We load the Hamiltonians from the general Hamiltonian evolutions.
We remind the shift of Darboux coordinates and the definition of the change of times.

```
> restart:
with(LinearAlgebra):
with(DifferentialGeometry):
with(Tools):

R1:=unapply((-s10-s20)/(xi-X1)-s11-s21+(-s12-s22)*xi,xi);
R2:=unapply(sX10*(-s10-s20-sX10)/(xi-X1)^2+s10*s22+s11*s21+s12*
s20+(s11*s22+s12*s21)*xi+s12*s22*xi^2,xi);

sX20:=-s10-s20-sX10;
KOldCoordinates :=unapply( (1/2)*(sX10+s20)*(sX10+s10)*ln((s12-
s22)/2)+(1/2)*X1*((X1*s12+2*s11)*s10+s20*(X1*s22+2*s21)),s11,
s21,s12,s22,X1);

Hams11 := (-Q^4*s12*s22+(2*X1*s12*s22+(-s12-s22)*P-s11*s22-s12*
s21)*Q^3+(-X1^2*s12*s22+((2*s12+2*s22)*P+2*s11*s22+2*s12*s21)*
X1-P^2+(-s11-s21)*P+(-s20-h)*s12+(h-s10)*s22-s11*s21)*Q^2+(((-
s12-s22)*P-s12*s21-s11*s22)*X1^2+(2*P^2+(2*s11+2*s21)*P+(2*
s20+2*h)*s12+(-h+2*s10)*s22+2*s11*s21)*X1+P*(sX10+sX20))*Q+((-
s20-h)*s12-P^2+(-s11-s21)*P-s10*s22-s11*s21)*X1^2-P*(sX10+sX20)
*X1-sX10*sX20)/((Q-X1)*(-s12+s22))
+
U11(s11,s21,s12,s22,X1) +diff(KOldCoordinates(s11,s21,s12,s22,
X1),s11):

Hams21 := (Q^4*s12*s22+(-2*X1*s12*s22+(s12+s22)*P+s11*s22+s12*
s21)*Q^3+(X1^2*s12*s22+((-2*s12-2*s22)*P-2*s11*s22-2*s12*s21)*
X1+P^2+(s21+s11)*P+s10*s22+s11*s21+s12*s20)*Q^2+(((s12+s22)*P+
s11*s22+s12*s21)*X1^2+(-2*P^2+(-2*s11-2*s21)*P+(-h-2*s20)*s12
-2*s10*s22-2*s11*s21)*X1-P*(sX10+sX20))*Q+((s20+h)*s12+P^2+
(s21+s11)*P+s10*s22+s11*s21)*X1^2+P*(sX10+sX20)*X1+sX10*sX20)/(
(Q-X1)*(-s12+s22))
+
U21(s11,s21,s12,s22,X1) +diff(KOldCoordinates(s11,s21,s12,s22,
X1),s21):

Hams12 := (-(s12*s22*Q^2+((s21+P)*s12+s22*(P+s11))*Q+(s20+h)*
s12+s10*s22+(P+s11)*(s21+P))*(-s12+s22)*X1^3+(2*s22*s12*(-s12+
s22)*Q^3+((-2*P-2*s21)*s12^2-3*s22*(-s21+s11)*s12+2*s22^2*(P+
s11))*Q^2+((-2*s20-2*h)*s12^2+((2*h-2*s10+2*s20)*s22-(2*(P+3*
```

```
s11*(1/2)-(1/2)*s21))*(s21+P))*s12+(2*(s10*s22+(P+s11)*(P-(1/2)
*s11+3*s21*(1/2))))*s22)*Q+((sX10+sX20-h)*P-(-s21+s11)*(s20+h))
*s12+((-sX10-sX20+h)*P-s10*(-s21+s11))*s22-(-s21+s11)*(s21+P)*
(P+s11))*X1^2+(-s22*s12*(-s12+s22)*Q^4+((s21+P)*s12^2+3*s22*(-
s21+s11)*s12-s22^2*(P+s11))*Q^3+((s20+h)*s12^2+((s10-s20)*s22+
(s21+P)*(P+3*s11-2*s21))*s12-s22*((h+s10)*s22+(P+s11)*(P-2*
