(f1_{1,1}\, dt11 - \frac{1}{2}\left(f1_{1,1}^2 + f1_{1,2}f1_{2,1} + f1_{1,3}f1_{3,1} - 2 f2_{1,1}\right) dt12 \right. \\
& + f1_{2,2}\, dt21 - \frac{1}{2}\left(f1_{1,2}f1_{2,1} + f1_{2,2}^2 + f1_{2,3}f1_{3,2} - 2 f2_{2,2}\right) dt22 + f1_{3,3}\, dt31 \\
& \left. - \frac{1}{2}\left(f1_{1,3}f1_{3,1} + f1_{2,3}f1_{3,2} + f1_{3,3}^2 - 2 f2_{3,3}\right) dt32 \right)
\end{aligned}
$$

```
> ToCancel0:=simplify(h*dtdPsidlambda-Multiply(tdL,tdPsi)):
> Cancel11:=series(simplify(series( simplify( ToCancel0[1,1]*exp
  (-((-2*t20-2*t30)*ln(lambda)+t12*lambda^2+2*lambda*t11)/(2*h))
  )),lambda=infinity,10)),lambda=infinity);
  f1[3,1]:=solve(residue(Cancel11 ,lambda=infinity),f1[3,1]);
  f2[3,1]:=solve(residue(Cancel11*lambda ,lambda=infinity),f2[3,
  1]);
  f3[3,1]:=solve(residue(Cancel11*lambda^2 ,lambda=infinity),f3
  [3,1]);
```

$$
Cancel11 := \frac{-a12\, f1_{2,1} - a13\, f1_{3,1} - t20 - t30}{\lambda}
$$
(3.16)
$$
+ \frac{-a12\, f2_{2,1} - a13\, f2_{3,1} - h f1_{1,1} - t20 f1_{1,1} - t30 f1_{1,1}}{\lambda^2}
$$
$$
+ \frac{-a12\, f3_{2,1} - a13\, f3_{3,1} - 2 h f2_{1,1} - t20 f2_{1,1} - t30 f2_{1,1}}{\lambda^3}
$$
$$
+ \frac{-a12\, f4_{2,1} - a13\, f4_{3,1} - 3 h f3_{1,1} - t20 f3_{1,1} - t30 f3_{1,1}}{\lambda^4}
$$
$$
+ \frac{-a12\, ff5_{2,1} - a13\, ff5_{3,1} - 4 h f4_{1,1} - t20 f4_{1,1} - t30 f4_{1,1}}{\lambda^5} + O\left(\frac{1}{\lambda^6}\right)
$$

$$
f1_{3,1} := -\frac{a12\, f1_{2,1} + t20 + t30}{a13}
$$
$$
f2_{3,1} := -\frac{a12\, f2_{2,1} + h f1_{1,1} + t20 f1_{1,1} + t30 f1_{1,1}}{a13}
$$
$$
f3_{3,1} := -\frac{a12\, f3_{2,1} + 2 h f2_{1,1} + t20 f2_{1,1} + t30 f2_{1,1}}{a13}
$$

```
> Cancel12:=series(simplify(series( simplify( ToCancel0[1,2]*exp
  (-(lambda^2*t22+2*t20*ln(lambda)+2*lambda*t21)/(2*h))
  )),lambda=infinity,10)),lambda=infinity):
```

```
f1[1,2]:=solve(residue(Cancel12/lambda,lambda=infinity),f1[1,2]
);
f2[1,2]:=solve(residue(Cancel12,lambda=infinity),f2[1,2]);
f3[1,2]:=solve(residue(Cancel12*lambda,lambda=infinity),f3[1,2]
);
```

$$f1_{1,2} := -\frac{a12}{t12-t22} \tag{3.17}$$

$$f2_{1,2} := \frac{1}{(t12-t22)^2}\left(-a12\,t12\,f1_{2,2}+a12\,t22\,f1_{2,2}-a13\,t12\,f1_{3,2}+a13\,t22\,f1_{3,2}\right.$$
$$\left.+a12\,t11-a12\,t21\right)$$

$$f3_{1,2} := \frac{1}{(t12-t22)^3}\left(a12\,t11\,t12\,f1_{2,2}-a12\,t11\,t22\,f1_{2,2}-a12\,t12^2\,f2_{2,2}\right.$$
$$-a12\,t12\,t21\,f1_{2,2}+2\,a12\,t12\,t22\,f2_{2,2}+a12\,t21\,t22\,f1_{2,2}-a12\,t22^2\,f2_{2,2}$$
$$+a13\,t11\,t12\,f1_{3,2}-a13\,t11\,t22\,f1_{3,2}-a13\,t12^2\,f2_{3,2}-a13\,t12\,t21\,f1_{3,2}$$
$$+2\,a13\,t12\,t22\,f2_{3,2}+a13\,t21\,t22\,f1_{3,2}-a13\,t22^2\,f2_{3,2}+a12\,h\,t12-a12\,h\,t22$$
$$\left.-a12\,t11^2+2\,a12\,t11\,t21-a12\,t12\,t20+a12\,t20\,t22-a12\,t21^2\right)$$

```
> Cancel13:=series(simplify(series( simplify( ToCancel0[1,3]*exp
  (-(lambda^2*t32+2*t30*ln(lambda)+2*lambda*t31)/(2*h))
    )),lambda=infinity,10)),lambda=infinity);
  f1[1,3]:=solve(residue(Cancel13/lambda,lambda=infinity),f1[1,3]
  );
  f2[1,3]:=solve(residue(Cancel13,lambda=infinity),f2[1,3]);
  f3[1,3]:=solve(residue(Cancel13*lambda,lambda=infinity),f3[1,3]
  );
```

$$Cancel13 := (-t12+t32)\,f1_{1,3}-a13 \tag{3.18}$$
$$+\frac{(-t11+t31)\,f1_{1,3}+(-t12+t32)\,f2_{1,3}-a12\,f1_{2,3}-f1_{3,3}\,a13}{\lambda}$$
$$+\frac{(-h+t30)\,f1_{1,3}+(-t11+t31)\,f2_{1,3}+(-t12+t32)\,f3_{1,3}-a12\,f2_{2,3}-f2_{3,3}\,a13}{\lambda^2}$$
$$+\frac{1}{\lambda^3}\left((-2\,h+t30)\,f2_{1,3}+(-t11+t31)\,f3_{1,3}+(-t12+t32)\,f4_{1,3}-a12\,f3_{2,3}\right.$$
$$\left.-f3_{3,3}\,a13\right)$$
$$+\frac{1}{\lambda^4}\left((-t12+t32)\,ff5_{1,3}+(-3\,h+t30)\,f3_{1,3}+(-t11+t31)\,f4_{1,3}-a12\,f4_{2,3}\right.$$
$$\left.-f4_{3,3}\,a13\right)+\frac{(-t11+t31)\,ff5_{1,3}+(-4\,h+t30)\,f4_{1,3}-a12\,ff5_{2,3}-a13\,ff5_{3,3}}{\lambda^5}$$
$$+O\left(\frac{1}{\lambda^6}\right)$$

$$f1_{1,3} := -\frac{a13}{t12-t32}$$

$$f2_{1,3} :=$$
$$-\frac{a12\,t12\,f1_{2,3} - a12\,t32\,f1_{2,3} + a13\,t12\,f1_{3,3} - a13\,t32\,f1_{3,3} - a13\,t11 + a13\,t31}{(t12 - t32)^2}$$

$$f3_{1,3} := \frac{1}{(t12 - t32)^3}\left(a12\,t11\,t12\,f1_{2,3} - a12\,t11\,t32\,f1_{2,3} - a12\,t12^2\,f2_{2,3}\right.$$
$$- a12\,t12\,t31\,f1_{2,3} + 2\,a12\,t12\,t32\,f2_{2,3} + a12\,t31\,t32\,f1_{2,3} - a12\,t32^2\,f2_{2,3}$$
$$+ a13\,t11\,t12\,f1_{3,3} - a13\,t11\,t32\,f1_{3,3} - a13\,t12^2\,f2_{3,3} - a13\,t12\,t31\,f1_{3,3}$$
$$+ 2\,a13\,t12\,t32\,f2_{3,3} + a13\,t31\,t32\,f1_{3,3} - a13\,t32^2\,f2_{3,3} + a13\,h\,t12 - a13\,h\,t32$$
$$\left. - a13\,t11^2 + 2\,a13\,t11\,t31 - a13\,t12\,t30 + a13\,t30\,t32 - a13\,t31^2\right)$$

```
> Cancel21:=series(simplify(series( simplify( ToCancel0[2,1]*exp
  (-((-2*t20-2*t30)*ln(lambda)+lambda^2*t12+2*lambda*t11)/(2*h)
  )),lambda=infinity,10)),lambda=infinity):
  f1[2,1]:=simplify(solve(  residue(Cancel21/lambda,lambda=
  infinity),f1[2,1]));
  f2[2,1]:=simplify(solve(  residue(Cancel21,lambda=infinity),f2
  [2,1]));
  f3[2,1]:=simplify(solve(  residue(Cancel21*lambda,lambda=
  infinity),f3[2,1])):
```

$$f1_{2,1} := \frac{1}{a12\,(t22 - t32)}\left(\left(q^2\,t32 - p\,q + q\,t31 - t20\right)t22 + \left(-p\,q + q\,t21 + t20\right)t32\right. \tag{3.19}$$
$$\left. + (p - t31)\,(p - t21)\right)$$

$$f2_{2,1} := \frac{1}{a12\,(-t22 + t32)\,(t12 - t22)}\left(\left(-q\,\left(f1_{1,1} + q\right)p + t32\,q^3 + \left(t32\,f1_{1,1}\right.\right.\right.$$
$$\left.+ t31\right)q^2 + \left(t31\,f1_{1,1} + t30\right)q - f1_{1,1}\,t20\right)t22^2 + \left(\left(f1_{1,1} + 2\,q\right)p^2 + \left(-2\,q^2\,t32\right.\right.$$
$$\left.+ \left((t12 - t32)\,f1_{1,1} - 2\,t31 - t11 - t21\right)q + (-t31 - t21)\,f1_{1,1} - t30\right)p$$
$$- t32\,\left(t12\,f1_{1,1} - t11 - t21\right)q^2 + \left(\left(-t12\,t31 + t21\,t32\right)f1_{1,1} - t30\,t32 + t31\,(t11\right.$$
$$\left.+ t21)\right)q + (t12\,t20 + t20\,t32 + t21\,t31)\,f1_{1,1} + t21\,(t20 + t30) - t11\,t20\right)t22 - p^3$$
$$+ \left(q\,t32 - t12\,f1_{1,1} + t11 + t21 + t31\right)p^2 + \left(t32\,\left(t12\,f1_{1,1} - t11 - t21\right)q + t12\,(t21\right.$$
$$\left.+ t31)\,f1_{1,1} + t30\,t32 + (-t11 - t31)\,t21 - t11\,t31\right)p - t21\,t32\,\left(t12\,f1_{1,1} - t11\right)q$$
$$\left.- t12\,(t20\,t32 + t21\,t31)\,f1_{1,1} + \left((-t20 - t30)\,t21 + t11\,t20\right)t32 + t11\,t21\,t31\right)$$

```
> Cancel22:=series(simplify(series( simplify( ToCancel0[2,2]*exp
  (-(lambda^2*t22+2*t20*ln(lambda)+2*lambda*t21)/(2*h))
     )),lambda=infinity,10)),lambda=infinity):
  f1[3,2]:=simplify(solve(residue(Cancel22,lambda=infinity),f1[3,
  2]));
  f2[3,2]:=simplify(solve(residue(Cancel22*lambda,lambda=
  infinity),f2[3,2]));
  f3[3,2]:=simplify(solve(residue(Cancel22*lambda^2,lambda=
  infinity),f3[3,2])):
```

$$\tag{3.20}$$

$$fl_{3,2} := -\frac{a12\,(-q\,t32 + p - t31)}{a13\,(t22 - t32)} \tag{3.20}$$

$$\begin{aligned}
f2_{3,2} := -\Big(a12\,\Big(fl_{2,2}\,(-q^2\,t32 + (p-t31)\,q + h)\,t22^3 + \Big(q^2\,(fl_{2,2} - q)\,t32^2 \\
+ \Big((t12fl_{2,2} + 2p - t11 + t21 - 2t31)\,q^2 + (-t21fl_{2,2} + t31fl_{2,2} + t20)\,q \\
- 2\,hfl_{2,2}\Big)\,t32 - (-p + t31)\,(-t12fl_{2,2} - p + t11 - t21 + t31)\,q - fl_{2,2}\,p^2 \\
+ (t21fl_{2,2} + t31fl_{2,2} - t20)\,p + (-t31fl_{2,2} - t20)\,t21 - hfl_{2,2}\,t12 + t20\,(t11 \\
+ t31)\Big)\,t22^2 + \Big((t12\,q^3 + (-t12fl_{2,2} + p + t11 - 2t21)\,q^2 + (-pfl_{2,2} + t21fl_{2,2} \\
- t20)\,q + hfl_{2,2}\Big)\,t32^2 + \Big(2\,t12\,(-p + t31)\,q^2 + (-2\,p^2 + (2\,t31 + 2\,t21)\,p + t21^2 \\
+ (t12fl_{2,2} - t11 - 3t31)\,t21 + (-t31fl_{2,2} - t20)\,t12 + t11\,t31\Big)\,q + fl_{2,2}\,p^2 + \Big(\\
-t21fl_{2,2} - t31fl_{2,2} + t20\Big)\,p + (t31fl_{2,2} + 2t20)\,t21 + 2\,hfl_{2,2}\,t12 - t20\,(t31 \\
+ 2t11)\Big)\,t32 + (-p + t31)\,\Big(t12\,(-p + t31)\,q - p^2 + (-t12fl_{2,2} + t11 + t31)\,p \\
+ t21^2 + (t12fl_{2,2} - t11 - t31)\,t21 - t20\,t12\Big)\Big)\,t22 + \Big(-(p - t21)\,t12\,q^2 \\
+ \Big((t12fl_{2,2} - t11 + t21)\,p - t21^2 + (-t12fl_{2,2} + t11)\,t21 + t20\,t12\Big)\,q \\
- hfl_{2,2}\,t12 + t11\,t20 - t21\,t20\Big)\,t32^2 + (-p + t31)\,\Big(-2\,(p - t21)\,t12\,q + (t12fl_{2,2} \\
- t11 + t21)\,p - t21^2 + (-t12fl_{2,2} + t11)\,t21 + t20\,t12\Big)\,t32 - t12\,(-p + t31)^2\,(p \\
- t21)\Big)\Big)\Big/\Big((t22 - t32)^2\,(-t12 + t22)\,a13\,(q\,t22 - p + t21)\Big)
\end{aligned}$$

```
> Cancel23:=series(simplify(series( simplify( ToCancel0[2,3]*exp
  (-(lambda^2*t32+2*t30*ln(lambda)+2*lambda*t31)/(2*h))
     )),lambda=infinity,10)),lambda=infinity):
  f1[2,3]:=simplify(solve( residue(Cancel23/lambda,lambda=
  infinity),f1[2,3]));
  f2[2,3]:=simplify(solve( residue(Cancel23,lambda=infinity),f2
  [2,3]));
  f3[2,3]:=simplify(solve( residue(Cancel23*lambda,lambda=
  infinity),f3[2,3])):
```

$$fl_{2,3} := \frac{a13\,(-q\,t22 + p - t21)}{a12\,(t22 - t32)} \tag{3.21}$$

$$\begin{aligned}
f2_{2,3} := -\frac{1}{a12\,(-t12 + t32)\,(-t22 + t32)^2}\,\Big(a13\,\Big(\big(q\,(fl_{3,3} - q)\,t32 - fl_{3,3}\,q\,t12 + p\,q \\
- q\,t31 + t20\big)\,t22^2 + \Big(-t32^2 fl_{3,3}\,q + (q\,(fl_{3,3} + q)\,t12 + (-fl_{3,3} + q)\,p + (\\
-2\,t21 + t31)\,q + fl_{3,3}\,t21 - t20\big)\,t32 + \big((fl_{3,3} - q)\,p - fl_{3,3}\,t21 + q\,t21 - t20\big)\,t12 \\
- (p - t31)\,(p - t21)\big)\,t22 + fl_{3,3}\,(p - t21)\,t32^2 + \big(\big((-fl_{3,3} - q)\,p + fl_{3,3}\,t21 \\
+ q\,t21 + t20\big)\,t12 + (t21 - t31)\,(p - t21)\big)\,t32 + t12\,(p - t21)^2\Big)\Big)
\end{aligned}$$

```
> Cancel31:=series(simplify(series( simplify( ToCancel0[3,1]*exp
  (-((-2*t20-2*t30)*ln(lambda)+lambda^2*t12+2*lambda*t11)/(2*h)
  )),lambda=infinity,10)),lambda=infinity):
  f1[1,1]:=simplify(solve(simplify(residue(Cancel31,lambda=
  infinity)),f1[1,1]));
```

```
f2[1,1]:=simplify(solve(simplify(residue(Cancel31*lambda,
lambda=infinity)),f2[1,1])):
f3[1,1]:=simplify(solve(simplify(residue(Cancel31*lambda^2,
lambda=infinity)),f3[1,1])):
```

$$f1_{1,\,1} := \frac{1}{(t12 - t22)\,(-t12 + t32)\,h}\Big(p^3 + \big((-t12 - t22 - t32)\,q - t31 - t11 - t21\big)\,p^2 \qquad \textbf{(3.22)}$$
$$+ \Big(\big((t22 + t32)\,t12 + t22\,t32\big)\,q^2 + \big(t12\,(t21 + t31) + t22\,(t11 + t31) + t32\,(t11$$
$$+ t21)\big)\,q + (t20 + t30)\,t12 - t20\,t22 - t30\,t32 + t11\,(t21 + t31) + t21\,t31\Big)\,p$$
$$- q^3\,t12\,t22\,t32 + \big((-t21\,t32 - t22\,t31)\,t12 - t22\,t32\,t11\big)\,q^2 + \big((-t20\,t32 - t21\,t31$$
$$- t22\,t30)\,t12 + \big((t20 + t30)\,t32 - t11\,t31\big)\,t22 - t32\,t11\,t21\big)\,q - (t20 + t30)\,(t31$$
$$- t11 + t21)\,t12 + (-t11\,t30 + t31\,(t20 + t30))\,t22 + (t21\,(t20 + t30) - t11\,t20)\,t32$$
$$- t11\,t21\,t31\Big)$$

```
> Cancel32:=series(simplify(series( simplify( ToCancel0[3,2]*exp
  (-(lambda^2*t22+2*t20*ln(lambda)+2*lambda*t21)/(2*h))
      ),lambda=infinity,10)),lambda=infinity):
  f1[2,2]:=solve(simplify(residue(Cancel32,lambda=infinity)),f1
  [2,2]);
  f2[2,2]:=solve(simplify(residue(Cancel32*lambda,lambda=
  infinity)),f2[2,2]):
  f3[2,2]:=solve(simplify(residue(Cancel32*lambda^2,lambda=
  infinity)),f3[2,2]):
```

$$f1_{2,\,2} := \frac{1}{(t12 - t22)\,(t22 - t32)\,h}\Big(-q^3\,t12\,t22\,t32 + p\,q^2\,t12\,t22 + p\,q^2\,t12\,t32 \qquad \textbf{(3.23)}$$
$$+ p\,q^2\,t22\,t32 - q^2\,t11\,t22\,t32 - q^2\,t12\,t21\,t32 - q^2\,t12\,t22\,t31 - p^2\,q\,t12 - p^2\,q\,t22$$
$$- p^2\,q\,t32 + p\,q\,t11\,t22 + p\,q\,t11\,t32 + p\,q\,t12\,t21 + p\,q\,t12\,t31 + p\,q\,t21\,t32$$
$$+ p\,q\,t22\,t31 - q\,t11\,t21\,t32 - q\,t11\,t22\,t31 - q\,t12\,t20\,t32 - q\,t12\,t21\,t31$$
$$- q\,t12\,t22\,t30 + q\,t20\,t22\,t32 + q\,t22\,t30\,t32 + p^3 - p^2\,t11 - p^2\,t21 - p^2\,t31$$
$$+ p\,t11\,t21 + p\,t11\,t31 + p\,t12\,t20 + p\,t12\,t30 - p\,t20\,t22 + p\,t21\,t31 - p\,t30\,t32$$
$$+ t11\,t20\,t22 - t11\,t20\,t32 - t11\,t21\,t31 - t12\,t20\,t31 - t12\,t21\,t30 - t20\,t21\,t22$$
$$+ t20\,t21\,t32 + t20\,t22\,t31 + t21\,t30\,t32\Big)$$

```
> Cancel33:=series(simplify(series( simplify( ToCancel0[3,3]*exp
  (-(lambda^2*t32+2*t30*ln(lambda)+2*lambda*t31)/(2*h))
      ),lambda=infinity,10)),lambda=infinity):
  f1[3,3]:=solve(simplify(residue(Cancel33*lambda,lambda=
  infinity)),f1[3,3]);
  f2[3,3]:=solve(simplify(residue(Cancel33*lambda^2,lambda=
  infinity)),f2[3,3]):
  f3[3,3]:=solve(simplify(residue(Cancel33*lambda^3,lambda=
  infinity)),f3[3,3]):
```

$$f1_{3,\,3} := -\frac{1}{(t22 - t32)\,(t12 - t32)\,h}\Big(-q^3\,t12\,t22\,t32 + p\,q^2\,t12\,t22 + p\,q^2\,t12\,t32 \qquad \textbf{(3.24)}$$
$$+ p\,q^2\,t22\,t32 - q^2\,t11\,t22\,t32 - q^2\,t12\,t21\,t32 - q^2\,t12\,t22\,t31 - p^2\,q\,t12 - p^2\,q\,t22$$
$$- p^2\,q\,t32 + p\,q\,t11\,t22 + p\,q\,t11\,t32 + p\,q\,t12\,t21 + p\,q\,t12\,t31 + p\,q\,t21\,t32$$