s11+3*s21)))*Q^2+(((-sX10-sX20+2*h)*P+(2*s20+2*h)*s11-s21*(h+2*
s20))*s12+((sX10+sX20-2*h)*P+(-h+2*s10)*s11-2*s21*s10)*s22+(2*
(-s21+s11))*(s21+P)*(P+s11))*Q+sX10*sX20*s12-sX10*sX20*s22-P*(-
s21+s11)*(sX10+sX20))*X1-s22*(-s21+s11)*s12*Q^4+((-s22*h-(-s21+
s11)*(s21+P))*s12-(-s22*h+(-s21+s11)*(P+s11))*s22)*Q^3+((-h*P+
(-s20-h)*s11+s20*s21)*s12+(h*P+(h-s10)*s11+s21*s10)*s22-(-s21+
s11)*(s21+P)*(P+s11))*Q^2+P*(-s21+s11)*(sX10+sX20)*Q-sX10*sX20*
(-s21+s11))/(2*(-s12+s22)^2*(Q-X1))
+
U12(s11,s21,s12,s22,X1) +diff(KOldCoordinates(s11,s21,s12,s22,
X1),s12):

Hams22 := ((s12*s22*Q^2+((s21+P)*s12+s22*(P+s11))*Q+(s20+h)*
s12+s10*s22+(P+s11)*(s21+P))*(-s12+s22)*X1^3+(-2*s22*s12*(-s12+
s22)*Q^3+((2*s21+2*P)*s12^2+3*s22*(-s21+s11)*s12-2*s22^2*(P+
s11))*Q^2+((2*s20+2*h)*s12^2+((-2*h+2*s10-2*s20)*s22+(2*(P+3*
s11*(1/2)-(1/2)*s21))*(s21+P))*s12-(2*(s10*s22+(P+s11)*(P-(1/2)
*s11+3*s21*(1/2))))*s22)*Q+((-sX10-sX20+h)*P+(-s21+s11)*(s20+h)
)*s12+((sX10+sX20-h)*P+s10*(-s21+s11))*s22+(-s21+s11)*(s21+P)*
(P+s11))*X1^2+(s22*s12*(-s12+s22)*Q^4+((-s21-P)*s12^2-3*s22*(-
s21+s11)*s12+s22^2*(P+s11))*Q^3+((-2*h-s20)*s12^2+((2*h-s10+
s20)*s22-(s21+P)*(P+3*s11-2*s21))*s12+s22*(s10*s22+(P+s11)*
(P-2*s11+3*s21)))*Q^2+(((sX10+sX20-2*h)*P+(-2*s20-2*h)*s11+s21*
(h+2*s20))*s12+((-sX10-sX20+2*h)*P+(h-2*s10)*s11+2*s21*s10)*s22
-(2*(-s21+s11))*(s21+P)*(P+s11))*Q-sX10*sX20*s12+sX10*sX20*s22+
P*(-s21+s11)*(sX10+sX20))*X1+s22*(-s21+s11)*s12*Q^4+(s12^2*h+(-
s22*h+(-s21+s11)*(s21+P))*s12+s22*(-s21+s11)*(P+s11))*Q^3+((h*
P+(s20+h)*s11-s20*s21)*s12+(-h*P+(-h+s10)*s11-s21*s10)*s22+(-
s21+s11)*(s21+P)*(P+s11))*Q^2-P*(-s21+s11)*(sX10+sX20)*Q+sX10*
sX20*(-s21+s11))/(2*(-s12+s22)^2*(Q-X1))
+
U22(s11,s21,s12,s22,X1) +diff(KOldCoordinates(s11,s21,s12,s22,
X1),s22):

HamX1 := (Q^4*s12*s22+(-2*X1*s12*s22+(s12+s22)*P+s11*s22+s12*
s21)*Q^3+(X1^2*s12*s22+((-2*s12-2*s22)*P-2*s11*s22-2*s12*s21)*
```

```
X1+(s20+h)*s12+P^2+(s21+s11)*P+s10*s22+s11*s21)*Q^2+(((s12+s22)
*P+s11*s22+s12*s21)*X1^2+(-2*P^2+(-2*s11-2*s21)*P+(-2*s20-2*h)*
s12-2*s10*s22-2*s11*s21)*X1-(sX10+sX20-h)*P)*Q+((s20+h)*s12+
P^2+(s21+s11)*P+s10*s22+s11*s21)*X1^2+(sX10+sX20-h)*P*X1+sX10*
sX20)/(Q-X1)
+
UX1(s11,s21,s12,s22,X1) +diff(KOldCoordinates(s11,s21,s12,s22,
X1),X1):
```