$$+ p\,q\,t22\,t31 - q\,t11\,t21\,t32 - q\,t11\,t22\,t31 - q\,t12\,t20\,t32 - q\,t12\,t21\,t31$$
$$- q\,t12\,t22\,t30 + q\,t20\,t22\,t32 + q\,t22\,t30\,t32 + p^3 - p^2\,t11 - p^2\,t21 - p^2\,t31$$
$$+ p\,t11\,t21 + p\,t11\,t31 + p\,t12\,t20 + p\,t12\,t30 - p\,t20\,t22 + p\,t21\,t31 - p\,t30\,t32$$
$$- t11\,t21\,t31 - t11\,t22\,t30 + t11\,t30\,t32 - t12\,t20\,t31 - t12\,t21\,t30 + t20\,t22\,t31$$
$$+ t21\,t30\,t32 + t22\,t30\,t31 - t30\,t31\,t32\big)$$

```
> omegaJMU:=simplify(omegaJMU):

> OmegaJMUdt11:=simplify(residue(omegaJMU/dt11^2,dt11=0)):
  OmegaJMUdt21:=simplify(residue(omegaJMU/dt21^2,dt21=0)):
  OmegaJMUdt31:=simplify(residue(omegaJMU/dt31^2,dt31=0)):

  OmegaJMUdt12:=simplify(residue(omegaJMU/dt12^2,dt12=0)):
  OmegaJMUdt22:=simplify(residue(omegaJMU/dt22^2,dt22=0)):
  OmegaJMUdt32:=simplify(residue(omegaJMU/dt32^2,dt32=0)):


  omegaJMUdt11:=1/((t12-t22)*(-t12+t32))*(-p^3+(t11+t21+t31+(t12+
  t22+t32)*q)*p^2+(((-t22-t32)*t12-t22*t32)*q^2+((-t31-t21)*t12+
  (-t11-t31)*t22-t32*(t11+t21))*q+t12*(-t20-t30)+t22*t20+t30*t32+
  (-t31-t21)*t11-t21*t31)*p+q^3*t12*t22*t32+((t21*t32+t22*t31)*
  t12+t22*t32*t11)*q^2+((t20*t32+t21*t31+t22*t30)*t12+((-t20-t30)
  *t32+t11*t31)*t22+t32*t11*t21)*q+(t20+t30)*(t31-t11+t21)*t12+
  (t11*t30-t31*(t20+t30))*t22+(t11*t20-t21*(t20+t30))*t32+t11*
  t21*t31);

  omegaJMUdt21:=1/((t12-t22)*(-t22+t32))*(p^3+((-t12-t22-t32)*q-
  t31-t11-t21)*p^2+(((t12+t32)*t22+t12*t32)*q^2+(t12*(t21+t31)+
  t22*(t11+t31)+t32*(t11+t21))*q-t22*t20-t30*t32+(t20+t30)*t12+
  (t11+t31)*t21+t11*t31)*p-q^3*t12*t22*t32+((-t11*t32-t12*t31)*
  t22-t32*t12*t21)*q^2+(((t20+t30)*t32-t11*t31-t12*t30)*t22+(-
  t11*t21-t12*t20)*t32-t12*t21*t31)*q+t20*(t31+t11-t21)*t22+(t21*
  (t20+t30)-t11*t20)*t32+(-t20*t31-t21*t30)*t12-t11*t21*t31);

  omegaJMUdt31:=1/((-t22+t32)*(-t12+t32))*(p^3+((-t12-t22-t32)*q-
  t31-t11-t21)*p^2+(((t12+t22)*t32+t12*t22)*q^2+(t12*(t21+t31)+
  t22*(t11+t31)+t32*(t11+t21))*q-t30*t32-t22*t20+(t20+t30)*t12+
  t31*(t11+t21)+t11*t21)*p-q^3*t12*t22*t32+((-t11*t22-t12*t21)*
  t32-t31*t12*t22)*q^2+(((t20+t30)*t22-t11*t21-t12*t20)*t32+(-
  t11*t31-t12*t30)*t22-t12*t21*t31)*q-t30*(t31-t11-t21)*t32+(-
  t11*t30+t31*(t20+t30))*t22+(-t20*t31-t21*t30)*t12-t11*t21*t31);

  omegaJMUdt12:=1/(2*(t12-t22)^2*(-t12+t32)^2)*(-(t20+t30)^2*
  t12^3+((-q^3*(t31-2*t11+t21)*t32+2*q*((-p+t31)*q+t30)*t11-(t21+
```

```
t31)*(-p+t31)*q^2-t30*(t21+t31)*q+(t20+2*t30)*(t20+t30))*t22+
(-2*q*((p-t21)*q-t20)*t11+(p-t21)*(t21+t31)*q^2-t20*(t21+t31)*
q+2*t20^2+3*t30*t20+t30^2)*t32+(-t20-t30)*t11^2+(-(2*(p-t21))*
(-p+t31)*q+(2*(t20+t30))*(t31-p+t21))*t11+(p-t21)*(t21+t31)*(-
p+t31)*q-(t20+t30)*((-t31-t21)*p+t31^2+t21*t31+t21^2))*t12^2+(
(q^3*(-t11+t31)*t32-q*((-p+t31)*q+t30)*t11+t31*(-p+t31)*q^2+
t31*t30*q-t30*(t20+t30))*t22^2+(-q^3*(t11-t21)*t32^2+(2*q^2*
t11^2-2*q*((t21+t31)*q+3*t20*(1/2)+3*t30*(1/2))*t11+2*q^2*t21*
t31+((t20+2*t30)*t21+2*t31*(t20+(1/2)*t30))*q-2*(t20+t30)^2)*
t32+((2*t31-2*p)*q+2*t30)*t11^2+(-(-p+t31)*(t31+2*t21+p)*q+
(t30+3*t20)*p-t21*t30-(3*(t20+4*t30*(1/3)))*t31)*t11+(-p^2*t21+
t21*t31^2)*q+(-t21*t20-2*t31*(t20+(1/2)*t30))*p+(2*(t20+t30))*
t31*((1/2)*t21+t31))*t22+(q*((p-t21)*q-t20)*t11-(p-t21)*t21*
q^2+t21*t20*q-t20*(t20+t30))*t32^2+(((-2*p+2*t21)*q+2*t20)*
t11^2+((2*(p-t21))*((1/2)*p+t31+(1/2)*t21)*q+(t20+3*t30)*p+(-4*
t20-3*t30)*t21-t31*t20)*t11+(-p^2*t31+t21^2*t31)*q+((-t20-2*
t30)*t21-t31*t30)*p+t21*(t20+t30)*(t31+2*t21))*t32-(p-t21)*(p-
t11)*(t31-2*t11+t21)*(-p+t31))*t12+((-q^2*t11^2+q*((t31+p)*q+
t20+t30)*t11-q^2*p*t31-(t20+t30)*t31*q+t30*(t20+t30))*t32-(-
t11+t31)*(((p-t31)*q-t30)*t11+p*(-p+t31)*q-p*t20+t31*(t20+t30))
)*t22^2+((-q^2*t11^2+((p+t21)*q+t20+t30)*q*t11-q^2*p*t21-t21*
(t20+t30)*q+t20*(t20+t30))*t32^2+(-q*(t31-2*p+t21)*t11^2+((-2*
p^2+2*t21*t31)*q+(-t20-t30)*p+t31*t20+t21*t30)*t11+((t21+t31)*
p-2*t21*t31)*p*q+(t20*t21+t30*t31)*p-t31*t21*(t20+t30))*t32+(p-
t11)*(-t11+t31)*(-p+t31)*(p-t21))*t22-((((-p+t21)*q+t20)*t11+p*
(p-t21)*q+p*t30-t21*(t20+t30))*t32+(p-t21)*(p-t11)*(-p+t31))*
(t11-t21)*t32);

omegaJMUdt22:=1/(2*(t12-t22)^2*(-t22+t32)^2)*(-t20^2*t22^3+((-
q^3*(t31+t11-2*t21)*t12-2*q*((p-t11)*q+t20+t30)*t21+(p-t11)*
(t11+t31)*q^2+(t20+t30)*(t11+t31)*q+2*t20^2+t30*t20)*t32+(2*q*(
(-p+t31)*q+t30)*t21-(t11+t31)*(-p+t31)*q^2-t30*(t11+t31)*q+t20*
(t20-t30))*t12+t20*t21^2+(-(2*(p-t11))*(-p+t31)*q-2*t20*(t31-p+
t11))*t21+(p-t11)*(t11+t31)*(-p+t31)*q+t20*((-t11-t31)*p+t31^2+
t11*t31+t11^2))*t22^2+((q^3*(t11-t21)*t12+q*((p-t11)*q+t20+t30)
*t21-(p-t11)*t11*q^2-t11*(t20+t30)*q-t20*(t20+t30))*t32^2+(q^3*
(-t21+t31)*t12^2+(2*q^2*t21^2-(2*((t11+t31)*q-3*t20*(1/2)))*q*
t21+2*q^2*t11*t31+(t11*(-t20+t30)-2*t31*(t20+(1/2)*t30))*q-2*
t20^2)*t12+((-2*p+2*t11)*q-2*t20-2*t30)*t21^2+((2*((1/2)*p+t31+
(1/2)*t11))*(p-t11)*q+(2*t30-t20)*p+(t30+4*t20)*t11+t31*(t20+
t30))*t21+(-p^2*t31+t11^2*t31)*q+((t20-t30)*t11-t31*t30)*p-t11*
t20*(t31+2*t11))*t32+(-q*((-p+t31)*q+t30)*t21+t31*(-p+t31)*q^2+
```

```
t31*t30*q+t30*t20)*t12^2+(((2*t31-2*p)*q+2*t30)*t21^2+(-(-p+
t31)*(t31+2*t11+p)*q+(-2*t30-3*t20)*p-t11*t30+3*t31*(t20-(1/3)*
t30))*t21+(-p^2*t11+t11*t31^2)*q+((t20+t30)*t11+2*t31*(t20+
(1/2)*t30))*p-(2*(t31+(1/2)*t11))*t31*t20)*t12-(p-t21)*(p-t11)*
(t31+t11-2*t21)*(-p+t31))*t22+((-q^2*t21^2+((p+t11)*q-t20)*q*
t21-q^2*p*t11+t20*t11*q+t20*(t20+t30))*t12+(t11-t21)*(((-p+t11)
*q-t20-t30)*t21+p*(p-t11)*q+p*t30+t11*t20))*t32^2+((-q^2*t21^2+
q*((t31+p)*q-t20)*t21-q^2*p*t31+t31*t20*q-t30*t20)*t12^2+(-q*
(-2*p+t31+t11)*t21^2+((-2*p^2+2*t11*t31)*q+p*t20+t11*t30-t31*
(t20+t30))*t21+((t11+t31)*p-2*t11*t31)*p*q+((-t20-t30)*t11+t31*
t30)*p+t31*t11*t20)*t12+(t11-t21)*(p-t21)*(p-t11)*(-p+t31))*
t32+t12*((((-p+t31)*q+t30)*t21-p*(-p+t31)*q+(-t20-t30)*p+t31*
t20)*t12+(p-t21)*(p-t11)*(-p+t31))*(-t21+t31));

omegaJMUdt32:=1/(2*(-t12+t32)^2*(-t22+t32)^2)*(-t30^2*t32^3+(
(2*q^3*(t31-(1/2)*t11-(1/2)*t21)*t12-2*q*((p-t11)*q+t20+t30)*
t31+(t11+t21)*(p-t11)*q^2+(t20+t30)*(t11+t21)*q+t30*(t20+2*t30)
)*t22+(-2*q*((p-t21)*q-t20)*t31+(t11+t21)*(p-t21)*q^2-t20*(t11+
t21)*q-t30*(t20-t30))*t12+t30*t31^2+((2*(p-t21))*(p-t11)*q+2*
t30*(p-t11-t21))*t31-(t11+t21)*(p-t21)*(p-t11)*q-t30*((t11+t21)
*p-t11^2-t11*t21-t21^2))*t32^2+((-q^3*(-t11+t31)*t12+q*((p-t11)
*q+t20+t30)*t31-(p-t11)*t11*q^2-t11*(t20+t30)*q-t30*(t20+t30))*
t22^2+(-q^3*(-t21+t31)*t12^2+(2*q^2*t31^2-2*q*((t11+t21)*q-3*
t30*(1/2))*t31+2*q^2*t11*t21+((t20-t30)*t11-(t20+2*t30)*t21)*
q-2*t30^2)*t12+((-2*p+2*t11)*q-2*t20-2*t30)*t31^2+((p-t11)*(p+
t11+2*t21)*q+(-t30+2*t20)*p+t11*(4*t30+t20)+t21*(t20+t30))*t31+
(-p^2*t21+t11^2*t21)*q+(t11*(-t20+t30)-t21*t20)*p-(2*(t11+(1/2)
*t21))*t11*t30)*t22+(q*((p-t21)*q-t20)*t31-(p-t21)*t21*q^2+t21*
t20*q+t30*t20)*t12^2+(((-2*p+2*t21)*q+2*t20)*t31^2+((p-t21)*
(p+2*t11+t21)*q+(-2*t20-3*t30)*p-t11*t20-t21*(t20-3*t30))*t31+
(-p^2*t11+t11*t21^2)*q+((t20+t30)*t11+(t20+2*t30)*t21)*p-t30*
t21*(t11+2*t21))*t12+(2*(t31-(1/2)*t11-(1/2)*t21))*(-p+t31)*(p-
t21)*(p-t11))*t32+((-q^2*t31^2+q*((p+t11)*q-t30)*t31-q^2*p*t11+
t30*t11*q+t30*(t20+t30))*t12+(-t11+t31)*(((p-t11)*q+t20+t30)*
t31-p*(p-t11)*q-p*t20-t11*t30))*t22^2+((-q^2*t31^2+q*((p+t21)*
q-t30)*t31-q^2*p*t21+t30*t21*q-t30*t20)*t12^2+(2*q*(p-(1/2)*t11
-(1/2)*t21)*t31^2+((-2*p^2+2*t11*t21)*q+p*t30+t11*t20-t21*(t20+
t30))*t31+((t11+t21)*p-2*t11*t21)*p*q+((-t20-t30)*t11+t21*t20)*
p+t21*t11*t30)*t12-(p-t11)*(-t11+t31)*(-p+t31)*(p-t21))*t22-
t12*((((-p+t21)*q+t20)*t31+p*(p-t21)*q+(-t20-t30)*p+t21*t30)*
t12+(p-t21)*(p-t11)*(-p+t31))*(-t21+t31));
```

```
simplify(OmegaJMUdt11-omegaJMUdt11);
simplify(OmegaJMUdt21-omegaJMUdt21);
simplify(OmegaJMUdt31-omegaJMUdt31);
simplify(OmegaJMUdt12-omegaJMUdt12);
simplify(OmegaJMUdt22-omegaJMUdt22);
simplify(OmegaJMUdt32-omegaJMUdt32);
```

$$omegaJMUdt11 := \frac{1}{(t12 - t22)\,(-t12 + t32)}\big(-p^3 + (t11 + t21 + t31 + (t12 + t22$$
$$+ t32)\,q)\,p^2 + \big(((-t22 - t32)\,t12 - t22\,t32)\,q^2 + ((-t31 - t21)\,t12 + (-t11$$
$$- t31)\,t22 - t32\,(t11 + t21))\,q + t12\,(-t20 - t30) + t20\,t22 + t30\,t32 + (-t31$$
$$- t21)\,t11 - t21\,t31)\,p + q^3\,t12\,t22\,t32 + ((t21\,t32 + t22\,t31)\,t12 + t22\,t32\,t11)\,q^2$$
$$+ ((t20\,t32 + t21\,t31 + t22\,t30)\,t12 + ((-t20 - t30)\,t32 + t11\,t31)\,t22$$
$$+ t32\,t11\,t21)\,q + (t20 + t30)\,(t31 - t11 + t21)\,t12 + (t11\,t30 - t31\,(t20 + t30))\,t22$$
$$+ (t11\,t20 - t21\,(t20 + t30))\,t32 + t11\,t21\,t31\big)$$

$$omegaJMUdt21 := \frac{1}{(t12 - t22)\,(-t22 + t32)}\big(p^3 + ((-t12 - t22 - t32)\,q - t31 - t11$$
$$- t21)\,p^2 + \big(((t12 + t32)\,t22 + t12\,t32)\,q^2 + (t12\,(t21 + t31) + t22\,(t11 + t31)$$
$$+ t32\,(t11 + t21))\,q - t20\,t22 - t30\,t32 + (t20 + t30)\,t12 + (t11 + t31)\,t21$$
$$+ t11\,t31)\,p - q^3\,t12\,t22\,t32 + ((-t11\,t32 - t12\,t31)\,t22 - t12\,t21\,t32)\,q^2 + (((t20$$
$$+ t30)\,t32 - t11\,t31 - t12\,t30)\,t22 + (-t11\,t21 - t12\,t20)\,t32 - t12\,t21\,t31)\,q$$
$$+ t20\,(t31 + t11 - t21)\,t22 + (t21\,(t20 + t30) - t11\,t20)\,t32 + (-t20\,t31$$
$$- t21\,t30)\,t12 - t11\,t21\,t31\big)$$

$$omegaJMUdt31 := \frac{1}{(-t22 + t32)\,(-t12 + t32)}\big(p^3 + ((-t12 - t22 - t32)\,q - t31 - t11$$
$$- t21)\,p^2 + \big(((t12 + t22)\,t32 + t12\,t22)\,q^2 + (t12\,(t21 + t31) + t22\,(t11 + t31)$$
$$+ t32\,(t11 + t21))\,q - t30\,t32 - t20\,t22 + (t20 + t30)\,t12 + t31\,(t11 + t21)$$
$$+ t11\,t21)\,p - q^3\,t12\,t22\,t32 + ((-t11\,t22 - t12\,t21)\,t32 - t31\,t12\,t22)\,q^2 + (((t20$$
$$+ t30)\,t22 - t11\,t21 - t20\,t12)\,t32 + (-t11\,t31 - t12\,t30)\,t22 - t12\,t21\,t31)\,q$$
$$- t30\,(t31 - t11 - t21)\,t32 + (-t11\,t30 + t31\,(t20 + t30))\,t22 + (-t20\,t31$$
$$- t21\,t30)\,t12 - t11\,t21\,t31\big)$$

$$omegaJMUdt12 := \frac{1}{2}\,\frac{1}{(t12 - t22)^2\,(-t12 + t32)^2}\bigg(-(t20 + t30)^2\,t12^3 + \Big(\big(-q^3\,(t31$$
$$- 2\,t11 + t21)\,t32 + 2\,q\,((-p + t31)\,q + t30)\,t11 - (t21 + t31)\,(-p + t31)\,q^2$$
$$- t30\,(t21 + t31)\,q + (t20 + 2\,t30)\,(t20 + t30)\big)\,t22 + \big(-2\,q\,((p - t21)\,q$$
$$- t20)\,t11 + (t21 + t31)\,(p - t21)\,q^2 - (t21 + t31)\,t20\,q + 2\,t20^2 + 3\,t30\,t20$$
$$+ t30^2\big)\,t32 + (-t20 - t30)\,t11^2 + (-2\,(p - t21)\,(-p + t31)\,q + 2\,(t20 + t30)\,(t31$$
$$- p + t21))\,t11 + (p - t21)\,(t21 + t31)\,(-p + t31)\,q - (t20 + t30)\,\big((-t31 - t21)\,p$$
$$+ t31^2 + t21\,t31 + t21^2\big)\Big)\,t12^2 + \bigg(\big(q^3\,(-t11 + t31)\,t32 - q\,((-p + t31)\,q + t30)\,t11$$
$$+ t31\,(-p + t31)\,q^2 + t31\,t30\,q - t30\,(t20 + t30)\big)\,t22^2 + \Big(-q^3\,(t11 - t21)\,t32^2$$
```

$$+ \left(2\,t11^2\,q^2 - 2\,q\left((t21+t31)\,q + \frac{3}{2}\,t20 + \frac{3}{2}\,t30 \right) t11 + 2\,q^2\,t21\,t31 + \left((t20 \right. \right.$$

$$\left. + 2\,t30)\,t21 + 2\,t31\left(t20 + \frac{1}{2}\,t30 \right) \right) q - 2\,(t20+t30)^2 \bigg) t32 + \big((2\,t31 - 2\,p)\,q$$

$$+ 2\,t30)\,t11^2 + \left(-(-p+t31)\,(p+2\,t21+t31)\,q + (t30+3\,t20)\,p - t21\,t30 \right.$$

$$\left. - 3\,t31\left(t20 + \frac{4}{3}\,t30 \right) \right) t11 + \left(-p^2\,t21 + t21\,t31^2 \right) q + \left(-t21\,t20 - 2\,t31\left(t20 \right. \right.$$

$$\left. \left. + \frac{1}{2}\,t30 \right) \right) p + 2\,(t20+t30)\,t31\left(t31 + \frac{1}{2}\,t21 \right) \bigg) t22 + \big(q\,((p-t21)\,q - t20)\,t11$$

$$- (p-t21)\,t21\,q^2 + t21\,t20\,q - t20\,(t20+t30) \big)\,t32^2 + \Big(((-2\,p+2\,t21)\,q$$

$$+ 2\,t20)\,t11^2 + \left(2\,(p-t21)\left(\frac{1}{2}\,p + t31 + \frac{1}{2}\,t21 \right) q + (t20+3\,t30)\,p + (-4\,t20 \right.$$

$$\left. - 3\,t30)\,t21 - t31\,t20 \right) t11 + \left(-p^2\,t31 + t21^2\,t31 \right) q + ((-t20-2\,t30)\,t21$$

$$- t31\,t30)\,p + t21\,(t20+t30)\,(t31+2\,t21) \bigg) t32 - (p-t21)\,(p-t11)\,(t31-2\,t11$$

$$+ t21)\,(-p+t31) \bigg) t12 + \Big(\big(-t11^2\,q^2 + q\,((p+t31)\,q + t20 + t30)\,t11 - t31\,q^2\,p$$

$$- t31\,(t20+t30)\,q + t30\,(t20+t30) \big)\,t32 - (-t11+t31)\,(((p-t31)\,q - t30)\,t11$$

$$+ p\,(-p+t31)\,q - t20\,p + t31\,(t20+t30)) \big)\,t22^2 + \big((-t11^2\,q^2 + q\,((p+t21)\,q$$

$$+ t20 + t30)\,t11 - t21\,q^2\,p - t21\,(t20+t30)\,q + t20\,(t20+t30))\,t32^2 + (-q\,(t31$$

$$- 2\,p + t21)\,t11^2 + ((-2\,p^2 + 2\,t21\,t31)\,q + (-t20-t30)\,p + t31\,t20 + t21\,t30)\,t11$$

$$+ ((t21+t31)\,p - 2\,t21\,t31)\,p\,q + (t20\,t21 + t30\,t31)\,p - t21\,t31\,(t20+t30))\,t32$$

$$+ (p-t11)\,(-t11+t31)\,(-p+t31)\,(p-t21) \big)\,t22 - ((((-p+t21)\,q + t20)\,t11$$

$$+ p\,(p-t21)\,q + p\,t30 - t21\,(t20+t30))\,t32 + (p-t21)\,(p-t11)\,(-p+t31))$$

$$(t11-t21)\,t32 \big)$$

$$omegaJMUdt22 := \frac{1}{2}\,\frac{1}{(t12-t22)^2\,(-t22+t32)^2}\Big(-t20^2\,t22^3 + \big((-q^3\,(t11 - 2\,t21$$

$$+ t31)\,t12 - 2\,((p-t11)\,q + t20 + t30)\,q\,t21 + (t11+t31)\,(p-t11)\,q^2 + (t20$$

$$+ t30)\,(t11+t31)\,q + 2\,t20^2 + t30\,t20 \big)\,t32 + \big(2\,q\,((-p+t31)\,q + t30)\,t21 - (t11$$

$$+ t31)\,(-p+t31)\,q^2 - t30\,(t11+t31)\,q + t20\,(t20-t30) \big)\,t12 + t20\,t21^2 + (-2\,(p$$

$$- t11)\,(-p+t31)\,q - 2\,t20\,(t31 - p + t11))\,t21 + (p-t11)\,(t11+t31)\,(-p$$

$$+ t31)\,q + t20\,((-t11-t31)\,p + t31^2 + t11\,t31 + t11^2) \big)\,t22^2 + \Big(\big(q^3\,(t11 - t21)\,t12$$

$$+ ((p-t11)\,q + t20 + t30)\,q\,t21 - t11\,(p-t11)\,q^2 - t11\,(t20+t30)\,q - t20\,(t20$$

$$+ t30) \big)\,t32^2 + \left(q^3\,(-t21+t31)\,t12^2 + \left(2\,t21^2\,q^2 - 2\,q\left((t11+t31)\,q \right. \right. \right.$$

$$\left. \left. - \frac{3}{2}\,t20 \right) t21 + 2\,t31\,t11\,q^2 + \left(t11\,(-t20+t30) - 2\,t31\left(t20 + \frac{1}{2}\,t30 \right) \right) q \right.$$

$$-2\,t20^2\Big)\,t12 + ((-2\,p + 2\,t11)\,q - 2\,t20 - 2\,t30)\,t21^2 + \Big(2\,\Big(\tfrac{1}{2}\,p + t31$$

$$+ \tfrac{1}{2}\,t11\Big)\,(p - t11)\,q + (2\,t30 - t20)\,p + (t30 + 4\,t20)\,t11 + t31\,(t20 + t30)\Big)\,t21$$

$$+ \big(-p^2\,t31 + t11^2\,t31\big)\,q + ((t20 - t30)\,t11 - t31\,t30)\,p - t11\,t20\,(t31 + 2\,t11)\Big)\,t32$$

$$+ \Big(-q\,((-p + t31)\,q + t30)\,t21 + t31\,(-p + t31)\,q^2 + t31\,t30\,q + t30\,t20\Big)\,t12^2$$

$$+ \Big(((2\,t31 - 2\,p)\,q + 2\,t30)\,t21^2 + \Big(-(-p + t31)\,(p + 2\,t11 + t31)\,q + (-2\,t30$$

$$- 3\,t20)\,p - t11\,t30 + 3\,t31\,\Big(t20 - \tfrac{1}{3}\,t30\Big)\Big)\,t21 + \big(-p^2\,t11 + t11\,t31^2\big)\,q + \Big((t20$$

$$+ t30)\,t11 + 2\,t31\,\Big(t20 + \tfrac{1}{2}\,t30\Big)\Big)\,p - 2\,t20\,t31\,\Big(t31 + \tfrac{1}{2}\,t11\Big)\Big)\,t12 - (p$$

$$- t21)\,(p - t11)\,(t11 - 2\,t21 + t31)\,(-p + t31)\Big)\,t22 + \Big(\big(-t21^2\,q^2 + q\,((p + t11)\,q$$

$$- t20)\,t21 - t11\,q^2\,p + t20\,t11\,q + t20\,(t20 + t30)\big)\,t12 + (((-p + t11)\,q - t20$$
$$- t30)\,t21 + p\,(p - t11)\,q + p\,t30 + t11\,t20)\,(t11 - t21)\big)\,t32^2 + \Big(\big(-t21^2\,q^2 + ((p$$
$$+ t31)\,q - t20)\,q\,t21 - t31\,q^2\,p + t31\,t20\,q - t30\,t20\big)\,t12^2 + \big(-q\,(-2\,p + t31$$
$$+ t11)\,t21^2 + \big((-2\,p^2 + 2\,t11\,t31)\,q + t20\,p + t11\,t30 - t31\,(t20 + t30)\big)\,t21$$
$$+ p\,((t11 + t31)\,p - 2\,t11\,t31)\,q + ((-t20 - t30)\,t11 + t31\,t30)\,p + t31\,t11\,t20\big)\,t12$$
$$+ (t11 - t21)\,(p - t21)\,(p - t11)\,(-p + t31)\big)\,t32 + t12\,((((-p + t31)\,q + t30)\,t21$$
$$- p\,(-p + t31)\,q + (-t20 - t30)\,p + t31\,t20)\,t12 + (p - t21)\,(p - t11)\,(-p$$
$$+ t31))\,(-t21 + t31)\big)$$

$$omegaJMUdt32 := \frac{1}{2}\,\frac{1}{(-t12 + t32)^2\,(-t22 + t32)^2}\Big(-t32^3\,t30^2 + \Big(\Big(2\,q^3\,\Big(t31 - \tfrac{1}{2}\,t11$$

$$- \tfrac{1}{2}\,t21\Big)\,t12 - 2\,((p - t11)\,q + t20 + t30)\,q\,t31 + (t11 + t21)\,(p - t11)\,q^2 + (t20$$

$$+ t30)\,(t11 + t21)\,q + t30\,(t20 + 2\,t30)\Big)\,t22 + \big(-2\,q\,((p - t21)\,q - t20)\,t31$$

$$+ (t11 + t21)\,(p - t21)\,q^2 - t20\,(t11 + t21)\,q - t30\,(t20 - t30)\big)\,t12 + t31^2\,t30$$

$$+ (2\,(p - t21)\,(p - t11)\,q + 2\,t30\,(p - t11 - t21))\,t31 - (t11 + t21)\,(p - t21)\,(p$$

$$- t11)\,q - \big((t11 + t21)\,p - t11^2 - t11\,t21 - t21^2\big)\,t30\Big)\,t32^2 + \Big(\big(-q^3\,(-t11$$

$$+ t31)\,t12 + ((p - t11)\,q + t20 + t30)\,q\,t31 - t11\,(p - t11)\,q^2 - t11\,(t20 + t30)\,q$$

$$- t30\,(t20 + t30)\big)\,t22^2 + \Big(-q^3\,(-t21 + t31)\,t12^2 + \Big(2\,t31^2\,q^2 - 2\,q\,\Big((t11 + t21)\,q$$

$$- \tfrac{3}{2}\,t30\Big)\,t31 + 2\,t21\,t11\,q^2 + ((t20 - t30)\,t11 - (t20 + 2\,t30)\,t21)\,q - 2\,t30^2\Big)\,t12$$

$$+ ((-2\,p + 2\,t11)\,q - 2\,t20 - 2\,t30)\,t31^2 + ((p - t11)\,(p + 2\,t21 + t11)\,q + (-t30$$

$$+ 2\,t20)\,p + t11\,(t20 + 4\,t30) + t21\,(t20 + t30))\,t31 + \big(-p^2\,t21 + t11^2\,t21\big)\,q$$

$+ (t11 (-t20 + t30) - t21\,t20)\,p - 2\,t30\,t11\left(t11 + \frac{1}{2}\,t21\right)\bigg)\,t22 + \bigg(q\,((p - t21)\,q$

$- t20)\,t31 - (p - t21)\,t21\,q^2 + t21\,t20\,q + t30\,t20\big)\,t12^2 + \big(((-2\,p + 2\,t21)\,q$

$+ 2\,t20)\,t31^2 + ((p - t21)\,(p + 2\,t11 + t21)\,q + (-2\,t20 - 3\,t30)\,p - t11\,t20$

$- t21\,(t20 - 3\,t30))\,t31 + \left(-p^2\,t11 + t11\,t21^2\right)q + ((t20 + t30)\,t11 + (t20$

$+ 2\,t30)\,t21)\,p - t30\,t21\,(t11 + 2\,t21)\big)\,t12 + 2\,(p - t21)\left(t31 - \frac{1}{2}\,t11\right.$

$\left. - \frac{1}{2}\,t21\right)(p - t11)\,(-p + t31)\bigg)\,t32 + \bigg(\left(-t31^2\,q^2 + q\,((p + t11)\,q - t30)\,t31\right.$

$- t11\,q^2\,p + t30\,t11\,q + t30\,(t20 + t30)\big)\,t12 + (-t11 + t31)\,(((p - t11)\,q + t20$

$+ t30)\,t31 - p\,(p - t11)\,q - t20\,p - t11\,t30)\big)\,t22^2 + \bigg(\left(-t31^2\,q^2 + ((p + t21)\,q\right.$

$- t30)\,q\,t31 - t21\,q^2\,p + t30\,t21\,q - t30\,t20\big)\,t12^2 + \left(2\,q\left(p - \frac{1}{2}\,t11 - \frac{1}{2}\,t21\right)t31^2\right.$

$+ \left(\left(-2\,p^2 + 2\,t11\,t21\right)q + p\,t30 + t11\,t20 - t21\,(t20 + t30)\right)t31 + p\,((t11 + t21)\,p$
$- 2\,t11\,t21)\,q + ((-t20 - t30)\,t11 + t21\,t20)\,p + t21\,t11\,t30\big)\,t12 - (p - t11)\,(-t11$
$+ t31)\,(-p + t31)\,(p - t21)\big)\,t22 - t12\,((((-p + t21)\,q + t20)\,t31 + p\,(p - t21)\,q$
$+ (-t20 - t30)\,p + t21\,t30)\,t12 + (p - t21)\,(p - t11)\,(-p + t31))\,(-t21 + t31)\bigg)$

0
0
0
0
0
0

> `simplify(Trace(2*F2-F1^2));`

0

(3.26)

Computation of the general auxiliary matrix \td{A} and of the Hamiltonian evolutions for (q,p)

The evolutions relatively to the irregular times are denoted dott12, dott22, dott32, dott11, dott21, dott31

> ```
tdA:=Matrix(3,3,0):
tdA[1,1]:=tdA112*lambda^2+tdA111*lambda+tdA110:
tdA[1,2]:=tdA122*lambda^2+tdA121*lambda+tdA120:
tdA[1,3]:=tdA132*lambda^2+tdA131*lambda+tdA130:
tdA[2,1]:=tdA212*lambda^2+tdA211*lambda+tdA210:
tdA[2,2]:=tdA222*lambda^2+tdA221*lambda+tdA220:
tdA[2,3]:=tdA232*lambda^2+tdA231*lambda+tdA230:
tdA[3,1]:=tdA312*lambda^2+tdA311*lambda+tdA310:
tdA[3,2]:=tdA322*lambda^2+tdA321*lambda+tdA320:
tdA[3,3]:=tdA332*lambda^2+tdA331*lambda+tdA330:
```

```
tdA;

tdAdt:=Matrix(3,3,0):
tdAdt[1,1]:=tdAdt112*lambda^2+tdAdt111*lambda+tdAdt110:
tdAdt[1,2]:=tdAdt122*lambda^2+tdAdt121*lambda+tdAdt120:
tdAdt[1,3]:=tdAdt132*lambda^2+tdAdt131*lambda+tdAdt130:
tdAdt[2,1]:=tdAdt212*lambda^2+tdAdt211*lambda+tdAdt210:
tdAdt[2,2]:=tdAdt222*lambda^2+tdAdt221*lambda+tdAdt220:
tdAdt[2,3]:=tdAdt232*lambda^2+tdAdt231*lambda+tdAdt230:
tdAdt[3,1]:=tdAdt312*lambda^2+tdAdt311*lambda+tdAdt310:
tdAdt[3,2]:=tdAdt322*lambda^2+tdAdt321*lambda+tdAdt320:
tdAdt[3,3]:=tdAdt332*lambda^2+tdAdt331*lambda+tdAdt330:
tdAdt;

tdAdlambda:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do tdAdlambda[i,j]:=diff
(tdA[i,j],lambda): od: od:
tdAdlambda;

Anew:=simplify(Multiply(Multiply(Gnormalized,tdA),Gnormalized^
(-1))+Multiply(LGnormalized,Gnormalized^(-1))):
```

$$\left[\left[\lambda^2\, tdA112 + \lambda\, tdA111 + tdA110,\ \lambda^2\, tdA122 + \lambda\, tdA121 + tdA120,\ \lambda^2\, tdA132 + \lambda\, tdA131 \right. \right. \tag{4.1}$$
$$\left. + tdA130\right],$$
$$\left[\lambda^2\, tdA212 + \lambda\, tdA211 + tdA210,\ \lambda^2\, tdA222 + \lambda\, tdA221 + tdA220,\ \lambda^2\, tdA232 \right.$$
$$\left. + \lambda\, tdA231 + tdA230\right],$$
$$\left[\lambda^2\, tdA312 + \lambda\, tdA311 + tdA310,\ \lambda^2\, tdA322 + \lambda\, tdA321 + tdA320,\ \lambda^2\, tdA332 \right.$$
$$\left.\left. + \lambda\, tdA331 + tdA330\right]\right]$$
$$\left[\left[\lambda^2\, tdAdt112 + \lambda\, tdAdt111 + tdAdt110,\ \lambda^2\, tdAdt122 + \lambda\, tdAdt121 + tdAdt120,\ \lambda^2\, tdAdt132 \right.\right.$$
$$\left. + \lambda\, tdAdt131 + tdAdt130\right],$$
$$\left[\lambda^2\, tdAdt212 + \lambda\, tdAdt211 + tdAdt210,\ \lambda^2\, tdAdt222 + \lambda\, tdAdt221 + tdAdt220, \right.$$
$$\left. \lambda^2\, tdAdt232 + \lambda\, tdAdt231 + tdAdt230\right],$$
$$\left[\lambda^2\, tdAdt312 + \lambda\, tdAdt311 + tdAdt310,\ \lambda^2\, tdAdt322 + \lambda\, tdAdt321 + tdAdt320, \right.$$
$$\left.\left. \lambda^2\, tdAdt332 + \lambda\, tdAdt331 + tdAdt330\right]\right]$$

$$\begin{bmatrix} 2\,\lambda\, tdA112 + tdA111 & 2\,\lambda\, tdA122 + tdA121 & 2\,\lambda\, tdA132 + tdA131 \\ 2\,\lambda\, tdA212 + tdA211 & 2\,\lambda\, tdA222 + tdA221 & 2\,\lambda\, tdA232 + tdA231 \\ 2\,\lambda\, tdA312 + tdA311 & 2\,\lambda\, tdA322 + tdA321 & 2\,\lambda\, tdA332 + tdA331 \end{bmatrix}$$

```
> dtdLdt:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do dtdLdt[i,j]:=diff(tdL
[i,j],t12)*dott12+diff(tdL[i,j],t22)*dott22+ diff(tdL[i,j],t32)
```

```
    *dott32+diff(tdL[i,j],a12)*dota12+diff(tdL[i,j],a13)*dota13
    +diff(tdL[i,j],q)*dotq+ diff(tdL[i,j],p)*dotp
    +diff(tdL[i,j],t11)*dott11+ diff(tdL[i,j],t21)*dott21+diff(tdL
    [i,j],t31)*dott31: od: od:
    dtdLdt:
    Compatibilite:=simplify(h*tdAdlambda-h*dtdLdt+Multiply(tdA,tdL)
    -Multiply(tdL,tdA)):
```

Solving the compatibility equations to obtain \td{A} and the general evolutions for (q,p)

```
> residue(Compatibilite[1,1]*(lambda-q),lambda=q);
  residue(Compatibilite[1,1],lambda=q);
  factor(residue(Compatibilite[1,1]/lambda^4,lambda=infinity));
  Eq1:= factor(residue(Compatibilite[1,1]/lambda^3,lambda=
  infinity)):
  Eq2:= factor(residue(Compatibilite[1,1]/lambda^2,lambda=
  infinity)):
  Eq3:= factor(residue(Compatibilite[1,1]/lambda^1,lambda=
  infinity)):
```

$$0$$
$$0 \qquad\qquad (4.2)$$
$$0$$

```
> residue(Compatibilite[1,2]*(lambda-q),lambda=q);
  residue(Compatibilite[1,2],lambda=q);
  factor(residue(Compatibilite[1,2]/lambda^5,lambda=infinity));
  tdA122:=0;
  factor(residue(Compatibilite[1,2]/lambda^4,lambda=infinity));
  Eq4:= factor(residue(Compatibilite[1,2]/lambda^3,lambda=
  infinity));
  Eq5:= factor(residue(Compatibilite[1,2]/lambda^2,lambda=
  infinity));
  Eq6:= factor(residue(Compatibilite[1,2]/lambda^1,lambda=
  infinity));
```

$$0$$
$$0 \qquad\qquad (4.3)$$
$$0$$
$$tdA122 := 0$$
$$0$$

$$Eq4 := -\frac{1}{a13}\big(a12\, q\, t32\, tdA132 + a12\, a13\, tdA112 - a12\, a13\, tdA222 - a12\, p\, tdA132$$
$$+ a12\, t31\, tdA132 - a13^2\, tdA322 - a13\, t12\, tdA121 + a13\, t22\, tdA121\big)$$

$$Eq5 := -\frac{1}{a13}\big(a12\, q\, t32\, tdA131 + a12\, a13\, tdA111 - a12\, a13\, tdA221 - a12\, p\, tdA131$$
$$+ a12\, t31\, tdA131 - a13^2\, tdA321 - a13\, t11\, tdA121 - a13\, t12\, tdA120 + a13\, t21\, tdA121$$
$$+ a13\, t22\, tdA120\big)$$

$$Eq6 := -\frac{1}{a13}\big(a12\,q\,t32\,tdA130 + a12\,a13\,tdA110 - a12\,a13\,tdA220 - a12\,p\,tdA130$$
$$+ a12\,t31\,tdA130 - a13^2\,tdA320 - a13\,dota12\,h + a13\,h\,tdA121 - a13\,t11\,tdA120$$
$$+ a13\,t21\,tdA120\big)$$

```
> residue(Compatibilite[1,3]*(lambda-q),lambda=q);
  residue(Compatibilite[1,3],lambda=q);
  factor(residue(Compatibilite[1,3]/lambda^5,lambda=infinity));
  tdA132:=0;
  factor(residue(Compatibilite[1,3]/lambda^4,lambda=infinity));
  Eq7:= factor(residue(Compatibilite[1,3]/lambda^3,lambda=
  infinity));
  Eq8:= factor(residue(Compatibilite[1,3]/lambda^2,lambda=
  infinity));
  Eq9:= factor(residue(Compatibilite[1,3]/lambda^1,lambda=
  infinity));
```

$$0 \tag{4.4}$$
$$0$$
$$0$$
$$tdA132 := 0$$
$$0$$

$$Eq7 := a12\,tdA232 - a13\,tdA112 + a13\,tdA332 + t12\,tdA131 - t32\,tdA131$$

$$Eq8 := \frac{1}{a12}\big(-a13\,q\,t22\,tdA121 + a12^2\,tdA231 - a12\,a13\,tdA111 + a12\,a13\,tdA331$$
$$+ a12\,t11\,tdA131 + a12\,t12\,tdA130 - a12\,t31\,tdA131 - a12\,t32\,tdA130$$
$$+ a13\,p\,tdA121 - a13\,t21\,tdA121\big)$$

$$Eq9 := \frac{1}{a12}\big(-a13\,q\,t22\,tdA120 + a12^2\,tdA230 - a12\,a13\,tdA110 + a12\,a13\,tdA330$$
$$+ a12\,dota13\,h - a12\,h\,tdA131 + a12\,t11\,tdA130 - a12\,t31\,tdA130 + a13\,p\,tdA120$$
$$- a13\,t21\,tdA120\big)$$

```
> residue(Compatibilite[2,1]*(lambda-q),lambda=q);
  residue(Compatibilite[2,1],lambda=q);
  factor(residue(Compatibilite[2,1]/lambda^5,lambda=infinity));
  tdA212:=0;
  factor(residue(Compatibilite[2,1]/lambda^4,lambda=infinity));
  Eq10:= factor(residue(Compatibilite[2,1]/lambda^3,lambda=
  infinity)):
  Eq11:= factor(residue(Compatibilite[2,1]/lambda^2,lambda=
  infinity)):
  Eq12:= factor(residue(Compatibilite[2,1]/lambda^1,lambda=
  infinity)):
```

$$0 \tag{4.5}$$
$$0$$
$$0$$
$$tdA212 := 0$$
$$0$$

```
> residue(Compatibilite[2,2]*(lambda-q),lambda=q);
  residue(Compatibilite[2,2],lambda=q);
  factor(residue(Compatibilite[2,2]/lambda^5,lambda=infinity));
  factor(residue(Compatibilite[2,2]/lambda^4,lambda=infinity));
  Eq13:= factor(residue(Compatibilite[2,2]/lambda^3,lambda=
  infinity));
  Eq14:= factor(residue(Compatibilite[2,2]/lambda^2,lambda=
  infinity));
  Eq15:= factor(residue(Compatibilite[2,2]/lambda^1,lambda=
  infinity));
```

$$0$$
$$0$$
$$0$$
$$0$$

(4.6)

$$Eq13 := \frac{1}{a13\ a12}\left(-a12^2\ q\ t32\ tdA232 + a13^2\ q\ t22\ tdA322 + a12^2\ p\ tdA232\right.$$
$$\left. - a12^2\ t31\ tdA232 - a13^2\ p\ tdA322 + a13^2\ t21\ tdA322\right)$$

$$Eq14 := -\frac{1}{a12\ (t22 - t32)\ a13}\left(-a13\ q^2\ t12\ t22\ t32\ tdA121 + a13\ q^2\ t22^2\ t32\ tdA121\right.$$
$$+ a12^2\ q\ t22\ t32\ tdA231 - a12^2\ q\ t32^2\ tdA231 - a13^2\ q\ t22^2\ tdA321$$
$$+ a13^2\ q\ t22\ t32\ tdA321 + a13\ p\ q\ t12\ t22\ tdA121 + a13\ p\ q\ t12\ t32\ tdA121$$
$$- a13\ p\ q\ t22^2\ tdA121 - a13\ p\ q\ t22\ t32\ tdA121 - a13\ q\ t12\ t21\ t32\ tdA121$$
$$- a13\ q\ t12\ t22\ t31\ tdA121 + a13\ q\ t21\ t22\ t32\ tdA121 + a13\ q\ t22^2\ t31\ tdA121$$
$$+ a12^2\ a13\ t22\ tdA211 - a12^2\ a13\ t32\ tdA211 - a12^2\ p\ t22\ tdA231 + a12^2\ p\ t32\ tdA231$$
$$+ a12^2\ t22\ t31\ tdA231 - a12^2\ t31\ t32\ tdA231 - a12\ a13\ dott22\ h\ t22$$
$$+ a12\ a13\ dott22\ h\ t32 + 2\ a12\ a13\ h\ t22\ tdA222 - 2\ a12\ a13\ h\ t32\ tdA222$$
$$+ a13^2\ p\ t22\ tdA321 - a13^2\ p\ t32\ tdA321 - a13^2\ t21\ t22\ tdA321 + a13^2\ t21\ t32\ tdA321$$
$$- a13\ p^2\ t12\ tdA121 + a13\ p^2\ t22\ tdA121 + a13\ p\ t12\ t21\ tdA121$$
$$+ a13\ p\ t12\ t31\ tdA121 - a13\ p\ t21\ t22\ tdA121 - a13\ p\ t22\ t31\ tdA121$$
$$+ a13\ t12\ t20\ t22\ tdA121 - a13\ t12\ t20\ t32\ tdA121 - a13\ t12\ t21\ t31\ tdA121$$
$$\left. - a13\ t20\ t22^2\ tdA121 + a13\ t20\ t22\ t32\ tdA121 + a13\ t21\ t22\ t31\ tdA121\right)$$

$$Eq15 := -\frac{1}{a12\ (t22 - t32)\ a13}\left(-a13\ q^2\ t12\ t22\ t32\ tdA120 + a13\ q^2\ t22^2\ t32\ tdA120\right.$$
$$+ a12^2\ q\ t22\ t32\ tdA230 - a12^2\ q\ t32^2\ tdA230 - a13^2\ q\ t22^2\ tdA320$$
$$+ a13^2\ q\ t22\ t32\ tdA320 + a13\ p\ q\ t12\ t22\ tdA120 + a13\ p\ q\ t12\ t32\ tdA120$$
$$- a13\ p\ q\ t22^2\ tdA120 - a13\ p\ q\ t22\ t32\ tdA120 - a13\ q\ t12\ t21\ t32\ tdA120$$
$$- a13\ q\ t12\ t22\ t31\ tdA120 + a13\ q\ t21\ t22\ t32\ tdA120 + a13\ q\ t22^2\ t31\ tdA120$$
$$+ a12^2\ a13\ t22\ tdA210 - a12^2\ a13\ t32\ tdA210 - a12^2\ p\ t22\ tdA230 + a12^2\ p\ t32\ tdA230$$
$$+ a12^2\ t22\ t31\ tdA230 - a12^2\ t31\ t32\ tdA230 - a12\ a13\ dott21\ h\ t22$$
$$+ a12\ a13\ dott21\ h\ t32 + a12\ a13\ h\ t22\ tdA221 - a12\ a13\ h\ t32\ tdA221$$
$$+ a13^2\ p\ t22\ tdA320 - a13^2\ p\ t32\ tdA320 - a13^2\ t21\ t22\ tdA320 + a13^2\ t21\ t32\ tdA320$$
$$- a13\ p^2\ t12\ tdA120 + a13\ p^2\ t22\ tdA120 + a13\ p\ t12\ t21\ tdA120$$
$$+ a13\ p\ t12\ t31\ tdA120 - a13\ p\ t21\ t22\ tdA120 - a13\ p\ t22\ t31\ tdA120$$
$$+ a13\ t12\ t20\ t22\ tdA120 - a13\ t12\ t20\ t32\ tdA120 - a13\ t12\ t21\ t31\ tdA120$$

$$-a13\,t20\,t22^2\,tdA120 + a13\,t20\,t22\,t32\,tdA120 + a13\,t21\,t22\,t31\,tdA120\big)$$

```
> residue(Compatibilite[2,3]*(lambda-q),lambda=q);
  residue(Compatibilite[2,3],lambda=q);
  factor(residue(Compatibilite[2,3]/lambda^5,lambda=infinity));
  tdA232:=0;
  factor(residue(Compatibilite[2,3]/lambda^4,lambda=infinity));
  Eq16:= factor(residue(Compatibilite[2,3]/lambda^3,lambda=
  infinity));
  Eq17:= factor(residue(Compatibilite[2,3]/lambda^2,lambda=
  infinity)):
  Eq18:= factor(residue(Compatibilite[2,3]/lambda^1,lambda=
  infinity)):
```

$$0 \tag{4.7}$$
$$0$$
$$0$$
$$tdA232 := 0$$
$$0$$

$$Eq16 := \frac{1}{a12}\big(-a13\,q\,t22\,tdA222 + a13\,q\,t22\,tdA332 + a12\,t22\,tdA231 - a12\,t32\,tdA231$$
$$+ a13\,p\,tdA222 - a13\,p\,tdA332 - a13\,t21\,tdA222 + a13\,t21\,tdA332\big)$$

```
> residue(Compatibilite[3,1]*(lambda-q),lambda=q);
  residue(Compatibilite[3,1],lambda=q);
  factor(residue(Compatibilite[3,1]/lambda^5,lambda=infinity));
  tdA312:=0;
  factor(residue(Compatibilite[3,1]/lambda^4,lambda=infinity));
  Eq19:= factor(residue(Compatibilite[3,1]/lambda^3,lambda=
  infinity)):
  Eq20:= factor(residue(Compatibilite[3,1]/lambda^2,lambda=
  infinity)):
  Eq21:= factor(residue(Compatibilite[3,1]/lambda^1,lambda=
  infinity)):
```

$$0 \tag{4.8}$$
$$0$$
$$0$$
$$tdA312 := 0$$
$$0$$

```
> residue(Compatibilite[3,2]*(lambda-q),lambda=q);
  residue(Compatibilite[3,2],lambda=q);
  factor(residue(Compatibilite[3,2]/lambda^5,lambda=infinity));
  tdA322:=0;
  factor(residue(Compatibilite[3,2]/lambda^4,lambda=infinity));
  Eq22:= factor(residue(Compatibilite[3,2]/lambda^3,lambda=
  infinity));
  Eq23:= factor(residue(Compatibilite[3,2]/lambda^2,lambda=
```

```
    infinity)):
    Eq24:= factor(residue(Compatibilite[3,2]/lambda^1,lambda=
    infinity)):
```

$$0$$
$$0$$
$$0$$
$$tdA322 := 0$$
$$0$$

(4.9)

$$Eq22 := -\frac{1}{a13}\left(-a12\,q\,t32\,tdA222 + a12\,q\,t32\,tdA332 + a12\,p\,tdA222 - a12\,p\,tdA332\right.$$
$$\left. - a12\,t31\,tdA222 + a12\,t31\,tdA332 + a13\,t22\,tdA321 - a13\,t32\,tdA321\right)$$

```
> residue(Compatibilite[3,3]*(lambda-q),lambda=q);
  residue(Compatibilite[3,3],lambda=q);
  factor(residue(Compatibilite[3,3]/lambda^5,lambda=infinity));

  factor(residue(Compatibilite[3,3]/lambda^4,lambda=infinity));
  factor(residue(Compatibilite[3,3]/lambda^3,lambda=infinity));
  Eq25:= factor(residue(Compatibilite[3,3]/lambda^2,lambda=
  infinity));
  Eq26:= factor(residue(Compatibilite[3,3]/lambda^1,lambda=
  infinity));
```

$$0$$
$$0$$
$$0$$
$$0$$
$$0$$

(4.10)

$$Eq25 := -\frac{1}{a12\,(t22-t32)\,a13}\left(a12\,q^2\,t12\,t22\,t32\,tdA131 - a12\,q^2\,t22\,t32^2\,tdA131\right.$$
$$- a12^2\,q\,t22\,t32\,tdA231 + a12^2\,q\,t32^2\,tdA231 - a12\,p\,q\,t12\,t22\,tdA131$$
$$- a12\,p\,q\,t12\,t32\,tdA131 + a12\,p\,q\,t22\,t32\,tdA131 + a12\,p\,q\,t32^2\,tdA131$$
$$+ a12\,q\,t12\,t21\,t32\,tdA131 + a12\,q\,t12\,t22\,t31\,tdA131 - a12\,q\,t21\,t32^2\,tdA131$$
$$- a12\,q\,t22\,t31\,t32\,tdA131 + a13^2\,q\,t22^2\,tdA321 - a13^2\,q\,t22\,t32\,tdA321$$
$$+ a12^2\,p\,t22\,tdA231 - a12^2\,p\,t32\,tdA231 - a12^2\,t22\,t31\,tdA231 + a12^2\,t31\,t32\,tdA231$$
$$+ a12\,a13^2\,t22\,tdA311 - a12\,a13^2\,t32\,tdA311 - a12\,a13\,dott32\,h\,t22$$
$$+ a12\,a13\,dott32\,h\,t32 + 2\,a12\,a13\,h\,t22\,tdA332 - 2\,a12\,a13\,h\,t32\,tdA332$$
$$+ a12\,p^2\,t12\,tdA131 - a12\,p^2\,t32\,tdA131 - a12\,p\,t12\,t21\,tdA131$$
$$- a12\,p\,t12\,t31\,tdA131 + a12\,p\,t21\,t32\,tdA131 + a12\,p\,t31\,t32\,tdA131$$
$$+ a12\,t12\,t21\,t31\,tdA131 + a12\,t12\,t22\,t30\,tdA131 - a12\,t12\,t30\,t32\,tdA131$$
$$- a12\,t21\,t31\,t32\,tdA131 - a12\,t22\,t30\,t32\,tdA131 + a12\,t30\,t32^2\,tdA131$$
$$\left. - a13^2\,p\,t22\,tdA321 + a13^2\,p\,t32\,tdA321 + a13^2\,t21\,t22\,tdA321 - a13^2\,t21\,t32\,tdA321\right)$$

$$Eq26 := -\frac{1}{a12\,(t22-t32)\,a13}\left(a12\,q^2\,t12\,t22\,t32\,tdA130 - a12\,q^2\,t22\,t32^2\,tdA130\right.$$
$$- a12^2\,q\,t22\,t32\,tdA230 + a12^2\,q\,t32^2\,tdA230 - a12\,p\,q\,t12\,t22\,tdA130$$
$$- a12\,p\,q\,t12\,t32\,tdA130 + a12\,p\,q\,t22\,t32\,tdA130 + a12\,p\,q\,t32^2\,tdA130$$
$$+ a12\,q\,t12\,t21\,t32\,tdA130 + a12\,q\,t12\,t22\,t31\,tdA130 - a12\,q\,t21\,t32^2\,tdA130$$