$$R1 := \xi \rightarrow \frac{-s10-s20}{\xi - X1} - s11 - s21 + (-s12 - s22)\,\xi \tag{1.1}$$

$$R2 := \xi \rightarrow \frac{sX10\,(-s10-s20-sX10)}{(\xi - X1)^2} + s10\,s22 + s11\,s21 + s12\,s20 + (s11\,s22 + s12\,s21)\,\xi + s12\,s22\,\xi^2$$

$$sX20 := -s10 - s20 - sX10$$

$$KOldCoordinates := (s11, s21, s12, s22, X1) \rightarrow \frac{1}{2}\,(sX10 + s20)\,(sX10 + s10)\,\ln\left(\frac{1}{2}\,s12 - \frac{1}{2}\,s22\right) + \frac{1}{2}\,X1\,((X1\,s12 + 2\,s11)\,s10 + s20\,(X1\,s22 + 2\,s21))$$

We load the purely time-dependent terms

```
> U11 := unapply( ((X1*s22+s21)*s10+(X1*s12+s11)*s20)/(s12-s22)
  +s22*X1/(s12-s22)*h,s11,s21,s12,s22,X1);
  U21 := unapply(-((X1*s22+s21)*s10+(X1*s12+s11)*s20)/(s12-s22)
  -s12*X1/(s12-s22)*h,s11,s21,s12,s22,X1);
  U12 := unapply( (X1*(s12-s22)-s11+s21)*((s10*s22+s12*s20)*X1+
  s21*s10+s11*s20)/(2*(s12-s22)^2)
  +(-s22^2*X1+(X1*s12-s11)*s22+s12*s21)*X1/(2*(s12-s22)^2)*h
  ,s11,s21,s12,s22,X1);
  U22 := unapply(-(X1*(s12-s22)-s11+s21)*((s10*s22+s12*s20)*X1+
  s21*s10+s11*s20)/(2*(s12-s22)^2)
  -h*(X1*s12^2+(-X1*s22+s21)*s12-s11*s22)*X1/(2*(s12-s22)^2)
  ,s11,s21,s12,s22,X1);
  UX1 := unapply( (X1*s22+s21)*s10+(X1*s12+s11)*s20,s11,s21,s12,
  s22,X1);
```

$$U11 := (s11, s21, s12, s22, X1) \rightarrow \frac{(X1\,s22 + s21)\,s10 + (X1\,s12 + s11)\,s20}{s12 - s22} + \frac{s22\,X1\,h}{s12 - s22} \tag{1.2}$$

$$U21 := (s11, s21, s12, s22, X1) \rightarrow -\frac{(X1\,s22 + s21)\,s10 + (X1\,s12 + s11)\,s20}{s12 - s22} - \frac{s12\,X1\,h}{s12 - s22}$$

$$U12 := (s11, s21, s12, s22, X1) \rightarrow \frac{1}{2}\,\frac{(X1\,(s12 - s22) - s11 + s21)\,((s10\,s22 + s12\,s20)\,X1 + s21\,s10 + s11\,s20)}{(s12 - s22)^2}$$