$$- a12\ q\ t22\ t31\ t32\ tdA130 + a13^2\ q\ t22^2\ tdA320 - a13^2\ q\ t22\ t32\ tdA320$$
$$+ a12^2\ p\ t22\ tdA230 - a12^2\ p\ t32\ tdA230 - a12^2\ t22\ t31\ tdA230 + a12^2\ t31\ t32\ tdA230$$
$$+ a12\ a13^2\ t22\ tdA310 - a12\ a13^2\ t32\ tdA310 - a12\ a13\ dott31\ h\ t22$$
$$+ a12\ a13\ dott31\ h\ t32 + a12\ a13\ h\ t22\ tdA331 - a12\ a13\ h\ t32\ tdA331$$
$$+ a12\ p^2\ t12\ tdA130 - a12\ p^2\ t32\ tdA130 - a12\ p\ t12\ t21\ tdA130$$
$$- a12\ p\ t12\ t31\ tdA130 + a12\ p\ t21\ t32\ tdA130 + a12\ p\ t31\ t32\ tdA130$$
$$+ a12\ t12\ t21\ t31\ tdA130 + a12\ t12\ t22\ t30\ tdA130 - a12\ t12\ t30\ t32\ tdA130$$
$$- a12\ t21\ t31\ t32\ tdA130 - a12\ t22\ t30\ t32\ tdA130 + a12\ t30\ t32^2\ tdA130$$
$$- a13^2\ p\ t22\ tdA320 + a13^2\ p\ t32\ tdA320 + a13^2\ t21\ t22\ tdA320 - a13^2\ t21\ t32\ tdA320 \big)$$

```
> Eq1:=simplify(Eq1):
  Eq2:=simplify(Eq2):
  Eq3:=simplify(Eq3):
  Eq4:=simplify(Eq4):
  Eq5:=simplify(Eq5):
  Eq6:=simplify(Eq6):
  Eq7:=simplify(Eq7):
  Eq8:=simplify(Eq8):
  Eq9:=simplify(Eq9):
  Eq10:=simplify(Eq10):
  Eq11:=simplify(Eq11):
  Eq12:=simplify(Eq12):
  Eq13:=simplify(Eq13):
  Eq14:=simplify(Eq14):
  Eq15:=simplify(Eq15):
  Eq16:=simplify(Eq16):
  Eq17:=simplify(Eq17):
  Eq18:=simplify(Eq18):
  Eq19:=simplify(Eq19):
  Eq20:=simplify(Eq20):
  Eq21:=simplify(Eq21):
  Eq22:=simplify(Eq22):
  Eq23:=simplify(Eq23):
  Eq24:=simplify(Eq24):
  Eq25:=simplify(Eq25):
  Eq26:=simplify(Eq26):
```

```
> tdA131:=a13*(tdA112-tdA332)/(t12-t32):
  simplify(Eq7);
  tdA130:=-(-q*a13*t12*t22*tdA121+q*a13*t22*t32*tdA121+p*a13*t12*
  tdA121-p*a13*t32*tdA121+a12^2*t12*tdA231-a12^2*t32*tdA231-a12*
  a13*t12*tdA111+a12*a13*t12*tdA331+a12*a13*t32*tdA111-a12*a13*
  t32*tdA331+a12*a13*t11*tdA112-a12*a13*t11*tdA332-a12*a13*t31*
  tdA112+a12*a13*t31*tdA332-a13*t12*t21*tdA121+a13*t32*t21*
```

```
    tdA121)/((t12-t32)^2*a12):
    simplify(Eq8);
    tdA321:=-(-q*a12*t32*tdA112+q*a12*t32*tdA332+p*a12*tdA112-p*
    a12*tdA332-a12*t12*tdA111+a12*t12*tdA221+a12*t32*tdA111-a12*
    t32*tdA221-a12*t31*tdA112+a12*t31*tdA332+t12^2*tdA120-t12*t22*
    tdA120-t12*t32*tdA120+t12*t11*tdA121-t12*t21*tdA121+t22*t32*
    tdA120-t32*t11*tdA121+t32*t21*tdA121)/((t12-t32)*a13):
    simplify(Eq5);
```
$$0$$
$$0$$
$$0$$
<div align="right">(4.11)</div>

```
>  tdA121:=a12*(tdA112-tdA222)/(t12-t22):
    simplify(Eq4);
    tdA320:=(((( tdA110-tdA220)*t12^3+((q*(tdA111-tdA331)-2*
    tdA110+2*tdA220)*t32+(-tdA110+tdA220)*t22+(-tdA111+tdA331)*p+
    (tdA111-tdA331)*t31+h*(tdA112-tdA222))*t12^2+(((-tdA111+tdA331)
    *q+tdA110-tdA220)*t32^2+(((tdA112-tdA222)*q^2+(-tdA111+tdA331)*
    q+2*tdA110-2*tdA220)*t22+((-tdA112+tdA222)*q+tdA111-tdA331)*p+(
    (tdA112-tdA332)*t31+(-t11+t21)*tdA112-tdA222*t21+tdA332*t11)*q+
    (-tdA111+tdA331)*t31-2*h*(tdA112-tdA222))*t32+(((-tdA112+
    tdA222)*q+tdA111-tdA331)*t22+(tdA112-tdA222)*p+(-tdA112+tdA332)
    *t31+(t11-t21)*tdA112+tdA222*t21-tdA332*t11)*(p-t31))*t12+(((-
    tdA112+tdA222)*q^2+q*(tdA111-tdA331)-tdA110+tdA220)*t22+(tdA112
    -tdA222)*(p*q-q*t21+h))*t32^2+(((( tdA112-tdA222)*q-tdA111+
    tdA331)*p+((-2*tdA112+tdA222+tdA332)*t31+t11*(tdA112-tdA332))*
    q+(tdA111-tdA331)*t31)*t22-(tdA112-tdA222)*(p-t31)*(p-t21))*
    t32+t22*(tdA112-tdA332)*(-t11+t31)*(p-t31))*a12+(t12-t32)^2*
    (t12-t22)*(-dota12*h-t11*tdA120+t21*tdA120))*a13+tdA231*a12^2*
    (t12-t32)*(t12-t22)*(-q*t32+p-t31))/((t12-t22)*(t12-t32)^2*
    a13^2):
    simplify(Eq6);
    tdA211:=((-q^2*(tdA222-tdA332)*t32+(p*q-q*t31+t20)*tdA222+(-p*
    q+q*t31+t30)*tdA332+(-t30+2*h-t20)*tdA112-dott12*h-tdA311*a13)*
    t22+((p*q-q*t21-t20)*tdA222+(-p*q+q*t21-t30)*tdA332+(t30-2*h+
    t20)*tdA112+dott12*h+tdA311*a13)*t32-(tdA222-tdA332)*(p-t21)*
    (p-t31))/((t22-t32)*a12):
    simplify(Eq2);
```
$$0$$
$$0$$
$$0$$
<div align="right">(4.12)</div>

```
>  tdA210:=(-tdA310*(t22-t32)*a13^2*(t12-t22)*(t12-t32)*a12+(((((
    (-tdA112+tdA222)*q+tdA111-tdA331)*q^2*t32-((-tdA112+tdA222)*q+
    tdA111-tdA331)*q*p-t31*(tdA112-tdA222)*q^2+((tdA111-tdA331)*t31
```

```
-t30*(tdA112-tdA222))*q+t30*(tdA111-tdA331)+h*(dott11-tdA111))*
t12-((-tdA112+tdA222)*q+tdA111-tdA331)*q^2*t32^2+(((-tdA112+
tdA222)*q+tdA111-tdA331)*q*p+((2*tdA112-tdA222-tdA332)*t31-t11*
(tdA112-tdA332))*q^2+((-tdA111+tdA331)*t31+t30*(tdA112-tdA222))
*q+(-tdA111+tdA331)*t30-h*(dott11-tdA111))*t32-(tdA112-tdA332)*
(-t11+t31)*(p*q-q*t31-t30))*t22^2+((-q^2*(tdA111-tdA331)*t32+q*
(tdA111-tdA331)*p-t31*(tdA111-tdA331)*q+(-tdA111+tdA331)*t30-h*
(dott11-tdA111))*t12^2+(q^2*(tdA111-tdA331)*t32^2-2*q*(((-
tdA112+tdA222)*q+tdA111-tdA331)*p+(((1/2)*tdA112-(1/2)*tdA332)*
t31+(tdA112-tdA222)*t21-(1/2)*t11*(tdA112-tdA332))*q+(-(1/2)*
tdA111+(1/2)*tdA331)*t31+(-(1/2)*tdA111+(1/2)*tdA331)*t21-(1/2)
*t30*(tdA112-tdA222))*t32+((-2*tdA112+2*tdA222)*q+tdA111-
tdA331)*p^2+(((3*tdA112-2*tdA222-tdA332)*t31+(2*tdA112-2*
tdA222)*t21-t11*(tdA112-tdA332))*q+(-tdA111+tdA331)*t31+(-
tdA111+tdA331)*t21+t30*(tdA112-tdA222))*p-2*t31*(((1/2)*tdA112-
(1/2)*tdA332)*t31+(tdA112-tdA222)*t21-(1/2)*t11*(tdA112-tdA332)
)*q+((tdA111-tdA331)*t21-t30*(tdA112-tdA332))*t31-t30*((tdA112-
tdA222)*t21-t11*(tdA112-tdA332)))*t12+(q*((-2*tdA112+2*tdA222)*
q+tdA111-tdA331)*p+(2*(tdA112-tdA222))*t21*q^2+((-tdA111+
tdA331)*t21-t30*(tdA112-tdA222))*q+t30*(tdA111-tdA331)+h*
(dott11-tdA111))*t32^2+(((2*tdA112-2*tdA222)*q-tdA111+tdA331)*
p^2+(((-3*tdA112+2*tdA222+tdA332)*t31+(-2*tdA112+2*tdA222)*t21+
t11*(tdA112-tdA332))*q+(tdA111-tdA331)*t31+(tdA111-tdA331)*t21-
t30*(tdA112-tdA222))*p+3*t21*((tdA112-2*tdA222*(1/3)-(1/3)*
tdA332)*t31-(1/3)*t11*(tdA112-tdA332))*q+((-tdA111+tdA331)*t21-
t30*(tdA112-tdA332))*t31+t30*((tdA112-tdA222)*t21+t11*(tdA112-
tdA332)))*t32+(tdA112-tdA332)*(-t11+t31)*(p-t31)*(p-t21))*t22+(
(q*(tdA111-tdA331)*p-t21*(tdA111-tdA331)*q+t30*(tdA111-tdA331)+
h*(dott11-tdA111))*t32-(tdA111-tdA331)*(p-t31)*(p-t21))*t12^2+(
(-q*(tdA111-tdA331)*p+t21*(tdA111-tdA331)*q+(-tdA111+tdA331)*
t30-h*(dott11-tdA111))*t32^2+(((-tdA112+tdA222)*q+tdA111-
tdA331)*p^2+(((tdA112-tdA332)*t31+(2*tdA112-2*tdA222)*t21-t11*
(tdA112-tdA332))*q+(-tdA111+tdA331)*t31+(-tdA111+tdA331)*t21-
t30*(tdA112-tdA222))*p-((tdA112-tdA332)*t31+(tdA112-tdA222)*t21
-t11*(tdA112-tdA332))*t21*q+((tdA111-tdA331)*t21+t30*(tdA112-
tdA332))*t31+t30*((tdA112-tdA222)*t21-t11*(tdA112-tdA332)))*
t32+(p-t21)*(p-t31)*((tdA112-tdA222)*p+(-tdA112+tdA332)*t31+(-
tdA112+tdA222)*t21+t11*(tdA112-tdA332)))*t12-(tdA112-tdA222)*(
(-p*q+q*t21-t30)*t32+(p-t31)*(p-t21))*(p-t21)*t32)*a12+((q^2*
t32-p*q+q*t31-t20)*t22+(-p*q+q*t21+t20)*t32+(p-t31)*(p-t21))*
(t12-t32)*(t12-t22)^2*tdA120)*a13+tdA231*a12^2*(t12-t32)*(t12-
t22)*((q^2*t32-p*q+q*t31+t30)*t22+(-p*q+q*t21-t30)*t32+(p-t31)*
```

```
        (p-t21)))/(a12^2*(t12-t22)*(t12-t32)*a13*(t22-t32)):
     simplify(Eq3);
     simplify(Eq1);
```

$$0$$

```
> tdA230:=(((((tdA110-tdA330)*t12^3+((-2*tdA110+2*tdA330)*t32+(-
     tdA110+tdA330)*t22+(-tdA111+tdA331)*t11+(tdA111-tdA331)*t31+h*
     (tdA112-tdA332))*t12^2+((tdA110-tdA330)*t32^2+((2*tdA110-2*
     tdA330)*t22+(tdA111-tdA331)*t11+(-tdA111+tdA331)*t31-h*(tdA112-
     tdA332))*t32+((-q*tdA112+q*tdA222+tdA111-tdA331)*t11+(q*tdA112-
     q*tdA222-tdA111+tdA331)*t31-h*(tdA112-tdA332))*t22-(t11*(tdA112
     -tdA332)+(-tdA112+tdA332)*t31+(tdA112-tdA222)*(p-t21))*(-t11+
     t31))*t12-t22*(tdA110-tdA330)*t32^2+(((q*tdA112-q*tdA222-
     tdA111+tdA331)*t11+(-q*tdA112+q*tdA222+tdA111-tdA331)*t31+h*
     (tdA112-tdA332))*t22+(tdA112-tdA222)*(-t11+t31)*(p-t21))*t32-
     t22*(-t11+t31)^2*(tdA112-tdA332))*a12-tdA120*(t12-t32)^2*(t12-
     t22)*(-q*t22+p-t21))*a13-(t12-t32)*a12*(tdA231*(-t11+t31)*a12+
     dota13*h*(t12-t32))*(t12-t22))/((t12-t22)*(t12-t32)^2*a12^2):
     simplify(Eq9);
     tdA311:=(((-q^2*t32+p*q-q*t31+2*h-t30)*t22+(p*q-q*t21-2*h+t30)*
     t32-(p-t31)*(p-t21))*tdA112+((q^2*t32-p*q+q*t31+t30)*t22+(-p*q+
     q*t21-t30)*t32+(p-t31)*(p-t21))*tdA332-dott12*h*(t22-t32))/
     (a13*(t22-t32)):
     simplify(Eq10);
     tdA231:=-(tdA222-tdA332)*(-q*t22+p-t21)*a13/(a12*(t22-t32)):
     simplify(Eq16);
     tdA112:=(1/2)*dott12:
     simplify(Eq19);
     tdA120:=-((((q*dott12-2*q*tdA222-2*tdA111+2*tdA221)*t12+(-q*
     dott12+2*q*tdA222+2*tdA111-2*tdA221)*t22+(t11-t21)*(dott12-2*
     tdA222))*t32^2+((2*q*tdA222-2*q*tdA332+2*tdA111-2*tdA221)*
     t12^2+(-q*(dott12+2*tdA222-4*tdA332)*t22-(dott12-2*tdA222)*(p-
     t31+t11-t21))*t12+((q*dott12-2*q*tdA332-2*tdA111+2*tdA221)*t22+
     (dott12-2*tdA222)*(p-t31-t11+t21))*t22)*t32+((-2*tdA111+2*
     tdA221)*t22-(2*(tdA222-tdA332))*(p-t31))*t12^2+((2*tdA111-2*
     tdA221)*t22+(p-t31+t11-t21)*dott12+(2*p-2*t31-2*t11+2*t21)*
     tdA222-4*tdA332*(p-t31))*t22*t12-t22^2*(dott12-2*tdA332)*(p-
     t31))*a12)/(2*(t12-t22)^2*(t12-t32)*(t22-t32)):
     simplify(Eq22);
     tdA332:=(1/2)*dott32:
     simplify(Eq25);
     tdA222:=(1/2)*dott22:
     simplify(Eq14);
```

```
tdA220:=simplify(solve(simplify(Eq23),tdA220)):
simplify(Eq23);
tdA310:=factor(solve(simplify(Eq20),tdA310)):
simplify(Eq20);
tdA111:=dott11:
simplify(Eq11);
tdA330:=factor(solve(simplify(Eq17),tdA330)):
simplify(Eq17);
tdA331:=dott31:
simplify(Eq26);
tdA221:=dott21:
simplify(Eq15);
```

$$
\begin{aligned}
&0\\
&0\\
&0\\
&0\\
&0\\
&0\\
&0\\
&0\\
&0\\
&0\\
&0\\
&0
\end{aligned}
$$

(4.14)

```
> Eq1:=simplify(Eq1);
  Eq2:=simplify(Eq2);
  Eq3:=simplify(Eq3);
  Eq4:=simplify(Eq4);
  Eq5:=simplify(Eq5);
  Eq6:=simplify(Eq6);
  Eq7:=simplify(Eq7);
  Eq8:=simplify(Eq8);
  Eq9:=simplify(Eq9);
  Eq10:=simplify(Eq10);
  Eq11:=simplify(Eq11);

  Eq13:=simplify(Eq13);
  Eq14:=simplify(Eq14);
  Eq15:=simplify(Eq15);
  Eq16:=simplify(Eq16);
  Eq17:=simplify(Eq17);

  Eq19:=simplify(Eq19);
  Eq20:=simplify(Eq20);
```

```
Eq22:=simplify(Eq22);
Eq23:=simplify(Eq23);

Eq25:=simplify(Eq25);
Eq26:=simplify(Eq26);
```

$$Eq1 := 0$$
$$Eq2 := 0$$
$$Eq3 := 0$$
$$Eq4 := 0$$
$$Eq5 := 0$$
$$Eq6 := 0$$
$$Eq7 := 0$$
$$Eq8 := 0$$
$$Eq9 := 0$$
$$Eq10 := 0$$
$$Eq11 := 0$$
$$Eq13 := 0$$
$$Eq14 := 0$$
$$Eq15 := 0$$
$$Eq16 := 0$$
$$Eq17 := 0$$
$$Eq19 := 0$$
$$Eq20 := 0$$
$$Eq22 := 0$$
$$Eq23 := 0$$
$$Eq25 := 0$$
$$Eq26 := 0$$

(4.15)

```
> Eq12:=simplify(Eq12):
  Eq18:=simplify(Eq18):
  Eq21:=simplify(Eq21):
  Eq24:=simplify(Eq24):
```

There are only 4 remaining non trivial equations. Solving the first two gives the evolutions for (q,p) and we check that these evolutions are Hamiltonian

```
> dotq:=factor(solve(simplify(Eq12),dotq)):
  simplify(Eq12);
  dotp:=solve(simplify(Eq18),dotp):
  simplify(Eq18);
  simplify(Eq21);
  simplify(Eq24);
  dotq:=factor(simplify(dotq)):
```

$$0$$
$$0$$
$$0$$
$$0$$

(4.16)

```
> H:=unapply( int(h*dotq,p),q,p):
  f:=unapply( factor(-simplify(int(simplify(diff(H(q,p),q)+h*
  dotp),q)))), q):
```

```
Hamiltonian:=unapply(simplify(H(q,p)+f(q)),q,p):
simplify(diff(Hamiltonian(q,p),p)-h*dotq);
simplify(diff(Hamiltonian(q,p),q)+h*dotp);
```

$$0 \qquad\qquad (4.17)$$
$$0$$

```
> dotqfunction:=unapply(dotq,t12,t22,t32,t11,t21,t31,dott12,
  dott22,dott32,dott11,dott21,dott31):
  dotpfunction:=unapply(dotp,t12,t22,t32,t11,t21,t31,dott12,
  dott22,dott32,dott11,dott21,dott31):
  P1function:=unapply(P1(lambda),lambda,t12,t22,t32,t11,t21,t31):
  dP1function:=unapply(dP1(lambda),lambda,t12,t22,t32,t11,t21,
  t31,dott12,dott22,dott32,dott11,dott21,dott31):
```

Expression of the Hamiltonian evolutions in the trivial directions

```
> factor(h*dotqfunction(t12,t22,t32,t11,t21,t31,0,0,0,1,1,1));
  factor(h*dotpfunction(t12,t22,t32,t11,t21,t31,0,0,0,1,1,1));
  Hamv1:=(q,p)->-h*q;
  simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,0,0,0,1,1,1)-
  diff(Hamv1(q,p),p));
  simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,0,0,0,1,1,1)+
  diff(Hamv1(q,p),q));
```

$$0 \qquad\qquad (5.1)$$
$$h$$
$$Hamv1 := (q, p) \rightarrow -h\,q$$
$$0$$
$$0$$

```
> factor(h*dotqfunction(t12,t22,t32,t11,t21,t31,1,1,1,0,0,0));
  factor(h*dotpfunction(t12,t22,t32,t11,t21,t31,1,1,1,0,0,0));
  Hamv2:=(q,p)->-h*q^2/2;
  simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,1,1,1,0,0,0)-
  diff(Hamv2(q,p),p));
  simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,1,1,1,0,0,0)+
  diff(Hamv2(q,p),q));
```

$$0 \qquad\qquad (5.2)$$
$$h\,q$$
$$Hamv2 := (q, p) \rightarrow -\frac{1}{2}\,h\,q^2$$
$$0$$
$$0$$

```
> simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,0,0,0,t12,t22,
  t32));
  simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,0,0,0,t12,t22,
```

```
    t32));
  Hamu1:=(q,p)->-h*p;
  simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,0,0,0,t12,t22,
  t32)- diff(Hamu1(q,p),p));
  simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,0,0,0,t12,t22,
  t32)+ diff(Hamu1(q,p),q));
```

$$-h \quad \text{(5.3)}$$
$$0$$
$$Hamu1 := (q, p) \rightarrow -h\,p$$
$$0$$
$$0$$

```
> simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,2*t12,2*t22,2*
  t32,t11,t21,t31));
  simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,2*t12,2*t22,2*
  t32,t11,t21,t31));
  Hamu2:=(q,p)->-h*q*p;
  simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,2*t12,2*t22,2*
  t32,t11,t21,t31)- diff(Hamu2(q,p),p));
  simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,2*t12,2*t22,2*
  t32,t11,t21,t31)+ diff(Hamu2(q,p),q));
```

$$-h\,q \quad \text{(5.4)}$$
$$p\,h$$
$$Hamu2 := (q, p) \rightarrow -h\,q\,p$$
$$0$$
$$0$$

```
> simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,2*(t12-t32)*
  (t12-t22),0,0,t12*(3*t11-t21-t31)-(t12+t22+t32)*t11+ t22*t31+
  t32*t21,0,0));
  simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,2*(t12-t32)*
  (t12-t22),0,0,t12*(3*t11-t21-t31)-(t12+t22+t32)*t11+t22*t31+
  t32*t21,0,0));
  Hama1:=(q,p)->h*( (t22+t32)*q*p-p^2+(t21+t31)*p -t32*t22*q^2 -
  (t22*t31+t32*t21)*q);
  simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,2*(t12-t32)*
  (t12-t22),0,0,t12*(3*t11-t21-t31)-(t12+t22+t32)*t11+ t22*t31+
  t32*t21,0,0)- diff(Hama1(q,p),p));
  simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,2*(t12-t32)*
  (t12-t22),0,0,t12*(3*t11-t21-t31)-(t12+t22+t32)*t11+ t22*t31+
  t32*t21,0,0)+ diff(Hama1(q,p),q));
```

$$(t22 + t32)\,h\,q - (-t31 + 2\,p - t21)\,h \quad \text{(5.5)}$$
$$-h\,((-2\,q\,t32 + p - t31)\,t22 + t32\,(p - t21))$$
$$Hama1 := (q, p) \rightarrow h\,\big((t22 + t32)\,q\,p - p^2 + (t21 + t31)\,p - t32\,t22\,q^2 - (t21\,t32$$
$$+ t22\,t31)\,q\big)$$
$$0$$
$$0$$

```
> simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,0,2*(t22-t12)*
  (t22-t32),0,0,t22*(3*t21-t11-t31)- (t12+t22+t32)*t21+t12*t31+
  t32*t11,0));
  simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,0,2*(t22-t12)*
  (t22-t32),0,0,t22*(3*t21-t11-t31)- (t12+t22+t32)*t21+t12*t31+
  t32*t11,0));
  Hama2:=(q,p)->h*( (t12+t32)*q*p-p^2+(t11+t31)*p -t32*t12*q^2 -
  (t12*t31+t32*t11)*q);
  simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,0,2*(t22-t12)*
  (t22-t32),0,0,t22*(3*t21-t11-t31)- (t12+t22+t32)*t21+t12*t31+
  t32*t11,0)- diff(Hama2(q,p),p));
  simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,0,2*(t22-t12)*
  (t22-t32),0,0,t22*(3*t21-t11-t31)- (t12+t22+t32)*t21+t12*t31+
  t32*t11,0)+ diff(Hama2(q,p),q));
```

$$(t12 + t32)\, h\, q - (-t31 + 2\, p - t11)\, h \qquad\qquad \textbf{(5.6)}$$
$$-h\left((-2\, q\, t32 + p - t31)\, t12 + t32\,(p - t11)\right)$$

$Hama2 := (q, p) \rightarrow h\left((t12 + t32)\, q\, p - p^2 + (t11 + t31)\, p - t32\, t12\, q^2 - (t11\, t32\right.$
$\left. + t12\, t31)\, q\right)$

$$0$$
$$0$$

```
> simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,0,0,2*(t32-t12)
  *(t32-t22),0,0,t32*(3*t31-t11-t21)- (t12+t22+t32)*t31+t12*t21+
  t22*t11));
  simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,0,0,2*(t32-t12)
  *(t32-t22),0,0,t32*(3*t31-t11-t21)- (t12+t22+t32)*t31+t12*t21+
  t22*t11));
  Hama3:=(q,p)->h*( (t12+t22)*q*p-p^2+(t11+t21)*p -t22*t12*q^2 -
  (t12*t21+t22*t11)*q);
  simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,0,0,2*(t32-t12)
  *(t32-t22),0,0,t32*(3*t31-t11-t21)- (t12+t22+t32)*t31+t12*t21+
  t22*t11)- diff(Hama3(q,p),p));
  simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,0,0,2*(t32-t12)
  *(t32-t22),0,0,t32*(3*t31-t11-t21)- (t12+t22+t32)*t31+t12*t21+
  t22*t11)+ diff(Hama3(q,p),q));
```

$$(t12 + t22)\, h\, q - (2\, p - t11 - t21)\, h \qquad\qquad \textbf{(5.7)}$$
$$-h\left((-2\, q\, t22 + p - t21)\, t12 + t22\,(p - t11)\right)$$