$$+\frac{1}{2}\frac{\left(-s22^2 X1+(X1\, s12-s11)\, s22+s12\, s21\right) X1\, h}{(s12-s22)^2}$$

$$U22 := (s11, s21, s12, s22, X1) \rightarrow$$

$$-\frac{1}{2}\frac{(X1\,(s12-s22)-s11+s21)\,((s10\, s22+s12\, s20)\, X1+s21\, s10+s11\, s20)}{(s12-s22)^2}$$

$$-\frac{1}{2}\frac{h\left(X1\, s12^2+(-X1\, s22+s21)\, s12-s11\, s22\right) X1}{(s12-s22)^2}$$

$$UX1 := (s11, s21, s12, s22, X1) \rightarrow (X1\, s22+s21)\, s10+(X1\, s12+s11)\, s20$$

```
> HamtdX1checkQcheckPbis :=  (checkQ-tdX1)*checkP^2+h*checkP-
  checkQ^3+tdX1*checkQ^2-(-h+s10-s20)*checkQ-(s10+s20+2*sX10)^2/
  (-4*tdX1+4*checkQ) ;
```

$$HamtdX1checkQcheckPbis := (checkQ-tdX1)\, checkP^2+h\, checkP-checkQ^3+tdX1\, checkQ^2-(-h+s10-s20)\, checkQ-\frac{(s10+s20+2\, sX10)^2}{-4\, tdX1+4\, checkQ} \tag{1.3}$$

```
> Sinfty2function:=s12+s22;
  Sinfty1function:=s11+s21;
  S2function:=sqrt(s12-s22)/sqrt(2);
  S1function:=(s11-s21)/sqrt(2)/sqrt(s12-s22);
  tdX1function:=X1*S2function+S1function;
  solve( {s12+s22=Sinfty2,s11+s21=Sinfty1, S2=sqrt(s12-s22)/sqrt
  (2),
  S1=(s11-s21)/sqrt(2)/sqrt(s12-s22),
  tdX1=X1*S2function+S1function},{s12,s22,s11,s21,X1});
  X1function := unapply( -(S1-tdX1)/S2 ,Sinfty1,Sinfty2,S1,S2,
  tdX1);
  s11function:= unapply( S2*S1+(1/2)*Sinfty1, Sinfty1,Sinfty2,S1,
  S2,tdX1);
  s12function:= unapply( S2^2+(1/2)*Sinfty2, Sinfty1,Sinfty2,S1,
  S2,tdX1);
  s21function:= unapply( -S2*S1+(1/2)*Sinfty1, Sinfty1,Sinfty2,
  S1,S2,tdX1);
  s22function:= unapply( -S2^2+(1/2)*Sinfty2, Sinfty1,Sinfty2,S1,
  S2,tdX1 );
  simplify(X1function(Sinfty1function,Sinfty2function,S1function,
  S2function,tdX1function));
  simplify(s11function(Sinfty1function,Sinfty2function,
  S1function,S2function,tdX1function));
  simplify(s12function(Sinfty1function,Sinfty2function,
  S1function,S2function,tdX1function));
  simplify(s21function(Sinfty1function,Sinfty2function,
  S1function,S2function,tdX1function));
  simplify(s22function(Sinfty1function,Sinfty2function,
```

```
S1function,S2function,tdX1function));

PartialtdX1function:=simplify(diff(s11function(Sinfty1,Sinfty2,
S1,S2,tdX1),tdX1))*Partials11+
simplify(diff(s21function(Sinfty1,Sinfty2,S1,S2,tdX1),tdX1))*
Partials21
+simplify(diff(s12function(Sinfty1,Sinfty2,S1,S2,tdX1),tdX1))*
Partials12
+simplify(diff(s22function(Sinfty1,Sinfty2,S1,S2,tdX1),tdX1))*
Partials22
+simplify(diff(X1function(Sinfty1,Sinfty2,S1,S2,tdX1),tdX1))*
PartialX1;
```