$Hama3 := (q, p) \rightarrow h\left((t12 + t22)\, q\, p - p^2 + (t11 + t21)\, p - t22\, t12\, q^2 - (t11\, t22\right.$
$\left. + t12\, t21)\, q\right)$

$$0$$
$$0$$

```
> simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,0,0,0,1,0,0));
  simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,0,0,0,1,0,0));
  Hame11 := unapply( (-p^3+((t12+t22+t32)*q+t11+t21+t31)*p^2+(((-
  t22-t32)*t12-t22*t32)*q^2+((-t31-t21)*t12+(-t11-t31)*t22-t32*
```

```
    (t11+t21))*q+t12*(-t20-t30)+t22*t20+t32*t30+(-t31-t21)*t11-t21*
    t31)*p+q*(q^2*t12*t22*t32+((t22*t31+t32*t21)*t12+t22*t32*t11)*
    q+(t22*t30+t32*t20+t21*t31)*t12+((-t20-t30)*t32+t11*t31)*t22+
    t32*t11*t21))/((t12-t32)*(t12-t22))  ,q,p);
    simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,0,0,0,1,0,0)-
    diff(Hame11(q,p),p));
    simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,0,0,0,1,0,0)+
    diff(Hame11(q,p),q));

    simplify( Hame11(q,p)-1/((t32-t12)*(t22-t12))*(-p^3+ P1(q)*p^2-
    P2(q)*p+P3(q)));
```

$$\frac{1}{(-t12+t32)\,(t12-t22)}\bigg(((t22+t32)\,t12+t22\,t32)\,q^2+\bigg((t31-2\,p+t21)\,t12+(\qquad \textbf{(5.8)}$$
$$-2\,p+t31+t11)\,t22-2\left(p-\frac{1}{2}\,t11-\frac{1}{2}\,t21\right)t32\bigg)q+(t20+t30)\,t12-t20\,t22$$
$$-t30\,t32+3\,p^2+(-2\,t31-2\,t11-2\,t21)\,p+t11\,(t21+t31)+t21\,t31\bigg)$$
$$\frac{1}{(-t12+t32)\,(t12-t22)}\bigg(\big((3\,q^2\,t32-2\,p\,q+2\,q\,t31+t30)\,t22+(-2\,p\,q+2\,q\,t21$$
$$+t20)\,t32+(p-t31)\,(p-t21)\big)\,t12+((-2\,p\,q+2\,q\,t11-t20-t30)\,t32+(p$$
$$-t31)\,(p-t11))\,t22+t32\,(p-t21)\,(p-t11)\bigg)$$

$$Hame11:=(q,p)\to\frac{1}{(t12-t32)\,(t12-t22)}\bigg(-p^3+((t12+t22+t32)\,q+t11+t21$$
$$+t31)\,p^2+\big(((-t22-t32)\,t12-t22\,t32)\,q^2+((-t31-t21)\,t12+(-t11-t31)\,t22$$
$$-t32\,(t11+t21))\,q+t12\,(-t20-t30)+t20\,t22+t30\,t32+(-t31-t21)\,t11$$
$$-t21\,t31\big)\,p+q\,(q^2\,t12\,t22\,t32+((t21\,t32+t22\,t31)\,t12+t22\,t32\,t11)\,q+(t20\,t32$$
$$+t21\,t31+t22\,t30)\,t12+((-t20-t30)\,t32+t11\,t31)\,t22+t32\,t11\,t21)\bigg)$$

$$0$$
$$0$$
$$0$$

```
> simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,0,0,0,0,1,0));
  simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,0,0,0,0,1,0));
  Hame21:= unapply( (p^3+((-t12-t22-t32)*q-t31-t11-t21)*p^2+((
  (t12+t32)*t22+t12*t32)*q^2+(t12*(t21+t31)+t22*(t11+t31)+t32*
  (t11+t21))*q+(h-t20)*t22+(t30-h+t20)*t12-t32*t30+t11*(t21+t31)+
  t21*t31)*p-q*(q^2*t12*t22*t32+((t12*t31+t32*t11)*t22+t12*t32*
  t21)*q+(t12*t30+(-t30+h-t20)*t32+t11*t31)*t22+((-h+t20)*t32+
  t21*t31)*t12+t32*t11*t21))/((t22-t32)*(t12-t22)),q,p);
  simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,0,0,0,0,1,0)-
  diff(Hame21(q,p),p));
  simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,0,0,0,0,1,0)+
  diff(Hame21(q,p),q));

  simplify( Hame21(q,p)- (1/((t32-t22)*(t12-t22))*(-p^3+ P1(q)*
```

```
   p^2-P2(q)*p+P3(q)) +(p-q*t32)*h/(t32-t22))  );
```

$$\frac{1}{(-t22+t32)\,(t12-t22)}\Bigg(\big((-t22-t32)\,t12-t22\,t32\big)\,q^2+\bigg((-t31+2\,p-t21)\,t12 \qquad\textbf{(5.9)}$$

$$+\,(-t31+2\,p-t11)\,t22+2\left(p-\frac{1}{2}\,t11-\frac{1}{2}\,t21\right)t32\bigg)q+(h-t20-t30)\,t12$$

$$+\,(-h+t20)\,t22+t30\,t32-3\,p^2+(2\,t31+2\,t11+2\,t21)\,p+(-t31-t21)\,t11$$

$$-\,t21\,t31\Bigg)$$

$$\frac{1}{(-t22+t32)\,(t12-t22)}\Big(\big((-3\,q^2\,t12+2\,p\,q-2\,q\,t11-h+t20+t30)\,t32+(2\,p\,q$$

$$-\,2\,q\,t31-t30)\,t12+(p-t11)\,(-p+t31)\big)\,t22+\big((2\,p\,q-2\,q\,t21+h-t20)\,t12$$

$$-\,(p-t21)\,(p-t11)\big)\,t32+t12\,(p-t21)\,(-p+t31)\Big)$$

$$Hame21:=(q,p)\rightarrow\frac{1}{(t22-t32)\,(t12-t22)}\Big(p^3+\big((-t12-t22-t32)\,q-t31-t11$$

$$-\,t21\big)\,p^2+\big(((t12+t32)\,t22+t12\,t32)\,q^2+(t12\,(t21+t31)+t22\,(t11+t31)$$

$$+\,t32\,(t11+t21))\,q+(h-t20)\,t22+(-h+t20+t30)\,t12-t30\,t32+t11\,(t21$$

$$+\,t31)+t21\,t31\big)\,p-q\,\big(q^2\,t12\,t22\,t32+((t11\,t32+t12\,t31)\,t22+t12\,t21\,t32)\,q$$

$$+\,(t12\,t30+(h-t20-t30)\,t32+t11\,t31)\,t22+((-h+t20)\,t32+t21\,t31)\,t12$$

$$+\,t32\,t11\,t21\big)\Big)$$

$$0$$
$$0$$
$$0$$

```
> simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,0,0,0,0,0,1));
  simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,0,0,0,0,0,1));
  Hame31:=unapply((-p^3+(t11+t21+t31+(t12+t22+t32)*q)*p^2+(((-t12
  -t22)*t32-t12*t22)*q^2+((-t21-t11)*t32+(-t31-t21)*t12-t22*(t11+
  t31))*q+(-h+t30)*t32+(-t30+h-t20)*t12+t22*t20+(-t31-t21)*t11-
  t21*t31)*p+q*(t12*t22*t32*q^2+((t12*t21+t22*t11)*t32+t31*t12*
  t22)*q+(t12*t20+(-t30+h-t20)*t22+t11*t21)*t32+((-h+t30)*t22+
  t21*t31)*t12+t22*t11*t31))/((t22-t32)*(t12-t32)),q,p);
  simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,0,0,0,0,0,1)-
  diff(Hame31(q,p),p));
  simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,0,0,0,0,0,1)+
  diff(Hame31(q,p),q));

  simplify( Hame31(q,p)- (1/((t22-t32)*(t12-t32))*(-p^3+ P1(q)*
  p^2-P2(q)*p+P3(q)) -(p-q*t22)*h/(t32-t22))  );
```

$$\frac{1}{(-t22+t32)\,(-t12+t32)}\Big(\big((-t22-t32)\,t12-t22\,t32\big)\,q^2+\big((-t31+2\,p-t21)\,t12 \qquad\textbf{(5.10)}$$

$$+\,t32\,(2\,p-t11-t21)-(-2\,p+t31+t11)\,t22\big)\,q+(h-t20-t30)\,t12+(-h$$

$$+\,t30)\,t32+t20\,t22-3\,p^2+(2\,t31+2\,t11+2\,t21)\,p+(-t31-t21)\,t11-t21\,t31\Big)$$

$$\frac{1}{(-t22+t32)\,(-t12+t32)}\Big(\big((-3\,q^2\,t12+2\,p\,q-2\,q\,t11-h+t20+t30)\,t32+(2\,p\,q$$

$$-\,2\,q\,t31+h-t30)\,t12+(p-t11)\,(-p+t31)\big)\,t22+\big((2\,p\,q-2\,q\,t21-t20)\,t12$$

$$- (p - t21) (p - t11)) t32 + t12 (p - t21) (-p + t31))$$

$$Hame31 := (q,p) \rightarrow \frac{1}{(t22 - t32)(t12 - t32)} \left(-p^3 + ((t12 + t22 + t32) q + t11 + t21 \right.$$
$$+ t31) p^2 + (((-t12 - t22) t32 - t12\, t22) q^2 + ((-t21 - t11) t32 + (-t31 - t21) t12$$
$$- t22 (t11 + t31)) q + (-h + t30) t32 + (h - t20 - t30) t12 + t20\, t22 + (-t31$$
$$- t21) t11 - t21\, t31) p + q \left(q^2\, t12\, t22\, t32 + ((t11\, t22 + t12\, t21) t32 + t31\, t12\, t22) q \right.$$
$$+ (t20\, t12 + (h - t20 - t30) t22 + t11\, t21) t32 + ((-h + t30) t22 + t21\, t31) t12$$
$$\left. \left. + t22\, t11\, t31 \right) \right)$$

$$0$$
$$0$$
$$0$$

```
> simplify(Hame31(q,p)- (t12/(t12-t32)*Hamv1(q,p)+1/(t32-t12)*
  Hamu1(q,p)+(t22-t12)/(t12-t32)*Hame21(q,p)));
```

$$0 \tag{5.11}$$

```
> simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,1,0,0,0,0,0)):
  simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,1,0,0,0,0,0)):
  Hame12:=unapply(((t32*t22*(t21+t31-2*t11)*q^3+(-(t22+t32)*(t21+
  t31-2*t11)*p+(-h*t32+t31*(t21+t31-2*t11))*t22+t32*t21*(t21+t31
  -2*t11))*q^2+((t21+t31-2*t11)*p^2+(h*t22+h*t32-(t21+t31)*(t21+
  t31-2*t11))*p+(-2*t30*t11+t21*t30+(-h+t30)*t31)*t22+(-2*t11*
  t20+(-h+t20)*t21+t31*t20)*t32+t31*t21*(t21+t31-2*t11))*q-(p*h+
  (-2*t30-2*t20)*t11+(t21+t31)*(t30-h+t20))*p)*t12^2+(-((-t11+
  t31)*t22-t32*(t11-t21))*t32*t22*q^3+(((-t11+t31)*t22^2-t32^2*
  (t11-t21))*p+(h*t32-t31*(-t11+t31))*t22^2+(2*((1/2)*h*t32+(t11-
  t21)*(-t11+t31)))*t32*t22+t21*t32^2*(t11-t21))*q^2+(((-t11+t21)
  *t22+t32*(-t11+t31))*p^2+(-h*t22^2+(-2*h*t32+2*t11*(t11-t21))*
  t22-h*t32^2-2*t11*(-t11+t31)*t32)*p+(t30*t11-(-h+t30)*t31)*
  t22^2+(((3*t20+3*t30)*t11+(h-t20-2*t30)*t21-(-h+2*t20+t30)*t31)
  *t32+t31*(t11-t21)*(-2*t11+t31))*t22+(2*(((1/2)*t11*t20+(1/2)*
  (h-t20)*t21)*t32+(-t11+t31)*t21*(-(1/2)*t21+t11)))*t32)*q-p*(
  (t21+t31-2*t11)*p^2+(-h*t22-h*t32-(t11+t21+t31)*(t21+t31-2*t11)
  )*p+((3*t20+t30)*t11+(h-t20)*t21-(-h+2*t20+t30)*t31)*t22+(t11*
  (t20+3*t30)+(h-t20-2*t30)*t21-(-h+t30)*t31)*t32+(t21+t31-2*t11)
  *(t11*(t21+t31)+t21*t31)))*t12+t32*(((-t11+t31)*t22-t32*(t11-
  t21))*p+(-h*t32-t11*(-t11+t31))*t22+t32*t11*(t11-t21))*t22*q^2+
  (-((-t11+t31)*t22-t32*(t11-t21))*(t22+t32)*p^2+((t32*h-t11^2+
  t31^2)*t22^2+(h*t32^2+(-2*t11^2+2*t21*t31)*t32)*t22+(-t11^2+
  t21^2)*t32^2)*p+(((-t20-t30)*t11+(t30-h+t20)*t31)*t32-t11*t31*
  (-t11+t31))*t22^2-t32*((t11*(t20+t30)-(t30-h+t20)*t21)*t32-
  (t11*(t21+t31)-2*t21*t31)*t11)*t22+t32^2*t11*t21*(t11-t21))*q+
  p*(((-t11+t31)*t22-t32*(t11-t21))*p^2+((-h*t32-(-t11+t31)*(t11+
  t21+t31))*t22+t32*(t11-t21)*(t11+t21+t31))*p-t20*(-t11+t31)*
  t22^2+((t11*(t20+t30)+(h-t20)*t21-(-h+t30)*t31)*t32+(-t11+t31)*
```

```
(t11*(t21+t31)+t21*t31))*t22+t32*((-t31-t21)*t11+t32*t30-t21*
t31)*(t11-t21)))/(2*(t12-t32)^2*(t12-t22)^2)
,q,p);
simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,1,0,0,0,0,0)-
diff(Hame12(q,p),p));
simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,1,0,0,0,0,0)+
diff(Hame12(q,p),q));
```

$Hame12 := (q, p) \rightarrow \dfrac{1}{2} \dfrac{1}{(t12 - t32)^2 (t12 - t22)^2} \Big((t32\, t22\, (t31 - 2\, t11 + t21)\, q^3 + ($ **(5.12)**

$- (t22 + t32)(t31 - 2\, t11 + t21)\, p + (-h\, t32 + t31\,(t31 - 2\, t11 + t21))\, t22$

$+ t32\, t21\, (t31 - 2\, t11 + t21))\, q^2 + \big((t31 - 2\, t11 + t21)\, p^2 + (h\, t22 + h\, t32 - (t21$

$+ t31)(t31 - 2\, t11 + t21))\, p + (-2\, t11\, t30 + t21\, t30 + (-h + t30)\, t31)\, t22 + ($

$-2\, t11\, t20 + (-h + t20)\, t21 + t31\, t20)\, t32 + t31\, t21\, (t31 - 2\, t11 + t21))\, q - (h\, p$

$+ (-2\, t20 - 2\, t30)\, t11 + (t21 + t31)(-h + t20 + t30))\, p)\, t12^2 + \Big(-((-t11$

$+ t31)\, t22 - t32\,(t11 - t21))\, t32\, t22\, q^3 + \Big(((-t11 + t31)\, t22^2 - t32^2\,(t11 - t21))\, p$

$+ (h\, t32 - t31\,(-t11 + t31))\, t22^2 + 2\Big(\dfrac{1}{2}\, h\, t32 + (t11 - t21)(-t11 + t31)\Big)\, t32\, t22$

$+ t21\, t32^2\,(t11 - t21)\Big)\, q^2 + \Big(((-t11 + t21)\, t22 + t32\,(-t11 + t31))\, p^2 + \big(-h\, t22^2$

$+ (-2\, h\, t32 + 2\, t11\,(t11 - t21))\, t22 - h\, t32^2 - 2\, t11\,(-t11 + t31)\, t32\big)\, p + (t11\, t30$

$- (-h + t30)\, t31)\, t22^2 + (((3\, t30 + 3\, t20)\, t11 + t21\,(h - t20 - 2\, t30) - (-h$

$+ 2\, t20 + t30)\, t31)\, t32 + t31\,(t11 - t21)(t31 - 2\, t11))\, t22 + 2\Big(\Big(\dfrac{1}{2}\, t11\, t20$

$+ \dfrac{1}{2}\,(h - t20)\, t21\Big)\, t32 + (-t11 + t31)\, t21\Big(t11 - \dfrac{1}{2}\, t21\Big)\Big)\, t32\Big)\, q - p\,((t31$

$- 2\, t11 + t21)\, p^2 + (-h\, t22 - h\, t32 - (t11 + t21 + t31)(t31 - 2\, t11 + t21))\, p$
$+ (t11\,(t30 + 3\, t20) + (h - t20)\, t21 - (-h + 2\, t20 + t30)\, t31)\, t22 + (t11\,(t20$
$+ 3\, t30) + t21\,(h - t20 - 2\, t30) - (-h + t30)\, t31)\, t32 + (t31 - 2\, t11$
$+ t21)(t11\,(t21 + t31) + t21\, t31)))\, t12 + t32\,(((-t11 + t31)\, t22 - t32\,(t11$
$- t21))\, p + (-h\, t32 - t11\,(-t11 + t31))\, t22 + t32\, t11\,(t11 - t21))\, t22\, q^2 + \big(-((t11$
$- t11 + t31)\, t22 - t32\,(t11 - t21))(t22 + t32)\, p^2 + \big((h\, t32 - t11^2 + t31^2)\, t22^2$
$+ \big(h\, t32^2 + (-2\, t11^2 + 2\, t21\, t31)\, t32\big)\, t22 + \big(-t11^2 + t21^2\big)\, t32^2\big)\, p + (((-t20$
$- t30)\, t11 + (-h + t20 + t30)\, t31)\, t32 - t11\, t31\,(-t11 + t31))\, t22^2 - t32\,(((t20$
$+ t30)\, t11 - (-h + t20 + t30)\, t21)\, t32 - (t11\,(t21 + t31) - 2\, t21\, t31)\, t11)\, t22$
$+ t32^2\, t11\, t21\,(t11 - t21))\, q + p\,(((-t11 + t31)\, t22 - t32\,(t11 - t21))\, p^2 + ((
$-h\, t32 - (-t11 + t31)(t11 + t21 + t31))\, t22 + t32\,(t11 - t21)(t11 + t21 + t31))\, p$
$- t20\,(-t11 + t31)\, t22^2 + (((t20 + t30)\, t11 + (h - t20)\, t21 - (-h + t30)\, t31)\, t32$
$+ (-t11 + t31)(t11\,(t21 + t31) + t21\, t31))\, t22 + t32\,((-t31 - t21)\, t11 + t30\, t32$

$$-t21\ t31)\ (t11-t21)))$$

$$0$$
$$0$$

```
> simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,0,1,0,0,0,0)):
  simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,0,1,0,0,0,0)):
  Hame22:=unapply( ((t12*t32*(t11-2*t21+t31)*q^3+(-(t12+t32)*(t11
  -2*t21+t31)*p+(-h*t12+t11*(t11-2*t21+t31))*t32+t12*t31*(t11-2*
  t21+t31))*q^2+((t11-2*t21+t31)*p^2+(h*t32+h*t12-(t11+t31)*(t11
  -2*t21+t31))*p+((2*t30-2*h+2*t20)*t21+(-t30+h-t20)*t31-t11*
  (t20+t30))*t32+(-2*t21*t30+(-h+t30)*t31+t30*t11)*t12+t11*t31*
  (t11-2*t21+t31))*q-(p*h+(-2*h+2*t20)*t21-t20*(t11+t31))*p)*
  t22^2+(-t12*t32*(t32*(t11-t21)+t12*(-t21+t31))*q^3+((t32^2*(t11
  -t21)+t12^2*(-t21+t31))*p+(h*t12-t11*(t11-t21))*t32^2-2*t12*(-
  (1/2)*h*t12+(t11-t21)*(-t21+t31))*t32-t31*(-t21+t31)*t12^2)*
  q^2+(((-t21+t31)*t32+t12*(t11-t21))*p^2+(-h*t32^2+(-2*h*t12-2*
  t21*(-t21+t31))*t32-t12*(h*t12+2*t21*(t11-t21)))*p+((-t30+h-
  t20)*t21+t11*(t20+t30))*t32^2+(((3*h-3*t20)*t21+(-h+2*t20+t30)*
  t31-(-t20+t30)*t11)*t12-t11*(-2*t21+t11)*(-t21+t31))*t32-t12*(
  (-t21*t30+(-h+t30)*t31)*t12+t31*(t11-t21)*(t31-2*t21)))*q-((t11
  -2*t21+t31)*p^2+(-h*t32-h*t12-(t11+t21+t31)*(t11-2*t21+t31))*p+
  ((2*t30+h-t20)*t21+(h-t30)*t31-(-t20+t30)*t11)*t32+((-2*t30+3*
  h-3*t20)*t21+(-h+2*t20+t30)*t31+t11*(t20+t30))*t12+(t11-2*t21+
  t31)*((t11+t31)*t21+t11*t31))*p)*t22+t12*((t32*(t11-t21)+t12*(-
  t21+t31))*p+(-h*t12-t21*(t11-t21))*t32-t12*t21*(-t21+t31))*t32*
  q^2+(-(t12+t32)*(t32*(t11-t21)+t12*(-t21+t31))*p^2+((t12*h+
  t11^2-t21^2)*t32^2+(h*t12^2+(2*t11*t31-2*t21^2)*t12)*t32+t12^2*
  (-t21^2+t31^2))*p+(((-h+t20)*t21-t11*t20)*t12-t11*t21*(t11-t21)
  )*t32^2-t12*(((h-t20)*t21+t31*t20)*t12+(-t11-t31)*t21^2+2*t11*
  t21*t31)*t32-t31*t12^2*t21*(-t21+t31))*q+((t32*(t11-t21)+t12*(-
  t21+t31))*p^2+((-h*t12-(t11-t21)*(t11+t21+t31))*t32-t12*(-t21+
  t31)*(t11+t21+t31))*p-t30*(t11-t21)*t32^2+(((h-t20)*t21+(h-t30)
  *t31+t11*(t20+t30))*t12+(t11-t21)*((t11+t31)*t21+t11*t31))*t32+
  t12*((t30-h+t20)*t12+(t11+t31)*t21+t11*t31)*(-t21+t31))*p)/(2*
  (t22-t32)^2*(t12-t22)^2),q,p);
  simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,0,1,0,0,0,0)-
  diff(Hame22(q,p),p));
  simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,0,1,0,0,0,0)+
  diff(Hame22(q,p),q));
```

$$Hame22 := (q, p) \rightarrow \frac{1}{2}\ \frac{1}{(t22-t32)^2\ (t12-t22)^2} \Big(\big(t12\ t32\ (t11-2\ t21+t31)\ q^3 + \big(\qquad \textbf{(5.13)}$$

$$- (t12+t32)\ (t11-2\ t21+t31)\ p + (-h\ t12 + t11\ (t11-2\ t21+t31))\ t32$$

$$+ t12\, t31\, (t11 - 2\, t21 + t31))\, q^2 + ((t11 - 2\, t21 + t31)\, p^2 + (h\, t32 + h\, t12 - (t11$$

$$+ t31)\, (t11 - 2\, t21 + t31))\, p + ((-2\, h + 2\, t30 + 2\, t20)\, t21 + (h - t20 - t30)\, t31$$

$$- (t20 + t30)\, t11)\, t32 + (-2\, t21\, t30 + (-h + t30)\, t31 + t11\, t30)\, t12 + t11\, t31\, (t11$$

$$- 2\, t21 + t31))\, q - (h\, p + (-2\, h + 2\, t20)\, t21 - t20\, (t11 + t31))\, p)\, t22^2 + \Big($$

$$-t12\, t32\, (t32\, (t11 - t21) + t12\, (-t21 + t31))\, q^3 + \Big((t32^2\, (t11 - t21) + t12^2\, (-t21$$

$$+ t31))\, p + (h\, t12 - t11\, (t11 - t21))\, t32^2 - 2\, t12 \Big(-\frac{1}{2}\, h\, t12 + (t11 - t21)\, (-t21$$

$$+ t31)\Big)\, t32 - t31\, (-t21 + t31)\, t12^2\Big)\, q^2 + (((-t21 + t31)\, t32 + t12\, (t11 - t21))\, p^2$$

$$+ (-h\, t32^2 + (-2\, h\, t12 - 2\, t21\, (-t21 + t31))\, t32 - t12\, (h\, t12 + 2\, t21\, (t11$$
$$- t21)))\, p + (t21\, (h - t20 - t30) + (t20 + t30)\, t11)\, t32^2 + (((3\, h - 3\, t20)\, t21 + ($$
$$-h + 2\, t20 + t30)\, t31 - t11\, (-t20 + t30))\, t12 - t11\, (t11 - 2\, t21)\, (-t21 + t31))\, t32$$
$$- t12\, ((-t21\, t30 + (-h + t30)\, t31)\, t12 + t31\, (t11 - t21)\, (-2\, t21 + t31)))\, q$$
$$- ((t11 - 2\, t21 + t31)\, p^2 + (-h\, t32 - h\, t12 - (t11 + t21 + t31)\, (t11 - 2\, t21$$
$$+ t31))\, p + ((h + 2\, t30 - t20)\, t21 + (h - t30)\, t31 - t11\, (-t20 + t30))\, t32 + (($$
$$-2\, t30 + 3\, h - 3\, t20)\, t21 + (-h + 2\, t20 + t30)\, t31 + (t20 + t30)\, t11)\, t12 + (t11$$
$$- 2\, t21 + t31)\, ((t11 + t31)\, t21 + t11\, t31))\, p)\, t22 + t12\, ((t32\, (t11 - t21) + t12\, ($$
$$-t21 + t31))\, p + (-h\, t12 - t21\, (t11 - t21))\, t32 - t12\, t21\, (-t21 + t31))\, t32\, q^2 + \Big($$
$$- (t12 + t32)\, (t32\, (t11 - t21) + t12\, (-t21 + t31))\, p^2 + \Big((h\, t12 + t11^2 - t21^2)\, t32^2$$
$$+ (h\, t12^2 + (2\, t11\, t31 - 2\, t21^2)\, t12)\, t32 + t12^2\, (-t21^2 + t31^2)\Big)\, p + (((-h$$
$$+ t20)\, t21 - t11\, t20)\, t12 - t11\, t21\, (t11 - t21))\, t32^2 - t12 \Big(((h - t20)\, t21$$
$$+ t31\, t20)\, t12 + (-t11 - t31)\, t21^2 + 2\, t11\, t21\, t31\Big)\, t32 - t31\, t12^2\, t21\, (-t21 + t31)\Big)\, q$$
$$+ ((t32\, (t11 - t21) + t12\, (-t21 + t31))\, p^2 + ((-h\, t12 - (t11 - t21)\, (t11 + t21$$
$$+ t31))\, t32 - t12\, (-t21 + t31)\, (t11 + t21 + t31))\, p - t30\, (t11 - t21)\, t32^2 + (((h$$
$$- t20)\, t21 + (h - t30)\, t31 + (t20 + t30)\, t11)\, t12 + (t11 - t21)\, ((t11 + t31)\, t21$$
$$+ t11\, t31))\, t32 + t12\, ((-h + t20 + t30)\, t12 + (t11 + t31)\, t21 + t11\, t31)\, (-t21$$
$$+ t31))\, p)$$

<div align="center">0</div>
<div align="center">0</div>

> `simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,0,0,1,0,0,0)):`
  `simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,0,0,1,0,0,0)):`
  `Hame32:=unapply(   ((-(2*(t31-(1/2)*t11-(1/2)*t21))*t12*t22*q^3+`
  `((2*(t31-(1/2)*t11-(1/2)*t21))*(t12+t22)*p+(-h*t12-(2*(t31-`
  `(1/2)*t11-(1/2)*t21))*t11)*t22-(2*(t31-(1/2)*t11-(1/2)*t21))*`
  `t12*t21)*q^2+((t11+t21-2*t31)*p^2+(h*t22+h*t12+(2*(t31-(1/2)*`
  `t11-(1/2)*t21))*(t11+t21))*p+((2*t30-2*h+2*t20)*t31+(-t30+h-`
  `t20)*t21-t11*(t20+t30))*t22+(-2*t31*t20+(-h+t20)*t21+t11*t20)*`
  `t12-2*t21*(t31-(1/2)*t11-(1/2)*t21)*t11)*q-p*(p*h+(2*t30-2*h)*`
  `t31-t30*(t11+t21)))*t32^2+(t12*((-t11+t31)*t22+t12*(-t21+t31))*`
  `t22*q^3+(((t11-t31)*t22^2-t12^2*(-t21+t31))*p+(h*t12+t11*(-t11+`

```
t31))*t22^2-2*t12*(-(1/2)*h*t12+(-t21+t31)*(-t11+t31))*t22+t21*
t12^2*(-t21+t31))*q^2+(((t21-t31)*t22-t12*(-t11+t31))*p^2+(-h*
t22^2+(-2*h*t12+2*t31*(-t21+t31))*t22-h*t12^2+2*t31*(-t11+t31)*
t12)*p+((-t30+h-t20)*t31+t11*(t20+t30))*t22^2+(((-3*t30+3*h)*
t31+(-h+t20+2*t30)*t21+(-t20+t30)*t11)*t12-(2*(-t21+t31))*(-
(1/2)*t11+t31)*t11)*t22-2*t12*((-(1/2)*t31*t20-(1/2)*(h-t20)*
t21)*t12+(-t11+t31)*t21*(t31-(1/2)*t21)))*q+(2*((t31-(1/2)*t11-
(1/2)*t21)*p^2+((1/2)*h*t22+(1/2)*h*t12-(t31-(1/2)*t11-(1/2)*
t21)*(t11+t21+t31))*p+((-(1/2)*h-t20+(1/2)*t30)*t31+t21*((1/2)*
t20-(1/2)*h)-(1/2)*(-t20+t30)*t11)*t22+((t20+3*t30*(1/2)-3*h*
(1/2))*t31+(-t30+(1/2)*h-(1/2)*t20)*t21-(1/2)*t11*(t20+t30))*
t12+(t31-(1/2)*t11-(1/2)*t21)*((t11+t21)*t31+t11*t21)))*p)*t32-
t12*t22*(((-t11+t31)*t22+t12*(-t21+t31))*p+(h*t12-t31*(-t11+
t31))*t22-t12*t31*(-t21+t31))*q^2+((t12+t22)*((-t11+t31)*t22+
t12*(-t21+t31))*p^2+((t12*h+t11^2-t31^2)*t22^2+(h*t12^2+(2*t11*
t21-2*t31^2)*t12)*t22+(t21^2-t31^2)*t12^2)*p+(((-h+t30)*t31-
t30*t11)*t12+t11*t31*(-t11+t31))*t22^2+((-t21*t30+(-h+t30)*t31)
*t12+t31*((t11+t21)*t31-2*t11*t21))*t12*t22+t31*t12^2*t21*(-
t21+t31))*q-(((-t11+t31)*t22+t12*(-t21+t31))*p^2+((h*t12-(-t11+
t31)*(t11+t21+t31))*t22-t12*(-t21+t31)*(t11+t21+t31))*p-t20*(-
t11+t31)*t22^2+(((-h+t30)*t31+(-h+t20)*t21-t11*(t20+t30))*t12+
(-t11+t31)*((t11+t21)*t31+t11*t21))*t22+t12*((t30-h+t20)*t12+
(t11+t21)*t31+t11*t21)*(-t21+t31))*p)/(2*(t22-t32)^2*(t12-t32)
^2)
,q,p);
simplify(h*dotqfunction(t12,t22,t32,t11,t21,t31,0,0,1,0,0,0)-
diff(Hame32(q,p),p));
simplify(h*dotpfunction(t12,t22,t32,t11,t21,t31,0,0,1,0,0,0)+
diff(Hame32(q,p),q));
```