$$Sinfty2function := s12 + s22 \tag{1.4}$$

$$Sinfty1function := s21 + s11$$

$$S2function := \frac{1}{2} \sqrt{s12 - s22} \sqrt{2}$$

$$S1function := \frac{1}{2} \frac{(-s21 + s11)\sqrt{2}}{\sqrt{s12 - s22}}$$

$$tdX1function := \frac{1}{2} X1 \sqrt{s12 - s22} \sqrt{2} + \frac{1}{2} \frac{(-s21 + s11)\sqrt{2}}{\sqrt{s12 - s22}}$$

$$\left\{X1 = -\frac{S1 - tdX1}{S2}, s11 = S1\ S2 + \frac{1}{2}\ Sinfty1, s12 = S2^2 + \frac{1}{2}\ Sinfty2, s21 = -S1\ S2 + \frac{1}{2}\ Sinfty1, s22 = -S2^2 + \frac{1}{2}\ Sinfty2\right\}$$

$$X1function := (Sinfty1, Sinfty2, S1, S2, tdX1) \rightarrow -\frac{S1 - tdX1}{S2}$$

$$s11function := (Sinfty1, Sinfty2, S1, S2, tdX1) \rightarrow S1\ S2 + \frac{1}{2}\ Sinfty1$$

$$s12function := (Sinfty1, Sinfty2, S1, S2, tdX1) \rightarrow S2^2 + \frac{1}{2}\ Sinfty2$$

$$s21function := (Sinfty1, Sinfty2, S1, S2, tdX1) \rightarrow -S1\ S2 + \frac{1}{2}\ Sinfty1$$

$$s22function := (Sinfty1, Sinfty2, S1, S2, tdX1) \rightarrow -S2^2 + \frac{1}{2}\ Sinfty2$$

$$X1$$

$$s11$$

$$s12$$

$$s21$$

$$s22$$

$$PartialtdX1function := \frac{PartialX1}{S2}$$

```
> checkQfunction:=S2*Q+S1;
checkPfunction=1/S2*(P-1/2*R1(Q));
SolQ:=-(S1-checkQ)/S2;
SolP:=checkP*S2+(1/2)*R1(-(S1-checkQ)/S2);
```

```
simplify(checkQ-(S2*SolQ+S1));
simplify(checkP-(1/S2*(SolP-1/2*R1(SolQ))));
checkSolQ:=unapply(S2function*Q+S1function,Q,P);
checkSolP:=unapply(1/S2function*(P-1/2*R1(Q)),Q,P);
```

$$checkQfunction := Q\, S2 + S1 \tag{1.5}$$

$$checkPfunction = \frac{P - \frac{1}{2}\,\frac{-s10 - s20}{Q - X1} + \frac{1}{2}\, s11 + \frac{1}{2}\, s21 - \frac{1}{2}\,(-s12 - s22)\, Q}{S2}$$

$$SolQ := -\frac{S1 - checkQ}{S2}$$

$$SolP := checkP\, S2 + \frac{1}{2}\,\frac{-s10 - s20}{-\frac{S1 - checkQ}{S2} - X1} - \frac{1}{2}\, s11 - \frac{1}{2}\, s21 - \frac{1}{2}\,\frac{(-s12 - s22)\,(S1 - checkQ)}{S2}$$

$$0$$

$$0$$

$$checkSolQ := (Q, P) \rightarrow \frac{1}{2}\, Q\, \sqrt{s12 - s22}\, \sqrt{2} + \frac{1}{2}\,\frac{(-s21 + s11)\,\sqrt{2}}{\sqrt{s12 - s22}}$$