$Hame32 := (q,p) \rightarrow \dfrac{1}{2} \dfrac{1}{(t22-t32)^2 (t12-t32)^2} \left(\left(-2 \left(t31 - \dfrac{1}{2} t11 \right. \right. \right.$ **(5.14)**

$\left. \left. - \dfrac{1}{2} t21 \right) t12\, t22\, q^3 + \left(2 \left(t31 - \dfrac{1}{2} t11 - \dfrac{1}{2} t21 \right) (t12+t22)\, p + \left(-h\, t12 \right. \right. \right.$

$\left. \left. - 2 \left(t31 - \dfrac{1}{2} t11 - \dfrac{1}{2} t21 \right) t11 \right) t22 - 2 \left(t31 - \dfrac{1}{2} t11 - \dfrac{1}{2} t21 \right) t12\, t21 \right) q^2$

$+ \left((t11 + t21 - 2\, t31)\, p^2 + \left(h\, t22 + h\, t12 + 2 \left(t31 - \dfrac{1}{2} t11 - \dfrac{1}{2} t21 \right) (t11 \right. \right.$

$\left. \left. + t21 \right) \right) p + ((-2\, h + 2\, t30 + 2\, t20)\, t31 + t21\, (h - t20 - t30) - (t20 + t30)\, t11)\, t22 \right.$

$+ \left(-2\, t31\, t20 + (-h + t20)\, t21 + t11\, t20 \right) t12 - 2\, t21 \left(t31 - \dfrac{1}{2} t11 - \dfrac{1}{2} t21 \right) t11 \right)$

$q - p\, (h\, p + (-2\, h + 2\, t30)\, t31 - t30\, (t11 + t21)) \bigg) t32^2 + \bigg(t12\, ((-t11 + t31)\, t22$

$$+ t12 \, (-t21 + t31)) \, t22 \, q^3 + \Big(\big((t11 - t31) \, t22^2 - t12^2 \, (-t21 + t31) \big) \, p + (h \, t12$$

$$+ t11 \, (-t11 + t31)) \, t22^2 - 2 \, t12 \left(-\frac{1}{2} \, h \, t12 + (-t21 + t31) \, (-t11 + t31) \right) t22$$

$$+ t21 \, t12^2 \, (-t21 + t31) \Big) \, q^2 + \Big(\big((t21 - t31) \, t22 - t12 \, (-t11 + t31) \big) \, p^2 + \big(-h \, t22^2$$

$$+ (-2 \, h \, t12 + 2 \, t31 \, (-t21 + t31)) \, t22 - h \, t12^2 + 2 \, t31 \, (-t11 + t31) \, t12 \big) \, p + ((h$$

$$- t20 - t30) \, t31 + (t20 + t30) \, t11) \, t22^2 + \Big(((3 \, h - 3 \, t30) \, t31 + (-h + t20$$

$$+ 2 \, t30) \, t21 + t11 \, (-t20 + t30)) \, t12 - 2 \, (-t21 + t31) \left(t31 - \frac{1}{2} \, t11 \right) t11 \Big) \, t22$$

$$- 2 \, t12 \left(\left(-\frac{1}{2} \, t31 \, t20 - \frac{1}{2} \, (h - t20) \, t21 \right) t12 + (-t11 + t31) \, t21 \left(t31 \right.$$

$$\left. - \frac{1}{2} \, t21 \right) \Big) \Big) q + 2 \left(\left(t31 - \frac{1}{2} \, t11 - \frac{1}{2} \, t21 \right) p^2 + \left(\frac{1}{2} \, h \, t22 + \frac{1}{2} \, h \, t12 - \left(t31 \right.\right.\right.$$

$$\left. - \frac{1}{2} \, t11 - \frac{1}{2} \, t21 \right) (t11 + t21 + t31) \Big) \, p + \left(\left(-t20 - \frac{1}{2} \, h + \frac{1}{2} \, t30 \right) t31 + t21 \left(\right.\right.$$

$$\left. - \frac{1}{2} \, h + \frac{1}{2} \, t20 \right) - \frac{1}{2} \, t11 \, (-t20 + t30) \Big) \, t22 + \left(\left(t20 + \frac{3}{2} \, t30 - \frac{3}{2} \, h \right) t31 + \left(-t30 \right.\right.$$

$$\left. + \frac{1}{2} \, h - \frac{1}{2} \, t20 \right) t21 - \frac{1}{2} \, (t20 + t30) \, t11 \Big) \, t12 + \left(t31 - \frac{1}{2} \, t11 \right.$$

$$\left. - \frac{1}{2} \, t21 \right) (t31 \, (t11 + t21) + t11 \, t21) \Big) \, p \Big) \, t32 - t12 \, t22 \, (((-t11 + t31) \, t22 + t12 \, ($$

$$-t21 + t31)) \, p + (h \, t12 - t31 \, (-t11 + t31)) \, t22 - t12 \, t31 \, (-t21 + t31)) \, q^2 + \big((t12$$

$$+ t22) \, ((-t11 + t31) \, t22 + t12 \, (-t21 + t31)) \, p^2 + \big(\big(h \, t12 + t11^2 - t31^2 \big) \, t22^2$$

$$+ \big(h \, t12^2 + \big(2 \, t11 \, t21 - 2 \, t31^2 \big) \, t12 \big) \, t22 + \big(t21^2 - t31^2 \big) \, t12^2 \big) \, p + (((-h + t30) \, t31$$

$$- t11 \, t30) \, t12 + t11 \, t31 \, (-t11 + t31)) \, t22^2 + ((-t21 \, t30 + (-h + t30) \, t31) \, t12$$

$$+ t31 \, (t31 \, (t11 + t21) - 2 \, t11 \, t21)) \, t12 \, t22 + t31 \, t12^2 \, t21 \, (-t21 + t31)) \, q - \big(((-t11$$

$$+ t31) \, t22 + t12 \, (-t21 + t31)) \, p^2 + ((h \, t12 - (-t11 + t31) \, (t11 + t21 + t31)) \, t22$$

$$- t12 \, (-t21 + t31) \, (t11 + t21 + t31)) \, p - t20 \, (-t11 + t31) \, t22^2 + (((-h + t30) \, t31$$

$$+ (-h + t20) \, t21 - (t20 + t30) \, t11) \, t12 + (-t11 + t31) \, (t31 \, (t11 + t21) + t11 \, t21))$$

$$t22 + t12 \, ((-h + t20 + t30) \, t12 + t31 \, (t11 + t21) + t11 \, t21) \, (-t21 + t31)) \, p)$$

$$0$$

$$0$$

```
> simplify(Hame21(q,p)-(1/((t32-t22)*(t12-t22))*(-p^3+ P1(q)*p^2-
  P2(q)*p+P3(q)) +(p-q*t32)*h/(t32-t22)));
```

$$0 \qquad\qquad\qquad \textbf{(5.15)}$$

Definition of the reduced Darboux coordiantes and Hamiltonian in the direction tau

```
> TT1:=unapply(t12+t22+t32,t11,t21,t31,t12,t22,t32);
  TT2:=unapply(t11+t21+t31,t11,t21,t31,t12,t22,t32);
  TT3:=unapply(t22,t11,t21,t31,t12,t22,t32);
  TT4:=unapply(t11,t11,t21,t31,t12,t22,t32);
```

```
TT5:=unapply(t32,t11,t21,t31,t12,t22,t32);
taufunction:=unapply(  ((t21-t31)*t12+(t31-t11)*t22+(t11-t21)*
t32)/(sqrt((t22-t12)*(t12-t32)*(t32-t22)))  ,t11,t21,t31,t12,
t22,t32);
solve({TT1(t11,t21,t31,t12,t22,t32)=T1,TT2(t11,t21,t31,t12,t22,
t32)=T2,TT3(t11,t21,t31,t12,t22,t32)=T3,TT4(t11,t21,t31,t12,
t22,t32)=T4,TT5(t11,t21,t31,t12,t22,t32)=T5,taufunction(t11,
t21,t31,t12,t22,t32)=tau },{t11,t21,t31,t12,t22,t32});

t11function := T4;
t12function := -T3-T5+T1;
t21function := (tau*sqrt((-2*T3-T5+T1)*(-T3-2*T5+T1)*(T3-T5))+
T1*T2-T1*T4-2*T3*T2-T5*T2+3*T4*T3)/(2*T1-3*T3-3*T5);
t22function := T3;
t31function := -(tau*sqrt((-2*T3-T5+T1)*(-T3-2*T5+T1)*(T3-T5))-
T1*T2+T1*T4+T3*T2+2*T5*T2-3*T4*T5)/(2*T1-3*T3-3*T5);
t32function := T5;
simplify(TT1(t11function,t21function,t31function,t12function,
t22function,t32function));
simplify(TT2(t11function,t21function,t31function,t12function,
t22function,t32function));
simplify(TT3(t11function,t21function,t31function,t12function,
t22function,t32function));
simplify(TT4(t11function,t21function,t31function,t12function,
t22function,t32function));
simplify(TT5(t11function,t21function,t31function,t12function,
t22function,t32function));
simplify(taufunction(t11function,t21function,t31function,
t12function,t22function,t32function));

partialtaufunction:=unapply( diff(t11function,tau)*partialt11 +
diff(t21function,tau)*partialt21+diff(t31function,tau)*
partialt31+diff(t12function,tau)*partialt12+diff(t22function,
tau)*partialt22+diff(t32function,tau)*partialt32,T1,T2,T3,T4,
T5,tau):
partialtau:=partialtaufunction(TT1(t11,t21,t31,t12,t22,t32),TT2
(t11,t21,t31,t12,t22,t32),TT3(t11,t21,t31,t12,t22,t32),TT4(t11,
t21,t31,t12,t22,t32),TT5(t11,t21,t31,t12,t22,t32),taufunction
(t11,t21,t31,t12,t22,t32)    ):

Coefft21:=residue(partialtau/partialt21^2,partialt21=0):
Coefft31:=residue(partialtau/partialt31^2,partialt31=0):
```

```
Hamtau:= unapply( simplify(Coefft21*Hame21(q,p)+Coefft31*Hame31
(q,p)),q,p):
Ltauq:=diff(Hamtau(q,p),p):
Ltaup:=-diff(Hamtau(q,p),q):

checkqfunction:=unapply( sqrt((t12-t32)/((t22-t12)*(t32-t22)))*
(-p+t22*q+t21),q,p);
checkpfunction:=unapply( sqrt((t32-t22)/((t12-t32)*(t22-t12)))*
(p-t12*q-t11),q,p);

SOLL:=solve({checkqfunction(q,p)=checkq,checkpfunction(q,p)=
checkp},{q,p}):
Solp:=simplify(rhs(SOLL[1])):
Solq:=simplify(rhs(SOLL[2])):
simplify(checkqfunction(Solq,Solp)-checkq);
simplify(checkpfunction(Solq,Solp)-checkp);

Ltaucheckqfunction:=unapply( simplify( diff(checkqfunction(q,
p),q)*Ltauq+diff(checkqfunction(q,p),p)*Ltaup+Coefft21*h*diff
(checkqfunction(q,p),t21)+Coefft31*h*diff(checkqfunction(q,p),
t31)),q,p):
Ltaucheckpfunction:=unapply(simplify( diff(checkpfunction(q,p),
q)*Ltauq+diff(checkpfunction(q,p),p)*Ltaup+Coefft21*h*diff
(checkpfunction(q,p),t21)+Coefft31*h*diff(checkpfunction(q,p),
t31)),q,p):
Ltaucheckqfunction2:=unapply(simplify(Ltaucheckqfunction(Solq,
Solp)),t11, t21, t31, t12, t22, t32):
Ltaucheckpfunction2:=unapply(simplify(Ltaucheckpfunction(Solq,
Solp)),t11, t21, t31, t12, t22, t32):
Ltaucheckq:=simplify(Ltaucheckqfunction2(t11function,
t21function ,t31function ,t12function ,t22function ,
t32function)):
Ltaucheckp:=simplify(Ltaucheckpfunction2(t11function,
t21function ,t31function ,t12function ,t22function ,
t32function)):
HamtaucheckCoordinates:=unapply(simplify(int(Ltaucheckq,checkp)
-int(simplify(diff(int(Ltaucheckq,checkp),checkq)+Ltaucheckp),
checkq),checkq,checkp)):
simplify(Ltaucheckq-diff(HamtaucheckCoordinates(checkq,checkp),
checkp));
simplify(Ltaucheckp+diff(HamtaucheckCoordinates(checkq,checkp),
checkq));
```

(5.16)

$$TT1 := (t11, t21, t31, t12, t22, t32) \rightarrow t12 + t22 + t32 \qquad \textbf{(5.16)}$$
$$TT2 := (t11, t21, t31, t12, t22, t32) \rightarrow t11 + t21 + t31$$
$$TT3 := (t11, t21, t31, t12, t22, t32) \rightarrow t22$$
$$TT4 := (t11, t21, t31, t12, t22, t32) \rightarrow t11$$
$$TT5 := (t11, t21, t31, t12, t22, t32) \rightarrow t32$$

$$taufunction := (t11, t21, t31, t12, t22, t32)$$
$$\rightarrow \frac{(t21 - t31)\, t12 + (-t11 + t31)\, t22 + t32\, (t11 - t21)}{\sqrt{(-t12 + t22)\,(t12 - t32)\,(-t22 + t32)}}$$

$$\left\{ t11 = T4,\ t12 = -T3 - T5 + T1,\ t21 \right.$$

$$= \frac{1}{2\,T1 - 3\,T3 - 3\,T5}\left(\tau\sqrt{(-T3 - 2\,T5 + T1)\,(T3 - T5)\,(-2\,T3 - T5 + T1)} + T1\,T2 \right.$$
$$\left. - T1\,T4 - 2\,T3\,T2 - T5\,T2 + 3\,T4\,T3\right),\ t22 = T3,\ t31 =$$

$$-\frac{1}{2\,T1 - 3\,T3 - 3\,T5}\left(\tau\sqrt{(-T3 - 2\,T5 + T1)\,(T3 - T5)\,(-2\,T3 - T5 + T1)} - T1\,T2 \right.$$
$$\left. + T1\,T4 + T3\,T2 + 2\,T5\,T2 - 3\,T4\,T5\right),\ t32 = T5 \left. \right\}$$

$$t11function := T4$$
$$t12function := -T3 - T5 + T1$$
$$t21function := \frac{1}{2\,T1 - 3\,T3 - 3\,T5}\Big(\tau$$
$$\sqrt{(-T3 - 2\,T5 + T1)\,(T3 - T5)\,(-2\,T3 - T5 + T1)} + T1\,T2 - T1\,T4 - 2\,T3\,T2$$
$$- T5\,T2 + 3\,T4\,T3 \Big)$$
$$t22function := T3$$
$$t31function :=$$
$$-\frac{1}{2\,T1 - 3\,T3 - 3\,T5}\Big(\tau\sqrt{(-T3 - 2\,T5 + T1)\,(T3 - T5)\,(-2\,T3 - T5 + T1)} - T1\,T2$$
$$+ T1\,T4 + T3\,T2 + 2\,T5\,T2 - 3\,T4\,T5 \Big)$$
$$t32function := T5$$
$$T1$$
$$T2$$
$$T3$$
$$T4$$
$$T5$$
$$\tau$$

$$checkqfunction := (q, p) \rightarrow \sqrt{\frac{t12 - t32}{(-t12 + t22)\,(-t22 + t32)}}\ (q\,t22 - p + t21)$$

$$checkpfunction := (q, p) \rightarrow \sqrt{\frac{-t22 + t32}{(-t12 + t22)\,(t12 - t32)}}\ (-q\,t12 + p - t11)$$
$$0$$
$$0$$
$$0$$
$$0$$

Verification that the reduced Darboux coordinates have trivial evolutions along the trivial directions

```
> hdotcheckqfunction:= unapply(h*simplify(diff(checkqfunction(q,
```

```
        p),q)*dotqfunction(t12,t22,t32,t11,t21,t31,dott12,dott22,
        dott32,dott11,dott21,dott31)+ diff(checkqfunction(q,p),p)*
        dotpfunction(t12,t22,t32,t11,t21,t31,dott12,dott22,dott32,
        dott11,dott21,dott31)+ diff(checkqfunction(q,p),t12)
        *dott12+diff(checkqfunction(q,p),t22)*dott22+diff
        (checkqfunction(q,p),t32)*dott32+diff(checkqfunction(q,p),t11)*
        dott11+diff(checkqfunction(q,p),t21)*dott21+diff(checkqfunction
        (q,p),t31)*dott31),t12,t22,t32,t11,t21,t31,dott12,dott22,
        dott32,dott11,dott21,dott31):
        hdotcheckpfunction:= unapply(h*simplify(diff(checkpfunction(q,
        p),q)*dotqfunction(t12,t22,t32,t11,t21,t31,dott12,dott22,
        dott32,dott11,dott21,dott31)+ diff(checkpfunction(q,p),p)*
        dotpfunction(t12,t22,t32,t11,t21,t31,dott12,dott22,dott32,
        dott11,dott21,dott31)+ diff(checkpfunction(q,p),t12)*dott12+
        diff(checkpfunction(q,p),t22)*dott22+diff(checkpfunction(q,p),
        t32)*dott32+diff(checkpfunction(q,p),t11)*dott11+diff
        (checkpfunction(q,p),t21)*dott21+diff(checkpfunction(q,p),t31)*
        dott31),t12,t22,t32,t11,t21,t31,dott12,dott22,dott32,dott11,
        dott21,dott31):

>   simplify(hdotcheckqfunction(t12,t22,t32,t11,t21,t31,1,1,1,0,0,
    0));
    simplify(hdotcheckqfunction(t12,t22,t32,t11,t21,t31,0,0,0,1,1,
    1));
    simplify(hdotcheckqfunction(t12,t22,t32,t11,t21,t31,2*t12,2*
    t22,2*t32,t11,t21,t31));
    simplify(hdotcheckqfunction(t12,t22,t32,t11,t21,t31,0,0,0,t12,
    t22,t32));
    simplify(hdotcheckqfunction(t12,t22,t32,t11,t21,t31,2*(t12-t32)
    *(t12-t22),0,0,t12*(3*t11-t21-t31)-(t12+t22+t32)*t11+ t22*t31+
    t32*t21,0,0));
    simplify(hdotcheckqfunction(t12,t22,t32,t11,t21,t31,0,2*(t22-
    t12)*(t22-t32),0,0,t22*(3*t21-t11-t31)- (t12+t22+t32)*t21+t12*
    t31+t32*t11,0));
    simplify(hdotcheckqfunction(t12,t22,t32,t11,t21,t31,0,0,2*(t32-
    t12)*(t32-t22),0,0,t32*(3*t31-t11-t21)- (t12+t22+t32)*t31+t12*
    t21+t22*t11));
    simplify(hdotcheckpfunction(t12,t22,t32,t11,t21,t31,1,1,1,0,0,
    0));
    simplify(hdotcheckpfunction(t12,t22,t32,t11,t21,t31,0,0,0,1,1,
    1));
    simplify(hdotcheckpfunction(t12,t22,t32,t11,t21,t31,2*t12,2*
```

```
t22,2*t32,t11,t21,t31));
simplify(hdotcheckpfunction(t12,t22,t32,t11,t21,t31,0,0,0,t12,
t22,t32));
simplify(hdotcheckpfunction(t12,t22,t32,t11,t21,t31,2*(t12-t32)
*(t12-t22),0,0,t12*(3*t11-t21-t31)-(t12+t22+t32)*t11+ t22*t31+
t32*t21,0,0));
simplify(hdotcheckpfunction(t12,t22,t32,t11,t21,t31,0,2*(t22-
t12)*(t22-t32),0,0,t22*(3*t21-t11-t31)- (t12+t22+t32)*t21+t12*
t31+t32*t11,0));
simplify(hdotcheckpfunction(t12,t22,t32,t11,t21,t31,0,0,2*(t32-
t12)*(t32-t22),0,0,t32*(3*t31-t11-t21)- (t12+t22+t32)*t31+t12*
t21+t22*t11));
```

$$
\begin{matrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{matrix}
\tag{5.17}
$$

```
> hdotcheckqtaufunction:=unapply(simplify(hdotcheckqfunction(t12,
  t22,t32,t11,t21,t31,0,0,0,0,Coefft21,Coefft31)),q,p):
  hdotcheckptaufunction:=unapply(simplify(hdotcheckpfunction(t12,
  t22,t32,t11,t21,t31,0,0,0,0,Coefft21,Coefft31)),q,p):
  hdotcheckqtaufunction2:=unapply(simplify(hdotcheckqtaufunction
  (Solq,Solp)),t11,t21,t31,t12,t22,t32):
  hdotcheckptaufunction2:=unapply(simplify(hdotcheckptaufunction
  (Solq,Solp)),t11,t21,t31,t12,t22,t32):
  hdotcheckqtau:=simplify(hdotcheckqtaufunction2(t11function,
  t21function ,t31function ,t12function ,t22function ,
  t32function));
  hdotcheckptau:=simplify(hdotcheckptaufunction2(t11function,
  t21function ,t31function ,t12function ,t22function ,
  t32function));
  simplify(hdotcheckqtau-Ltaucheckq);
```

```
simplify(hdotcheckptau-Ltaucheckp);
```

$$hdotcheckqtau := \left(\sqrt{(-T3 - 2\,T5 + T1)\,(T3 - T5)\,(-2\,T3 - T5 + T1)} \left(checkq \right. \right.$$ **(5.18)**

$$-2\,checkp\,(-T3 - 2\,T5 + T1)\,(-2\,T3 - T5$$

$$+ T1)\,\sqrt{\frac{T3 - T5}{(-T3 - 2\,T5 + T1)\,(-2\,T3 - T5 + T1)}}$$

$$+ \tau\,\sqrt{(-T3 - 2\,T5 + T1)\,(T3 - T5)\,(-2\,T3 - T5 + T1)} \bigg)$$

$$\sqrt{\frac{-T3 - 2\,T5 + T1}{(T3 - T5)\,(-2\,T3 - T5 + T1)}} + \left(-checkq^2 + t20\right)(-T3 - 2\,T5 + T1)\bigg) \bigg/$$

$$\left(\sqrt{\frac{-T3 - 2\,T5 + T1}{(T3 - T5)\,(-2\,T3 - T5 + T1)}}\,(-T3 - 2\,T5 + T1)\,(T3 - T5)\,(-2\,T3 - T5 \right.$$

$$\left. + T1) \right)$$

$$hdotcheckptau := \left(\left(2\,\sqrt{\frac{T3 - T5}{(-T3 - 2\,T5 + T1)\,(-2\,T3 - T5 + T1)}}\,checkp\,checkq\,(T3 \right. \right.$$

$$- T5)\,(-2\,T3 - T5 + T1)\,\sqrt{\frac{-T3 - 2\,T5 + T1}{(T3 - T5)\,(-2\,T3 - T5 + T1)}}$$

$$- \sqrt{\frac{T3 - T5}{(-T3 - 2\,T5 + T1)\,(-2\,T3 - T5 + T1)}}$$

$$\sqrt{(-T3 - 2\,T5 + T1)\,(T3 - T5)\,(-2\,T3 - T5 + T1)}\,checkp\,\tau + (T3 - T5)\left(checkp^2 \right.$$

$$+ h - t20 - t30 \bigg) \sqrt{(-T3 - 2\,T5 + T1)\,(T3 - T5)\,(-2\,T3 - T5 + T1)} \bigg) \bigg/$$

$$\left(\sqrt{\frac{T3 - T5}{(-T3 - 2\,T5 + T1)\,(-2\,T3 - T5 + T1)}}\,(-T3 - 2\,T5 + T1)\,(T3 - T5)\,(-2\,T3 \right.$$

$$\left. - T5 + T1) \right)$$

$$0$$
$$0$$

The Hamiltonian in the only non-trivial direction is independent of the values of the trivial times. Thus, we may take some arbitrary values to obtain it since Maple is struggling to simplify the square roots

```
> hdotcheckqtauValues:=unapply(hdotcheckqtau,T1,T2,T3,T4,T5):
  hdotcheckptauValues:=unapply(hdotcheckptau,T1,T2,T3,T4,T5):
  hdotcheckqtauFinal:=simplify(hdotcheckqtauValues(0,0,0,0,-1/2))
  ;
  hdotcheckptauFinal:=simplify(hdotcheckptauValues(0,0,0,0,-1/2))
  ;
  HamtaucheckCoordinatesTheo:=-(checkq*checkp^2+checkq^2*checkp-
  tau*checkq*checkp-t20*checkp+(t10+h)*checkq);
  simplify(hdotcheckqtauFinal-diff(HamtaucheckCoordinatesTheo,
```

```
  checkp));
  simplify(hdotcheckptauFinal+diff(HamtaucheckCoordinatesTheo,
  checkq));
```

$$hdotcheckqtauFinal := -checkq^2 + (\tau - 2\,checkp)\,checkq + t20 \tag{5.19}$$

$$hdotcheckptauFinal := checkp^2 + (-\tau + 2\,checkq)\,checkp + h - t20 - t30$$

$$HamtaucheckCoordinatesTheo := -checkq\,checkp^2 - checkq^2\,checkp + \tau\,checkq\,checkp$$
$$+ t20\,checkp - (h - t20 - t30)\,checkq$$

$$0$$
$$0$$

```
> S2:=1/sqrt(2)*sqrt(s12-s22);
  S1:=1/sqrt(2)*(s11-s21)/sqrt(s12-s22);
  tdX1function:=unapply(S2*X1+S1,s11,s21,s12,s22,X1);
  RES:=unapply( simplify(series(simplify(tdX1function(t31/(t32-
  t22),t11/(t12-t22),-1/(t32-t22),-1/(t12-t22),t21)   -1/sqrt(2)*
  taufunction(t11,t21,t31,t12,t22,t32)),t21)),t11,t21,t31,t12,
  t22,t32) ;
  RES(2,4,3,-1,0,1);
  RES(2,4,3,1,0,-1);
  tdX1:=1/sqrt(2)*taufunction(t11,t21,t31,t12,t22,t32):
```

$$S2 := \frac{1}{2}\,\sqrt{2}\,\sqrt{s12 - s22} \tag{5.20}$$

$$S1 := \frac{1}{2}\,\frac{\sqrt{2}\,(s11 - s21)}{\sqrt{s12 - s22}}$$

$$tdX1function := (s11, s21, s12, s22, X1) \rightarrow \frac{1}{2}\,\sqrt{2}\,\sqrt{s12 - s22}\,X1 + \frac{1}{2}\,\frac{\sqrt{2}\,(s11 - s21)}{\sqrt{s12 - s22}}$$

$$RES := (t11, t21, t31, t12, t22, t32) \rightarrow -\frac{1}{2}\left(\left(-(t22 - t32)\,(t12\right.\right.$$
$$-t22)\,\sqrt{\frac{t12 - t32}{(t22 - t32)\,(t12 - t22)}} + \sqrt{(t12 - t32)\,(t22 - t32)\,(t12 - t22)}\right)$$
$$\left.(t22\,(t11 - t31) + t31\,t12 - t11\,t32)\,\sqrt{2}\right) \Bigg/$$
$$\left(\sqrt{\frac{t12 - t32}{(t22 - t32)\,(t12 - t22)}}\,\sqrt{(t12 - t32)\,(t22 - t32)\,(t12 - t22)}\,(t22 - t32)\,(t12\right.$$
$$\left.- t22)\right) - \frac{1}{2}\left((t12 - t32)\,\sqrt{2}\left((t22 - t32)\,(t12\right.\right.$$
$$-t22)\,\sqrt{\frac{t12 - t32}{(t22 - t32)\,(t12 - t22)}} - \sqrt{(t12 - t32)\,(t22 - t32)\,(t12 - t22)}\right)\Bigg) \Bigg/$$
$$\left(\sqrt{\frac{t12 - t32}{(t22 - t32)\,(t12 - t22)}}\,\sqrt{(t12 - t32)\,(t22 - t32)\,(t12 - t22)}\,(t22 - t32)\,(t12\right.$$
$$\left.- t22)\right)\,t21$$

$$0$$
$$0$$

```
> R1:=unapply((sX10+sX20)/(xi-X1) -(s12+s22)*xi-s11-s21,xi);
  checkqfunction(q,p);
  checkqidfunction:=unapply(checkqfunction(p,q+t22*p),q,p):
  checkqidfunction2:=unapply( checkqidfunction((checkq-S1)/S2,S2*
  checkp+1/2*R1((checkq-S1)/S2)), s11,s21,s12,s22,X1,sX10,sX20):
  simplify(series(checkqidfunction2(t31/(t32-t22),t11/(t12-t22),
  -1/(t32-t22),-1/(t12-t22),t21,-s10-s20,0),checkq  ));
  RES2:=unapply( simplify(series(checkqidfunction2(t31/(t32-t22),
  t11/(t12-t22),-1/(t32-t22),-1/(t12-t22),t21,-s10-s20,0)- (-sqrt
  (2)*(checkq-1/sqrt(2)*taufunction(t11,t21,t31,t12,t22,t32))),
  checkq=0) ),t11,t21,t31,t12,t22,t32);
  RES2(2,4,3,-1,0,1);
  RES2(2,4,3,1,0,-1);
```

$$R1 := \xi \rightarrow \frac{sX10 + sX20}{\xi - X1} - (s12 + s22)\,\xi - s11 - s21 \tag{5.21}$$

$$-\frac{\left(t12\,(-t21+t31) + (-t11+t21)\,t32 + t22\,(t11-t31)\right)\sqrt{\dfrac{t12-t32}{(t22-t32)\,(t12-t22)}}}{t12-t32} \; \sqrt{\dfrac{t12-t32}{(-t12+t22)\,(-t22+t32)}}\,(q\,t22 - p + t21)$$

$$-\sqrt{2}\; checkq$$

$$RES2 := (t11,t21,t31,t12,t22,t32) \rightarrow \Bigg(\Bigg($$

$$-\sqrt{\frac{t12-t32}{(t22-t32)\,(t12-t22)}}\;\sqrt{(t12-t32)\,(t22-t32)\,(t12-t22)} + t12 - t32\Bigg)$$

$$\left(t12\,(-t21+t31) + (t21-t11)\,t32 + t22\,(t11-t31)\right)\Bigg) \Bigg/$$

$$\left(\sqrt{(t12-t32)\,(t22-t32)\,(t12-t22)}\;(t12-t32)\right)$$

$$0$$

$$0$$

```
> checkqfunction(q,p);
  checkpidfunction:=unapply(checkpfunction(p,q+t22*p),q,p):
  checkpidfunction2:=unapply( checkpidfunction((checkq-S1)/S2,S2*
  checkp+1/2*R1((checkq-S1)/S2)), s11,s21,s12,s22,X1,sX10,sX20):
  checkpid:=simplify(series(checkpidfunction2(t31/(t32-t22),t11/
  (t12-t22),-1/(t32-t22),-1/(t12-t22),t21,-s10-s20,0),checkp  )):

  checkpidbis:=-1/sqrt(2)*(checkp- (s10+s20)/2/(checkq-tdX1)-
  checkq );
  simplify(series(simplify(series(simplify
  (checkpid- checkpidbis),checkp),symbolic),checkq),symbolic);
```

$$\sqrt{\frac{t12-t32}{(-t12+t22)\,(-t22+t32)}}\,(q\,t22 - p + t21) \tag{5.22}$$

$$checkpidbis := -\frac{1}{2} \sqrt{2} \left(checkp \right.$$

$$-\frac{1}{2} \frac{s10 + s20}{checkq - \frac{1}{2} \dfrac{\sqrt{2} \left((t21 - t31) \, t12 + (-t11 + t31) \, t22 + t32 \, (t11 - t21) \right)}{\sqrt{(-t12 + t22)(t12 - t32)(-t22 + t32)}}}$$

$$\left. - checkq \right)$$

$$0$$

We find that under the spectral identification:
tdX1=1/sqrt(2)*tau
check{q}=-sqrt{2}*(\check{q}-\tdX1)
check{p}=-1/sqrt(2)*(checkp- (s10+s20)/2/(checkq-tdX1)-checkq)

```
> HamtaucheckCoordinatesTheofunction:=unapply
  (HamtaucheckCoordinatesTheo,checkq,checkp,tau);
  dtaufunction:=diff(taufunction(t11,t21,t31,t12,t22,t32),t11)*
  dt11+diff(taufunction(t11,t21,t31,t12,t22,t32),t21)*dt21+diff
  (taufunction(t11,t21,t31,t12,t22,t32),t31)*dt31+diff
  (taufunction(t11,t21,t31,t12,t22,t32),t12)*dt12+diff
  (taufunction(t11,t21,t31,t12,t22,t32),t22)*dt22+diff
  (taufunction(t11,t21,t31,t12,t22,t32),t32)*dt32:
  deltaHamtaucheckCoordinatesTheodeltatau:=unapply(diff
  (HamtaucheckCoordinatesTheo,tau),checkq,checkp,tau):
  OmegaHam:=unapply(HamtaucheckCoordinatesTheofunction(checkq,
  checkp,tau)*dtaufunction,checkq,checkp,tau):
  OmegaR2:=unapply(-int(deltaHamtaucheckCoordinatesTheodeltatau
  (checkq(s),checkp(s),s),s=0..tau)*dtaufunction,checkq,checkp,
  tau);
  OmegaHambis:=simplify(OmegaHam(checkqfunction(q,p),
  checkpfunction(q,p),taufunction(t11,t21,t31,t12,t22,t32))):
  OmegaHamNonReduced:=Hame11(q,p)*dt11+ Hame21(q,p)*dt21+Hame31
  (q,p)*dt31+Hame12(q,p)*dt12+Hame22(q,p)*dt22+Hame32(q,p)*dt32:
```

$$HamtaucheckCoordinatesTheofunction := (checkq, checkp, \tau) \rightarrow -checkq\ checkp^2 \tag{5.23}$$
$$- checkq^2\ checkp + checkq\ checkp\ \tau + t20\ checkp - (h - t20 - t30)\ checkq$$

$$OmegaR2 := (checkq, checkp, \tau) \rightarrow -\left(\int_0^\tau checkp(s)\ checkq(s) \right.$$

$$\left. ds \right) \left(\frac{(-t22 + t32)\ dt11}{\sqrt{(-t12 + t22)(t12 - t32)(-t22 + t32)}} \right.$$
$$\left. + \frac{(t12 - t32)\ dt21}{\sqrt{(-t12 + t22)(t12 - t32)(-t22 + t32)}} \right.$$

$$+ \frac{(-t12 + t22)\, dt31}{\sqrt{(-t12 + t22)\,(t12 - t32)\,(-t22 + t32)}}$$

$$+ \left(\frac{t21 - t31}{\sqrt{(-t12 + t22)\,(t12 - t32)\,(-t22 + t32)}} \right.$$

$$- \frac{1}{2} \frac{1}{((-t12 + t22)\,(t12 - t32)\,(-t22 + t32))^{3/2}}\, (((t21 - t31)\,t12 + (-t11$$

$$+ t31)\,t22 + t32\,(t11 - t21))\,(-(t12 - t32)\,(-t22 + t32) + (-t12 + t22)\,(-t22$$

$$\left. + t32))))\, dt12 + \left(\frac{-t11 + t31}{\sqrt{(-t12 + t22)\,(t12 - t32)\,(-t22 + t32)}} \right.$$

$$- \frac{1}{2} \frac{1}{((-t12 + t22)\,(t12 - t32)\,(-t22 + t32))^{3/2}}\, (((t21 - t31)\,t12 + (-t11$$

$$+ t31)\,t22 + t32\,(t11 - t21))\,((t12 - t32)\,(-t22 + t32) - (-t12 + t22)\,(t12$$

$$\left. - t32))))\, dt22 + \left(\frac{t11 - t21}{\sqrt{(-t12 + t22)\,(t12 - t32)\,(-t22 + t32)}} \right.$$

$$- \frac{1}{2} \frac{1}{((-t12 + t22)\,(t12 - t32)\,(-t22 + t32))^{3/2}}\, (((t21 - t31)\,t12 + (-t11$$

$$+ t31)\,t22 + t32\,(t11 - t21))\,(-(-t12 + t22)\,(-t22 + t32) + (-t12 + t22)\,(t12$$

$$\left. - t32))))\, dt32 \right)$$

```
> simplify(residue(simplify(OmegaHambis)/dt11^2,dt11=0),symbolic)
  ;
  simplify(Hame11(q,p));
  simplify(series(simplify(simplify(residue(simplify(OmegaHambis)
  /dt11^2,dt11=0),symbolic) -( -Hame11(q,p) -h*(p-t22*q)/(t12-
  t22)+h*t21/(t12-t22)-t11*t21*t31/((t12-t32)*(t12-t22))+(-t12*
  t21+(-t11+t21)*t32+t11*t22)*t20/((t12-t32)*(t12-t22))  -t21*
  t30/(t12-t22) )  ),t30=0));
```

$$\frac{1}{(t12 - t22)\,(-t12 + t32)} \left(-p^3 + (t11 + t21 + t31 + (t12 + t22 + t32)\,q)\,p^2 + \big((((-t22 \quad \textbf{(5.24)}\right.$$

$$- t32)\,t12 - t22\,t32)\,q^2 + ((-t21 - t11)\,t32 + (-t31 - t21)\,t12 - t22\,(t11 + t31))\,q$$

$$+ (h - t20 - t30)\,t12 + (-h + t30)\,t32 + t20\,t22 + (-t11 - t31)\,t21 - t11\,t31)\,p$$

$$+ q^3\,t12\,t22\,t32 + ((t21\,t32 + t22\,t31)\,t12 + t22\,t32\,t11)\,q^2 + ((t20\,t32 + (-h$$

$$+ t30)\,t22 + t21\,t31)\,t12 + ((h - t20 - t30)\,t22 + t11\,t21)\,t32 + t22\,t11\,t31)\,q$$

$$- t21\,(h - t20 - t30)\,t12 + (t21\,(h - t20 - t30) + t11\,t20)\,t32 + t11\,(-t20\,t22$$

$$+ t21\,t31))$$

$$\frac{1}{(-t12 + t32)\,(t12 - t22)} \left(p^3 + ((-t12 - t22 - t32)\,q - t31 - t11 - t21)\,p^2 + \big((((t22$$

$$+ t32)\,t12 + t22\,t32)\,q^2 + (t12\,(t21 + t31) + t22\,(t11 + t31) + t32\,(t11 + t21))\,q$$

$$+ (t20 + t30)\,t12 - t20\,t22 - t30\,t32 + t11\,(t21 + t31) + t21\,t31)\,p$$

$$- q\,(q^2\,t12\,t22\,t32 + ((t21\,t32 + t22\,t31)\,t12 + t22\,t32\,t11)\,q + (t20\,t32 + t21\,t31$$

$$+ t22\,t30)\,t12 + ((-t20 - t30)\,t32 + t11\,t31)\,t22 + t32\,t11\,t21))$$

$$0$$

```
> simplify(residue(simplify(OmegaHambis)/dt21^2,dt21=0),symbolic)
```

```
;
simplify(Hame21(q,p));
simplify(series(simplify(residue(simplify(OmegaHambis)/dt21^2,
dt21=0),symbolic) -( -Hame21(q,p) +h*(p-t12*q)/(t12-t22)) ,q=0)
);
```

$$\frac{1}{(t12-t22)\,(-t22+t32)}\Big(p^3 + ((-t12-t22-t32)\,q - t31 - t11 - t21)\,p^2 + \big(((t12 \quad\textbf{(5.25)}$$
$$+t22)\,t32 + t12\,t22)\,q^2 + (t12\,(t21+t31) + t22\,(t11+t31) + t32\,(t11+t21))\,q$$
$$+ (h-t30)\,t32 + (-h+t20+t30)\,t12 - t20\,t22 + (t11+t31)\,t21 + t11\,t31\big)\,p$$
$$- q^3\,t12\,t22\,t32 + ((-t11\,t22 - t12\,t21)\,t32 - t31\,t12\,t22)\,q^2 + ((-t20\,t12 + (-h$$
$$+t20+t30)\,t22 - t11\,t21)\,t32 + ((h-t30)\,t22 - t21\,t31)\,t12 - t22\,t11\,t31)\,q + ((\,$$
$$-h+t20+t30)\,t21 - t11\,t20)\,t32 + t21\,(h-t20-t30)\,t12 - t11\,(-t20\,t22$$
$$+t21\,t31)\Big)$$

$$\frac{1}{(t12-t22)\,(-t22+t32)}\Big(-p^3 + (t11+t21+t31+(t12+t22+t32)\,q)\,p^2 + \big(((-t12$$
$$-t32)\,t22 - t12\,t32)\,q^2 + ((-t31-t21)\,t12 + (-t11-t31)\,t22 - t32\,(t11+t21))\,q$$
$$+ (-h+t20)\,t22 + (h-t20-t30)\,t12 + t30\,t32 + (-t31-t21)\,t11 - t21\,t31\big)\,p$$
$$- \big(-q^2\,t12\,t22\,t32 + ((-t11\,t32 - t12\,t31)\,t22 - t12\,t21\,t32)\,q + (-t12\,t30 + (-h$$
$$+t20+t30)\,t32 - t11\,t31)\,t22 + ((h-t20)\,t32 - t21\,t31)\,t12 - t32\,t11\,t21\big)\,q\Big)$$
$$\frac{((h-t20-t30)\,t32 + (-h+t20+t30)\,t12 + t11\,t31)\,t21 - t11\,t20\,(t22-t32)}{(t22-t32)\,(t12-t22)}$$

```
> simplify(residue(simplify(OmegaHambis)/dt31^2,dt31=0),symbolic)
;
simplify(Hame31(q,p));
simplify(series(simplify(residue(simplify(OmegaHambis)/dt31^2,
dt31=0),symbolic) -( -Hame31(q,p)) ,q=0));
```

$$\frac{1}{(-t22+t32)\,(-t12+t32)}\Big(p^3 + ((-t12-t22-t32)\,q - t31 - t11 - t21)\,p^2 + \big(((t12 \quad\textbf{(5.26)}$$
$$+t22)\,t32 + t12\,t22)\,q^2 + (t12\,(t21+t31) + t22\,(t11+t31) + t32\,(t11+t21))\,q$$
$$+ (h-t30)\,t32 + (-h+t20+t30)\,t12 - t20\,t22 + (t11+t31)\,t21 + t11\,t31\big)\,p$$
$$- q^3\,t12\,t22\,t32 + ((-t11\,t22 - t12\,t21)\,t32 - t31\,t12\,t22)\,q^2 + ((-t20\,t12 + (-h$$
$$+t20+t30)\,t22 - t11\,t21)\,t32 + ((h-t30)\,t22 - t21\,t31)\,t12 - t22\,t11\,t31)\,q + ((\,$$
$$-h+t20+t30)\,t21 - t11\,t20)\,t32 + t21\,(h-t20-t30)\,t12 - t11\,(-t20\,t22$$
$$+t21\,t31)\Big)$$

$$\frac{1}{(-t22+t32)\,(-t12+t32)}\Big(-p^3 + (t11+t21+t31+(t12+t22+t32)\,q)\,p^2 + \big((($$
$$-t12-t22)\,t32 - t12\,t22)\,q^2 + ((-t21-t11)\,t32 + (-t31-t21)\,t12 - t22\,(t11$$
$$+t31))\,q + (-h+t30)\,t32 + (h-t20-t30)\,t12 + t20\,t22 + (-t31-t21)\,t11$$
$$-t21\,t31\big)\,p + q\,\big(q^2\,t12\,t22\,t32 + ((t11\,t22 + t12\,t21)\,t32 + t31\,t12\,t22)\,q + (t20\,t12$$
$$+ (h-t20-t30)\,t22 + t11\,t21)\,t32 + ((-h+t30)\,t22 + t21\,t31)\,t12 + t22\,t11\,t31)\big)\Big)$$
$$\frac{((-h+t20+t30)\,t32 + (h-t20-t30)\,t12 - t11\,t31)\,t21 + t11\,t20\,(t22-t32)}{(t22-t32)\,(t12-t32)}$$

```
> simplify(residue(simplify(OmegaHambis)/dt12^2,dt12=0),symbolic)
:
simplify(Hame12(q,p)):
```

```
simplify(series(simplify(residue(simplify(OmegaHambis)/dt12^2,
dt12=0),symbolic) -( -Hame12(q,p) -(1/2)*h*p^2/((t12-t22)*(t12-
t32))) - (1/2)*(-q*t22^2+(q*t12-q*t32-t11-t21)*t22+(q*t32+2*
t11)*t12-t32*(t11-t21))*h/((t12-t22)^2*(t12-t32))*p +(1/2)*h*
t22*t32*q^2/((t12-t22)*(t12-t32))     +(((t11-(1/2)*t21)*t22+
(1/2)*t32*t21)*t12-(1/2)*t11*t22*(t22+t32))*h*q/((t12-t22)^2*
(t12-t32)),q=0));
```

$$-\frac{1}{(t12-t32)^2\,(t12-t22)^2}\Bigg(\big(\big((-h+t20+t30)\,t32+(h-t20-t30)\,t12-t11\,t31\big)\,t21 \quad \text{(5.27)}$$
$$+\,t11\,t20\,(t22-t32)\big)\left(\left(-\frac{1}{2}\,t12+\frac{1}{2}\,t32\right)t21+\left(t11-\frac{1}{2}\,t31\right)t12-\frac{1}{2}\,t11\,t32\right.$$
$$\left.-\frac{1}{2}\,t22\,(t11-t31)\right)\Bigg)$$

```
> simplify(residue(simplify(OmegaHambis)/dt32^2,dt32=0),symbolic)
  :
  simplify(Hame32(q,p)):
  simplify(series(simplify(residue(simplify(OmegaHambis)/dt32^2,
  dt32=0),symbolic) -( -Hame32(q,p)
  -(1/2)*h*p^2/((t22-t32)*(t12-t32))+(1/2)*((t12+t22)*q+t11+t21)*
  h*p/((t22-t32)*(t12-t32))-(1/2)*h*t12*t22*q^2/((t22-t32)*(t12-
  t32))-(1/2)*h*(t11*t22+t12*t21)*q/((t22-t32)*(t12-t32))
  ),q=0));
```

$$-\frac{1}{2}\,\frac{1}{(t22-t32)^2\,(t12-t32)^2}\,\big(\big((t12-t32)\,t21+(-t11+2\,t31)\,t32+(-t12 \quad \text{(5.28)}$$
$$-t22)\,t31+t11\,t22\big)\,\big(((-h+t20+t30)\,t32+(h-t20-t30)\,t12-t11\,t31)\,t21$$
$$+\,t11\,t20\,(t22-t32)\big)\big)$$

We do not care about the term only proportional to times. Thus, we find OmegaHambis= - Hame11 –h*(p-t22*q-t21)/(t12-t22) +times

```
> simplify(checkqfunction(q,p)*checkpfunction(q,p));
  checkqcheckpbis:=-1/(t12-t22)*(p^2- ((t12+t22)*q+(t11+t21))*p
  +(t12*q+t11)*(t22*q+t21) );
  series(simplify(simplify(checkqfunction(q,p)*checkpfunction(q,
  p),symbolic)- checkqcheckpbis),q=0);
```

$$-\sqrt{\frac{t12-t32}{(t22-t32)\,(t12-t22)}}\,(-q\,t22+p-t21)\,\sqrt{\frac{t22-t32}{(t12-t32)\,(t12-t22)}}\,(-q\,t12 \quad \text{(5.29)}$$
$$+\,p-t11)$$

$$checkqcheckpbis:=-\frac{p^2-((t12+t22)\,q+t11+t21)\,p+(q\,t12+t11)\,(q\,t22+t21)}{t12-t22}$$

$$0$$

```
> diff(checkqcheckpbis,t11);
  diff(checkqcheckpbis,t21);
  diff(checkqcheckpbis,t31);
```

$$-\frac{q\,t22-p+t21}{t12-t22} \quad \text{(5.30)}$$

$$-\frac{q\,t12 - p + t11}{t12 - t22}$$
$$0$$

Expression of the Lax matrix in the coordinates (checkq,checkp)

```
> tdL21functioninter:=unapply(tdL[2,1],q,p);
  tdL21function:=unapply(simplify(tdL21functioninter(Solq,Solp)),
  t11,t21,t31,t12,t22,t32):
  tdL21check:=unapply(1/a12*(t22-t12)*(-checkq*checkp-checkq^2+
  tau*checkq+t20),tau):
  simplify(series(tdL21check(taufunction(t11,t21,t31,t12,t22,t32)
  )-tdL21function(t11,t21,t31,t12,t22,t32),checkp=0),symbolic);

  tdL23functioninter:=unapply(tdL[2,3],q,p);
  tdL23function:=unapply(simplify(tdL23functioninter(Solq,Solp)),
  t11,t21,t31,t12,t22,t32):
  tdL23check:=unapply(-a13/a12*sqrt(t32-t22)*sqrt(t22-t12)/sqrt
  (t12-t32)*checkq,tau):
  simplify(series(tdL23check(taufunction(t11,t21,t31,t12,t22,t32)
  )-tdL23function(t11,t21,t31,t12,t22,t32),checkp=0),symbolic);

  tdL31functioninter:=unapply(tdL[3,1],q,p);
  tdL31function:=unapply(simplify(tdL31functioninter(Solq,Solp)),
  t11,t21,t31,t12,t22,t32):
  tdL31check:=unapply( 1/a13*(t12-t32)*(-checkq*checkp-checkq^2+
  tau*checkq-t30)  ,tau):
  simplify(series(tdL31check(taufunction(t11,t21,t31,t12,t22,t32)
  )-tdL31function(t11,t21,t31,t12,t22,t32),checkp=0),symbolic);

  tdL32functioninter:=unapply(tdL[3,2],q,p);
  tdL32function:=unapply(simplify(tdL32functioninter(Solq,Solp)),
  t11,t21,t31,t12,t22,t32):
  tdL32check:=unapply( a12/a13*sqrt(t12-t32)*sqrt(t32-t22)/sqrt
  (t22-t12)*(checkp+1*checkq+1*tau)  ,tau):
  simplify(series(simplify(series(tdL32check(taufunction(t11,t21,
  t31,t12,t22,t32))-tdL32function(t11,t21,t31,t12,t22,t32),
  checkp=0),symbolic),checkq=0),symbolic);
```

$$tdL21functioninter := (q,p) \rightarrow \frac{1}{a12\,(t22-t32)}\big((t12-t22)\,((q^2\,t32 - p\,q + q\,t31 \qquad \textbf{(5.31)}$$
$$- t20)\,t22 + (-p\,q + q\,t21 + t20)\,t32 + (p - t31)\,(p - t21))\big)$$
$$0$$

$$tdL23functioninter := (q,p) \rightarrow -\frac{a13\,(-q\,t22 + p - t21)}{a12}$$
$$0$$

$$tdL31functioninter := (q,p) \rightarrow -\frac{1}{a13\,(t22-t32)}\left((t12-t32)\left(\left(q^2\,t22-p\,q+q\,t21\right.\right.\right.$$
$$\left.\left.\left.-\,t30\right)t32+(-p\,q+q\,t31+t30)\,t22+(p-t31)\,(p-t21)\right)\right)$$

$$0$$

$$tdL32functioninter := (q,p) \rightarrow -\frac{a12\,(-q\,t32+p-t31)}{a13}$$

$$-\frac{2\,(t22\,(t11-t31)+t12\,(-t21+t31)-t32\,(t11-t21))\,a12}{a13\,(t12-t22)}$$

$$-\frac{2\,a12\,\sqrt{t12-t32}\,\sqrt{-t22+t32}}{a13\,\sqrt{-t12+t22}}\quad checkq$$

```
> GeneralSpectralCurve:=simplify(Determinant(y*IdentityMatrix(3)-
  tdL)):
  GeneralSpectralCurvebis:=y^3-P1(lambda)*y^2+P2(lambda)*y-P3
  (lambda)- (p^3-P1(q)*p^2+P2(q)*p-P3(q)):
  simplify(series(simplify
  (GeneralSpectralCurve-GeneralSpectralCurvebis),y=0));
```

$$0 \tag{5.32}$$

▼ Definition of the auxiliary matrix A by gauge transformation of \td{A} and verification of the compatibility equation in the oper gauge. Verification of the gauge transformations

```
> LalphaG:=Matrix(3,3,0):
  for i from 1 to 3 do for j from 1 to 3 do
   LalphaG[i,j]:=simplify( h*diff(G[i,j],t12)*dott12+h*diff(G[i,
  j],t22)*dott22+h*diff(G[i,j],t32)*dott32+h*diff(G[i,j],t11)*
  dott11+h*diff(G[i,j],t21)*dott21+h*diff(G[i,j],t31)*dott31+h*
  diff(G[i,j],a12)*dota12+h*diff(G[i,j],a13)*dota13+ h*diff(G[i,
  j],q)*dotq+h*diff(G[i,j],p)*dotp): od: od:

  LalphatdL:=Matrix(3,3,0):
  for i from 1 to 3 do for j from 1 to 3 do
   LalphatdL[i,j]:=simplify( h*diff(tdL[i,j],t12)*dott12+h*diff
  (tdL[i,j],t22)*dott22+h*diff(tdL[i,j],t32)*dott32+h*diff(tdL[i,
  j],t11)*dott11+h*diff(tdL[i,j],t21)*dott21+h*diff(tdL[i,j],t31)
  *dott31+h*diff(tdL[i,j],a12)*dota12+h*diff(tdL[i,j],a13)*
  dota13+ h*diff(tdL[i,j],q)*dotq+h*diff(tdL[i,j],p)*dotp): od:
  od:

  LalphaLnew:=Matrix(3,3,0):
  for i from 1 to 3 do for j from 1 to 3 do
   LalphaLnew[i,j]:=simplify( h*diff(Lnew[i,j],t12)*dott12+h*diff
  (Lnew[i,j],t22)*dott22+h*diff(Lnew[i,j],t32)*dott32+h*diff
```

```
(Lnew[i,j],t11)*dott11+h*diff(Lnew[i,j],t21)*dott21+h*diff
(Lnew[i,j],t31)*dott31+h*diff(Lnew[i,j],a12)*dota12+h*diff
(Lnew[i,j],a13)*dota13+ h*diff(Lnew[i,j],q)*dotq+h*diff(Lnew[i,
j],p)*dotp): od: od:

dGdlambda:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do
dGdlambda[i,j]:=diff(G[i,j],lambda): od: od:

dtdAdlambda:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do dtdAdlambda[i,j]:=
diff(tdA[i,j],lambda): od: od:

LalphaL:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do
 LalphaL[i,j]:=simplify( h*diff(L[i,j],t12)*dott12+h*diff(L[i,
j],t22)*dott22+h*diff(L[i,j],t32)*dott32+h*diff(L[i,j],t11)*
dott11+h*diff(L[i,j],t21)*dott21+h*diff(L[i,j],t31)*dott31+h*
diff(L[i,j],a12)*dota12+h*diff(L[i,j],a13)*dota13+ h*diff(L[i,
j],q)*dotq+h*diff(L[i,j],p)*dotp): od: od:
```

```
> Verification0:= simplify(L- (  Multiply(Multiply(G,tdL),G^(-1))
+h*Multiply(dGdlambda,G^(-1)))));
Verification1:=simplify(LalphatdL-h*dtdLdt);
Verification2:=simplify(h*dtdAdlambda-LalphatdL- Multiply(tdL,
tdA)+Multiply(tdA,tdL) );
Verification2bis:=simplify(LalphatdL-h*dtdAdlambda-(Multiply
(tdA,tdL)- Multiply(tdL,tdA)) );
```

$$
\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

$$
\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

$$
\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

$$
\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

(6.1)

```
> A:=simplify( Multiply(Multiply(G,tdA),G^(-1))+Multiply(LalphaG,
  G^(-1))):
  dAdlambda:=Matrix(3,3,0):
  for i from 1 to 3 do for j from 1 to 3 do dAdlambda[i,j]:=diff
  (A[i,j],lambda): od: od:
> Verification3:= simplify(A- (  Multiply(Multiply(G,tdA),G^(-1))
  +Multiply(LalphaG,G^(-1))));
  Verification4:=simplify(h*dAdlambda-LalphaL- Multiply(L,A)+
  Multiply(A,L))  ;
  Verification4bis:=simplify( LalphaL-h*dAdlambda-(Multiply(A,L)-
  Multiply(L,A)) );
```