$$checkSolP := (Q, P) \rightarrow \frac{\sqrt{2}\,\left(P - \frac{1}{2}\,\frac{-s10 - s20}{Q - X1} + \frac{1}{2}\, s11 + \frac{1}{2}\, s21 - \frac{1}{2}\,(-s12 - s22)\, Q\right)}{\sqrt{s12 - s22}}$$

```
> HamtdX1checkQcheckPbisfunction:=unapply(HamtdX1checkQcheckPbis
,checkQ,checkP):
HamtdX1checkQcheckPbisfunction2:=unapply
(HamtdX1checkQcheckPbisfunction(checkSolQ(Q,P),checkSolP(Q,P)),
tdX1):
HamtdX1OldCoordinates:=simplify(HamtdX1checkQcheckPbisfunction2
(tdX1function));
```

$$HamtdX1OldCoordinates := \frac{1}{\sqrt{s12 - s22}\,(Q - X1)}\Bigg(\sqrt{2}\,\Big((Q - X1)\,(Q\,(Q - X1)\, s22 + P\,(Q - X1) + s21\,(Q - X1) + h + s20)\, Q\, s12 + (Q - X1)\, Q\,(P\,(Q - X1) + s11\,(Q - X1) + s10)\, s22 + (Q - X1)^2\, P^2 + (Q - X1)\,(s11\,(Q - X1) + s21\,(Q - X1) + h + s10 + s20)\, P + (Q - X1)\,(s21\,(Q - X1) + h + s20)\, s11 + s10\,(Q - X1)\, s21 + \left(\frac{1}{2}\, s10 + \frac{1}{2}\, s20\right) h - sX10\,(sX10 + s10 + s20)\Big)\Bigg) \tag{1.6}$$

Computation of the symplectic two-form Omega^{P4} using the differential geometry package to compute wedge products.

```
> DGsetup([s11,s21,s12,s22,X1,Q,P],B,verbose);
```

The following coordinates have been protected: (2.1)

$$[s11, s21, s12, s22, X1, Q, P]$$

The following vector fields have been defined and protected:

[_DG([["vector", B, [ ]], [[[1], 1]]]), _DG([["vector", B, [ ]], [[[2], 1]]]), _DG([["vector", B, [ ]], [[[3], 1]]]), _DG([["vector", B, [ ]], [[[4], 1]]]), _DG([["vector", B, [ ]], [[[5], 1]]]), _DG([["vector", B, [ ]], [[[6], 1]]]), _DG([["vector", B, [ ]], [[[7], 1]]])]

The following differential 1-forms have been defined and protected:

[_DG([["form", B, 1], [[[1], 1]]]), _DG([["form", B, 1], [[[2], 1]]]), _DG([["form", B, 1], [[[3], 1]]]), _DG([["form", B, 1], [[[4], 1]]]), _DG([["form", B, 1], [[[5], 1]]]), _DG([["form", B, 1], [[[6], 1]]]), _DG([["form", B, 1], [[[7], 1]]])]

frame name: B

```
B > dcheckQ:=(simplify(diff(checkSolQ(Q,P),s11)))*(DGform(s11))
    +(simplify(diff(checkSolQ(Q,P),s21)))*(DGform(s21))+
    (simplify(diff(checkSolQ(Q,P),s12)))*(DGform(s12))+
    (simplify(diff(checkSolQ(Q,P),s22)))*(DGform(s22))+
    (simplify(diff(checkSolQ(Q,P),X1)))*(DGform(X1))+(simplify
    (diff(checkSolQ(Q,P),Q)))*(DGform(Q))+(simplify(diff
    (checkSolQ(Q,P),P)))*(DGform(P));

    dcheckP:=(simplify(diff(checkSolP(Q,P),s11)))*(DGform(s11))
    +(simplify(diff(checkSolP(Q,P),s21)))*(DGform(s21))+
    (simplify(diff(checkSolP(Q,P),s12)))*(DGform(s12))+
    (simplify(diff(checkSolP(Q,P),s22)))*(DGform(s22))+
    (simplify(diff(checkSolP(Q,P),X1)))*(DGform(X1))+(simplify
    (diff(checkSolP(Q,P),Q)))*(DGform(Q))+(simplify(diff
    (checkSolP(Q,P),P)))*(DGform(P));
```