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(6.2)

```
> dAnewdlambda:=Matrix(3,3,0):
  for i from 1 to 3 do for j from 1 to 3 do dAnewdlambda[i,j]:=
  diff(Anew[i,j],lambda): od: od:

  Verification5:= simplify(Anew- (  Multiply(Multiply
  (Gnormalized,tdA),Gnormalized^(-1))+Multiply(LGnormalized,
  Gnormalized^(-1))));
  Verification6:=simplify(h*dAnewdlambda-LalphaLnew- Multiply
  (Lnew,Anew)+Multiply(Anew,Lnew))  ;
```

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(6.3)

▼ Expression of the auxiliary matrices in the various directions of deformations

```
> tdAfunction[1,1]:=unapply(tdA[1,1],dott12,dott22,dott32,dott11,
  dott21,dott31):
  tdAfunction[1,2]:=unapply(tdA[1,2],dott12,dott22,dott32,dott11,
  dott21,dott31):
  tdAfunction[1,3]:=unapply(tdA[1,3],dott12,dott22,dott32,dott11,
  dott21,dott31):
  tdAfunction[2,1]:=unapply(tdA[2,1],dott12,dott22,dott32,dott11,
  dott21,dott31):
  tdAfunction[2,2]:=unapply(tdA[2,2],dott12,dott22,dott32,dott11,
  dott21,dott31):
  tdAfunction[2,3]:=unapply(tdA[2,3],dott12,dott22,dott32,dott11,
  dott21,dott31):
  tdAfunction[3,1]:=unapply(tdA[3,1],dott12,dott22,dott32,dott11,
  dott21,dott31):
  tdAfunction[3,2]:=unapply(tdA[3,2],dott12,dott22,dott32,dott11,
  dott21,dott31):
  tdAfunction[3,3]:=unapply(tdA[3,3],dott12,dott22,dott32,dott11,
  dott21,dott31):

  Afunction[1,1]:=unapply(A[1,1],dott12,dott22,dott32,dott11,
  dott21,dott31):
  Afunction[1,2]:=unapply(A[1,2],dott12,dott22,dott32,dott11,
  dott21,dott31):
  Afunction[1,3]:=unapply(A[1,3],dott12,dott22,dott32,dott11,
  dott21,dott31):
  Afunction[2,1]:=unapply(A[2,1],dott12,dott22,dott32,dott11,
  dott21,dott31):
  Afunction[2,2]:=unapply(A[2,2],dott12,dott22,dott32,dott11,
  dott21,dott31):
  Afunction[2,3]:=unapply(A[2,3],dott12,dott22,dott32,dott11,
  dott21,dott31):
  Afunction[3,1]:=unapply(A[3,1],dott12,dott22,dott32,dott11,
  dott21,dott31):
  Afunction[3,2]:=unapply(A[3,2],dott12,dott22,dott32,dott11,
  dott21,dott31):
  Afunction[3,3]:=unapply(A[3,3],dott12,dott22,dott32,dott11,
  dott21,dott31):

  Anewfunction[1,1]:=unapply(Anew[1,1],dott12,dott22,dott32,
  dott11,dott21,dott31):
  Anewfunction[1,2]:=unapply(Anew[1,2],dott12,dott22,dott32,
  dott11,dott21,dott31):
```

```
Anewfunction[1,3]:=unapply(Anew[1,3],dott12,dott22,dott32,
dott11,dott21,dott31):
Anewfunction[2,1]:=unapply(Anew[2,1],dott12,dott22,dott32,
dott11,dott21,dott31):
Anewfunction[2,2]:=unapply(Anew[2,2],dott12,dott22,dott32,
dott11,dott21,dott31):
Anewfunction[2,3]:=unapply(Anew[2,3],dott12,dott22,dott32,
dott11,dott21,dott31):
Anewfunction[3,1]:=unapply(Anew[3,1],dott12,dott22,dott32,
dott11,dott21,dott31):
Anewfunction[3,2]:=unapply(Anew[3,2],dott12,dott22,dott32,
dott11,dott21,dott31):
Anewfunction[3,3]:=unapply(Anew[3,3],dott12,dott22,dott32,
dott11,dott21,dott31):
```

Specialisation in different directions of the tangent space. We also assume from now on that $a\_\{1,2\}$ and $a\_\{1,3\}$ are independent to the irregular times

```
> tdA110:=0:
dota12:=0;
dota13:=0;

tdAvinfty1:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do tdAvinfty1[i,j]:=
simplify(tdAfunction[i,j](0,0,0,1,1,1)): od: od:
tdAvinfty2:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do tdAvinfty2[i,j]:=
simplify(tdAfunction[i,j](1,1,1,0,0,0)): od: od:
tdAuinfty1:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do tdAuinfty1[i,j]:=
simplify(tdAfunction[i,j](0,0,0,t12,t22,t32)): od: od:
tdAuinfty2:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do tdAuinfty2[i,j]:=
simplify(tdAfunction[i,j](2*t12,2*t22,2*t32,t11,t21,t31)): od:
od:
tdAa1:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do tdAa1[i,j]:= simplify
(tdAfunction[i,j](2*(t12-t32)*(t12-t22),0,0,t12*(3*t11-t21-t31)
-(t12+t22+t32)*t11+ t22*t31+t32*t21,0,0)): od: od:
tdAa2:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do tdAa2[i,j]:= simplify
(tdAfunction[i,j](0,2*(t22-t12)*(t22-t32),0,0,t22*(3*t21-t11-
t31)- (t12+t22+t32)*t21+t12*t31+t32*t11,0)): od: od:
```

```
tdAa3:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do tdAa3[i,j]:= simplify
(tdAfunction[i,j](0,0,2*(t32-t12)*(t32-t22),0,0,t32*(3*t31-t11-
t21)- (t12+t22+t32)*t31+t12*t21+t22*t11)): od: od:
tdAe11:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do tdAe11[i,j]:=
simplify(tdAfunction[i,j](0,0,0,1,0,0)): od: od:
tdAe21:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do tdAe21[i,j]:=
simplify(tdAfunction[i,j](0,0,0,0,1,0)): od: od:
tdAe31:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do tdAe31[i,j]:=
simplify(tdAfunction[i,j](0,0,0,0,0,1)): od: od:


tdAe12:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do tdAe12[i,j]:=
simplify(tdAfunction[i,j](1,0,0,0,0,0)): od: od:
tdAe22:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do tdAe22[i,j]:=
simplify(tdAfunction[i,j](0,1,0,0,0,0)): od: od:
tdAe32:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do tdAe32[i,j]:=
simplify(tdAfunction[i,j](0,0,1,0,0,0)): od: od:



tdAtau:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do tdAtau[i,j]:=
simplify(tdAfunction[i,j](0,0,0,0,Coefftau,-Coefftau)): od: od:


Anewe2:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do Anewe2[i,j]:=
simplify(Anewfunction[i,j](0,0,0,0,1,0)): od: od:


dtdAtaudlambda:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do dtdAtaudlambda[i,j]:=
diff(tdAtau[i,j],lambda): od: od:


dtdAe31dlambda:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do dtdAe31dlambda[i,j]:=
diff(tdAe31[i,j],lambda): od: od:


dtdAe21dlambda:=Matrix(3,3,0):
```

```
for i from 1 to 3 do for j from 1 to 3 do dtdAe21dlambda[i,j]:=
diff(tdAe21[i,j],lambda): od: od:

dtdAnewdlambda:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do dtdAnewdlambda[i,j]:=
diff(Anewe2[i,j],lambda): od: od:


Avinfty1:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do Avinfty1[i,j]:=
simplify(Afunction[i,j](0,0,0,1,1,1)): od: od:
Avinfty2:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do Avinfty2[i,j]:=
simplify(Afunction[i,j](1,1,1,0,0,0)): od: od:
Auinfty1:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do Auinfty1[i,j]:=
simplify(Afunction[i,j](0,0,0,t12,t22,t32)): od: od:
Auinfty2:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do Auinfty2[i,j]:=
simplify(Afunction[i,j](2*t12,2*t22,2*t32,t11,t21,t31)): od:
od:
Aa1:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do Aa1[i,j]:= simplify
(Afunction[i,j](2*(t12-t32)*(t12-t22),0,0,t12*(3*t11-t21-t31)-
(t12+t22+t32)*t11+ t22*t31+t32*t21,0,0)): od: od:
Aa2:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do Aa2[i,j]:= simplify
(Afunction[i,j](0,2*(t22-t12)*(t22-t32),0,0,t22*(3*t21-t11-t31)
- (t12+t22+t32)*t21+t12*t31+t32*t11,0)): od: od:
Aa3:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do Aa3[i,j]:= simplify
(Afunction[i,j](0,0,2*(t32-t12)*(t32-t22),0,0,t32*(3*t31-t11-
t21)- (t12+t22+t32)*t31+t12*t21+t22*t11)): od: od:
Ae11:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do Ae11[i,j]:= simplify
(Afunction[i,j](0,0,0,1,0,0)): od: od:
Ae21:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do Ae21[i,j]:= simplify
(Afunction[i,j](0,0,0,0,1,0)): od: od:
Ae31:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do Ae31[i,j]:= simplify
(Afunction[i,j](0,0,0,0,0,1)): od: od:
```

```
Ae12:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do Ae12[i,j]:= simplify
(Afunction[i,j](1,0,0,0,0,0)): od: od:
Ae22:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do Ae22[i,j]:= simplify
(Afunction[i,j](0,1,0,0,0,0)): od: od:
Ae32:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do Ae32[i,j]:= simplify
(Afunction[i,j](0,0,1,0,0,0)): od: od:

Atau:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do Atau[i,j]:= simplify
(Afunction[i,j](0,0,0,0,Coefftau,-Coefftau)): od: od:

dAtaudlambda:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do dAtaudlambda[i,j]:=
diff(Atau[i,j],lambda): od: od:

dAe31dlambda:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do dAe3dlambda[i,j]:=
diff(Ae31[i,j],lambda): od: od:

dAe21dlambda:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do dAe2dlambda[i,j]:=
diff(Ae21[i,j],lambda): od: od:
```

$$dota12 := 0$$
$$dota13 := 0$$
(7.1)

```
> tdAvinfty1;
  tdAvinfty2;
  tdAuinfty1:
  tdAuinfty2:
  tdAe11:
  tdAe21:
  tdAe31:
  tdAa1:
  tdAa2:
  tdAa3:

  tdAtau:
```

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$(7.2)$$

$$\begin{bmatrix} \dfrac{1}{2}\lambda^2 & 0 & 0 \\[2mm] 0 & \dfrac{1}{2}\lambda^2 & 0 \\[2mm] 0 & 0 & \dfrac{1}{2}\lambda^2 \end{bmatrix}$$

```
> simplify(series(tdAa2[1,1],lambda)):
  simplify(series(tdAa2[1,2],lambda)):
  simplify(series(tdAa2[1,3],lambda)):
  simplify(series(tdAa2[2,1],lambda)):
  simplify(series(tdAa2[2,2],lambda)):
  simplify(series(tdAa2[2,3],lambda)):
  simplify(series(tdAa2[3,1],lambda)):
  simplify(series(tdAa2[3,2],lambda)):
  simplify(series(tdAa2[3,3],lambda)):
```

Verification for direction \partial_{\tau} and \partial_{e_31} and \partial_{e_21}

```
> LalphatautdL:=Matrix(3,3,0):
  for i from 1 to 3 do for j from 1 to 3 do
  LalphatdLfunction:=unapply(LalphatdL[i,j],dott12,dott22,dott32,
  dott11,dott21,dott31):
   LalphatautdL[i,j]:=simplify(LalphatdLfunction(0,0,0,0,
  Coefftau,-Coefftau)): od: od:

  Lalphae31tdL:=Matrix(3,3,0):
  for i from 1 to 3 do for j from 1 to 3 do
  LalphatdLfunction:=unapply(LalphatdL[i,j],dott12,dott22,dott32,
  dott11,dott21,dott31):
   Lalphae31tdL[i,j]:=simplify(LalphatdLfunction(0,0,0,0,0,1)):
  od: od:

  Lalphae21tdL:=Matrix(3,3,0):
  for i from 1 to 3 do for j from 1 to 3 do
  LalphatdLfunction:=unapply(LalphatdL[i,j],dott12,dott22,dott32,
  dott11,dott21,dott31):
   Lalphae21tdL[i,j]:=simplify(LalphatdLfunction(0,0,0,0,1,0)):
  od: od:
```

```
Lalphae2Lnew:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do
LalphaLnewfunction:=unapply(LalphaLnew[i,j],dott12,dott22,
dott32,dott11,dott21,dott31):
 Lalphae21Lnew[i,j]:=simplify(LalphaLnewfunction(0,0,0,0,1,0)):
od: od:

LalphatauL:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do
LalphaLfunction:=unapply(LalphaL[i,j],dott12,dott22,dott32,
dott11,dott21,dott31):
 LalphatauL[i,j]:=simplify(LalphaLfunction(0,0,0,0,Coefftau,
-Coefftau)): od: od:

Lalphae31L:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do
LalphaLfunction:=unapply(LalphaL[i,j],dott12,dott22,dott32,
dott11,dott21,dott31):
 Lalphae31L[i,j]:=simplify(LalphaLfunction(0,0,0,0,0,1)): od:
od:

Lalphae21L:=Matrix(3,3,0):
for i from 1 to 3 do for j from 1 to 3 do
LalphaLfunction:=unapply(LalphaL[i,j],dott12,dott22,dott32,
dott11,dott21,dott31):
 Lalphae21L[i,j]:=simplify(LalphaLfunction(0,0,0,0,1,0)): od:
od:

Verificationtau:=simplify(LalphatautdL-h*dtdAtaudlambda-
 (Multiply(tdAtau,tdL)- Multiply(tdL,tdAtau)) );
Verificatione3:=simplify(Lalphae31tdL-h*dtdAe31dlambda-
 (Multiply(tdAe31,tdL)- Multiply(tdL,tdAe31)) );
Verificatione2:=simplify(Lalphae21tdL-h*dtdAe21dlambda-
 (Multiply(tdAe21,tdL)- Multiply(tdL,tdAe21)) );
```

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(7.3)

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

▼ Canonical choice of trivial times: t12:=1/2, t22=0, t32=-1/2, t11=0 and t31=-t21 and t21=1/3*tau

```
> TT1(0,1/3*tau,-1/3*tau,1/2,0,-1/2);
  TT2(0,1/3*tau,-1/3*tau,1/2,0,-1/2);
  TT3(0,1/3*tau,-1/3*tau,1/2,0,-1/2);
  TT4(0,1/3*tau,-1/3*tau,1/2,0,-1/2);
  TT5(0,1/3*tau,-1/3*tau,1/2,0,-1/2);
  simplify(taufunction(0,1/3*tau,-1/3*tau,1/2,0,-1/2));

  partialtaufunction:=unapply(partialtau,t11,t21,t31,t12,t22,t32)
  :
  simplify(partialtaufunction(0,1/3*tau,-1/3*tau,1/2,0,-1/2));

  checkqfunctionT:=unapply(checkqfunction(q,p),t11,t21,t31,t12,
  t22,t32) ;
  simplify(checkqfunctionT(0,1/3*tau,-1/3*tau,1/2,0,-1/2));
  checkpfunctionT:=unapply(checkpfunction(q,p),t11,t21,t31,t12,
  t22,t32) ;
  simplify(checkpfunctionT(0,1/3*tau,-1/3*tau,1/2,0,-1/2));
  solve({-2*p+2*tau*(1/3)=checkq,-(1/2)*q+p=checkp },{q,p});
```

$$\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ -\dfrac{1}{2} \\ \tau \end{array} \tag{8.1}$$

$$\frac{1}{3}\,partialt21 - \frac{1}{3}\,partialt31$$

$$checkqfunctionT := (t11, t21, t31, t12, t22, t32) \rightarrow \sqrt{\frac{t12 - t32}{(-t12 + t22)\,(-t22 + t32)}}\;(q\,t22$$
$$-\,p + t21)$$

$$-2\,p + \frac{2}{3}\,\tau$$

$$checkpfunctionT := (t11, t21, t31, t12, t22, t32) \rightarrow \sqrt{\frac{-t22 + t32}{(-t12 + t22)\,(t12 - t32)}}\;(-q\,t12$$
$$+\,p - t11)$$

$$-\frac{1}{2}\, q + p$$

$$\left\{ p = \frac{1}{3}\,\tau - \frac{1}{2}\, checkq,\ q = \frac{2}{3}\,\tau - checkq - 2\, checkp \right\}$$

```
> a12:=1;
  a13:=1;
  t12:=1/2;
  t22:=0;
  t32:=-1/2;
  t11:=0;
  t31:=-t21;
  t21:=1/3*tau;
```

$$a12 := 1 \tag{8.2}$$
$$a13 := 1$$
$$t12 := \frac{1}{2}$$
$$t22 := 0$$
$$t32 := -\frac{1}{2}$$
$$t11 := 0$$
$$t31 := -t21$$
$$t21 := \frac{1}{3}\,\tau$$

```
> tdL[1,1];
  tdL[2,2];
  tdL[3,3];
  tdL21check(tau);
  tdL23check(tau);
  tdL31check(tau);
  tdL32check(tau);
```

$$\frac{1}{2}\,\lambda \tag{8.3}$$

$$\frac{1}{3}\,\tau$$

$$-\frac{1}{2}\,\lambda - \frac{1}{3}\,\tau$$

$$\frac{1}{2}\, checkp\, checkq + \frac{1}{2}\, checkq^2 - \frac{1}{2}\, checkq\,\tau - \frac{1}{2}\, t20$$

$$\frac{1}{2}\, checkq$$

$$-checkp\, checkq - checkq^2 + checkq\,\tau - t30$$

$$checkp + checkq + \tau$$

```
> Coefftau:=simplify(Coefft21);
  tdAtau:=simplify(tdAtau):
  tdAtau11function:=unapply(tdAtau[1,1],q,p):
```

```
tdAtau11:=simplify(tdAtau11function(2*tau*(1/3)-checkq-2*
checkp,(1/3)*tau-(1/2)*checkq));
tdAtau12function:=unapply(tdAtau[1,2],q,p):
tdAtau12:=simplify(tdAtau12function(2*tau*(1/3)-checkq-2*
checkp,(1/3)*tau-(1/2)*checkq));
tdAtau13function:=unapply(tdAtau[1,3],q,p):
tdAtau13:=simplify(tdAtau13function(2*tau*(1/3)-checkq-2*
checkp,(1/3)*tau-(1/2)*checkq));
tdAtau21function:=unapply(tdAtau[2,1],q,p):
tdAtau21:=simplify(tdAtau21function(2*tau*(1/3)-checkq-2*
checkp,(1/3)*tau-(1/2)*checkq));
tdAtau22function:=unapply(tdAtau[2,2],q,p):
tdAtau22:=simplify(tdAtau22function(2*tau*(1/3)-checkq-2*
checkp,(1/3)*tau-(1/2)*checkq));
tdAtau23function:=unapply(tdAtau[2,3],q,p):
tdAtau23:=simplify(tdAtau23function(2*tau*(1/3)-checkq-2*
checkp,(1/3)*tau-(1/2)*checkq));
tdAtau31function:=unapply(tdAtau[3,1],q,p):
tdAtau31:=simplify(tdAtau31function(2*tau*(1/3)-checkq-2*
checkp,(1/3)*tau-(1/2)*checkq));
tdAtau32function:=unapply(tdAtau[3,2],q,p):
tdAtau32:=simplify(tdAtau32function(2*tau*(1/3)-checkq-2*
checkp,(1/3)*tau-(1/2)*checkq));
tdAtau33function:=unapply(tdAtau[3,3],q,p):
tdAtau33:=simplify(tdAtau33function(2*tau*(1/3)-checkq-2*
checkp,(1/3)*tau-(1/2)*checkq));
```

$$Coefftau := \frac{1}{3}$$

<div align="right">(8.4)</div>

$$tdAtau11 := 0$$

$$tdAtau12 := -\frac{2}{3}$$

$$tdAtau13 := \frac{1}{3}$$

$$tdAtau21 := -\frac{1}{3}\,checkq^2 + \frac{1}{3}\,(\tau - checkp)\,checkq + \frac{1}{3}\,t20$$

$$tdAtau22 := \frac{7}{9}\,\tau - checkq - checkp + \frac{1}{3}\,\lambda$$

$$tdAtau23 := \frac{2}{3}\,checkq$$

$$tdAtau31 := -\frac{1}{3}\,checkq^2 + \frac{1}{3}\,(\tau - checkp)\,checkq - \frac{1}{3}\,t30$$

$$tdAtau32 := -\frac{4}{3}\,\tau + \frac{4}{3}\,checkq + \frac{4}{3}\,checkp$$

$$tdAtau33 := -\frac{1}{9}\,\tau - checkq - \frac{1}{3}\,\lambda$$

```
> P1(lambda);
  P2(lambda);
  P3(lambda);
```
$$0 \tag{8.5}$$
$$-\frac{1}{4}\,\lambda^2 - \frac{1}{6}\,\tau\,\lambda + t30 + \frac{1}{2}\,t20 - \frac{1}{9}\,\tau^2$$
$$-\frac{1}{12}\,\tau\,\lambda^2 + \left(-\frac{1}{4}\,t20 - \frac{1}{18}\,\tau^2\right)\lambda$$

```
> L[3,3];
  simplify(L[3,2]-(h*p/(lambda-q)-P2(lambda)+h/2));
  simplify(L[3,1]-(h*(p^2+P2(q)-h/2)/(lambda-q)+P3(lambda)+p^3+
  (P2(q)-h/2)*p-P3(q)+h/4*(lambda-q)));
```
$$-\frac{h}{-\lambda+q} \tag{8.6}$$
$$0$$
$$0$$

```
> Atau:=simplify(Atau):
  simplify( Atau[1,1]- ( (-2*p^2-2*P2(q)+h)/(lambda-q)+1/3*
  lambda+p+1/2*q)   );
  simplify( Atau[1,2]- ( -2*p/(lambda-q)+1/3)   );
  simplify( Atau[1,3]- ( -2/(lambda-q))   );
  simplify( Atau[2,1]- ( p*(-2*p^2-2*P2(q)+h)/(lambda-q)+1/6*tau*
  lambda+tau/6*q +tau^2/9+t20/2-h/6 )   );
  simplify( Atau[2,2]- ( -2*p^2/(lambda-q)-1/6*lambda)-p+1/3*tau
  );
  simplify( Atau[2,3]- ( -2*p/(lambda-q) +1/3)   );
  simplify( Atau[3,1]- (-1/3*(6*p^2-h)*(p^2+P2(q)-1/2*h)/(lambda-
  q)-(1/36)*tau*lambda^2
  +1/6*(tau*p-tau^2/9-t20/2+h/2)*lambda
  +1/6*(2*p^3-(q^2-4/3*tau*q-8/9*tau^2+8*h-8*t20-4*t30)*p/2+
  tau^2/9*q+tau/6*q^2+h*tau -h/2*q+t20/2*q)
  )   );
  simplify( Atau[3,2]- ( p*(-6*p^2+h)/3/(lambda-q)+(1/12)*
  lambda^2 +(-(1/2)*p+2*tau*(1/9))*lambda
  +1/6*(8/9*tau^2+(-2*p+q)*tau-3*p*q-h+2*t20-2*t30)
  )   );
  simplify( Atau[3,3]- ( (-6*p^2+h)/3/(lambda-q) -lambda/6+p-
  tau/3)   );
```
$$0 \tag{8.7}$$
$$0$$
$$0$$
$$0$$
$$0$$
$$0$$

0
0
0