$$\frac{1}{2}\,\frac{\sqrt{2}\ \_DG([["form", B, 1], [[[1], 1]]])}{\sqrt{s12-s22}} - \frac{1}{2}\,\frac{\sqrt{2}\ \_DG([["form", B, 1], [[[2], 1]]])}{\sqrt{s12-s22}} + \frac{1}{4}\,\frac{\sqrt{2}\ (Q\,(s12-s22)-s11+s21)\ \_DG([["form", B, 1], [[[3], 1]]])}{(s12-s22)^{3/2}} - \frac{1}{4}\,\frac{\sqrt{2}\ (Q\,(s12-s22)-s11+s21)\ \_DG([["form", B, 1], [[[4], 1]]])}{(s12-s22)^{3/2}} + \frac{1}{2}\sqrt{s12-s22}\,\sqrt{2}\ \_DG([["form", B, 1], [[[6], 1]]]) \tag{2.2}$$

$$\frac{1}{2}\,\frac{\sqrt{2}\ \_DG([["form", B, 1], [[[1], 1]]])}{\sqrt{s12-s22}} + \frac{1}{2}\,\frac{\sqrt{2}\ \_DG([["form", B, 1], [[[2], 1]]])}{\sqrt{s12-s22}} - \frac{1}{(s12-s22)^{3/2}\,(4\,Q-4\,X1)}\Bigg(2\sqrt{2}\,\bigg(\Big(-\frac{1}{2}\,s12+\frac{3}{2}\,s22\Big)\,Q^2+\bigg(\Big(\frac{1}{2}\,s12$$

$$- \frac{3}{2}\, s22\Big)\, X1 + P + \frac{1}{2}\, s11 + \frac{1}{2}\, s21\Big)\, Q + \Big(-P - \frac{1}{2}\, s11 - \frac{1}{2}\, s21\Big)\, X1 + \frac{1}{2}\, s10$$

$$+ \frac{1}{2}\, s20\Big)\, \_DG([[\text{"form"}, B, 1], [[[3], 1]]])\Big)$$

$$+ \frac{1}{(s12 - s22)^{3/2}\,(4\,Q - 4\,X1)}\,\Big(2\sqrt{2}\,\Big(\Big(\frac{3}{2}\, s12 - \frac{1}{2}\, s22\Big)\, Q^2 + \Big(\Big(-\frac{3}{2}\, s12$$

$$+ \frac{1}{2}\, s22\Big)\, X1 + P + \frac{1}{2}\, s11 + \frac{1}{2}\, s21\Big)\, Q + \Big(-P - \frac{1}{2}\, s11 - \frac{1}{2}\, s21\Big)\, X1 + \frac{1}{2}\, s10$$

$$+ \frac{1}{2}\, s20\Big)\, \_DG([[\text{"form"}, B, 1], [[[4], 1]]])\Big)$$

$$+ \frac{1}{2}\, \frac{\sqrt{2}\,(s10 + s20)\, \_DG([[\text{"form"}, B, 1], [[[5], 1]]])}{\sqrt{s12 - s22}\,(Q - X1)^2}$$

$$+ \frac{1}{2}\, \frac{1}{\sqrt{s12 - s22}\,(Q - X1)^2}\,\big(\big(s12\,(Q - X1)^2 + s22\,(Q - X1)^2 - s10$$

$$- s20\big)\sqrt{2}\, \_DG([[\text{"form"}, B, 1], [[[6], 1]]])\big)$$

$$+ \frac{\sqrt{2}\, \_DG([[\text{"form"}, B, 1], [[[7], 1]]])}{\sqrt{s12 - s22}}$$

```
B > dcheckQwedgedcheckP:=simplify((dcheckQ)&wedge(dcheckP)):
```

```
B > dtdX1:=simplify(
     (simplify(diff(tdX1function,s11)))*(DGform(s11))
    +(simplify(diff(tdX1function,s21)))*(DGform(s21))
    +(simplify(diff(tdX1function,s12)))*(DGform(s12))
    +(simplify(diff(tdX1function,s22)))*(DGform(s22))
    +(simplify(diff(tdX1function,X1)))*(DGform(X1))
    );

    dHamtdX1OldCoordinates:=simplify(
     (simplify(diff(HamtdX1OldCoordinates,s11)))*(DGform(s11))
    +(simplify(diff(HamtdX1OldCoordinates,s21)))*(DGform(s21))
    +(simplify(diff(HamtdX1OldCoordinates,s12)))*(DGform(s12))
    +(simplify(diff(HamtdX1OldCoordinates,s22)))*(DGform(s22))
    +(simplify(diff(HamtdX1OldCoordinates,X1)))*(DGform(X1))
    +(simplify(diff(HamtdX1OldCoordinates,Q)))*(DGform(Q))
    +(simplify(diff(HamtdX1OldCoordinates,P)))*(DGform(P))   ):

    dtdX1dwedgedHamtdX1:= simplify((dtdX1)&wedge
    (dHamtdX1OldCoordinates)):
```

$$-\frac{1}{2}\, \frac{1}{(s12 - s22)^{3/2}}\,\Big(\sqrt{2}\,\Big(\Big(-\frac{1}{2}\, s12\, X1 + \frac{1}{2}\, s22\, X1 + \frac{1}{2}\, s11 \tag{2.3}$$

$$- \frac{1}{2}\, s21\Big)\, \_DG([[\text{"form"}, B, 1], [[[3], 1]]]) + \Big(\frac{1}{2}\, s12\, X1 - \frac{1}{2}\, s22\, X1 - \frac{1}{2}\, s11$$

$$+ \frac{1}{2}\, s21\Big)\, \_DG([[\text{"form"}, B, 1], [[[4], 1]]]) + \big((-s12 + s22)\, \_DG([[\text{"form"}, B,$$

1], [[[5], 1]]]) + _DG([["form", B, 1], [[[2], 1]]]) − _DG([["form", B, 1], [[[1], 1]]])) ($s12 - s22$)))

```
B > Omega2:=simplify( (h&mult(dcheckQwedgedcheckP))&minus
    (dtdX1dwedgedHamtdX1) ):
```

```
B > dHams11:=
    (simplify(diff(Hams11,s11)))*(DGform(s11))+
    (simplify(diff(Hams11,s21)))*(DGform(s21))+
    (simplify(diff(Hams11,s12)))*(DGform(s12))+
    (simplify(diff(Hams11,s22)))*(DGform(s22))+
    (simplify(diff(Hams11,X1)))*(DGform(X1))+
    (simplify(diff(Hams11,Q)))*(DGform(Q))+
    (simplify(diff(Hams11,P)))*(DGform(P)):

    dHams21:=
    (simplify(diff(Hams21,s11)))*(DGform(s11))+
    (simplify(diff(Hams21,s21)))*(DGform(s21))+
    (simplify(diff(Hams21,s12)))*(DGform(s12))+
    (simplify(diff(Hams21,s22)))*(DGform(s22))+
    (simplify(diff(Hams21,X1)))*(DGform(X1))+
    (simplify(diff(Hams21,Q)))*(DGform(Q))+
    (simplify(diff(Hams21,P)))*(DGform(P)):

    dHams12:=
    (simplify(diff(Hams12,s11)))*(DGform(s11))+
    (simplify(diff(Hams12,s21)))*(DGform(s21))+
    (simplify(diff(Hams12,s12)))*(DGform(s12))+
    (simplify(diff(Hams12,s22)))*(DGform(s22))+
    (simplify(diff(Hams12,X1)))*(DGform(X1))+
    (simplify(diff(Hams12,Q)))*(DGform(Q))+
    (simplify(diff(Hams12,P)))*(DGform(P)):

    dHams22:=
    (simplify(diff(Hams22,s11)))*(DGform(s11))+
    (simplify(diff(Hams22,s21)))*(DGform(s21))+
    (simplify(diff(Hams22,s12)))*(DGform(s12))+
    (simplify(diff(Hams22,s22)))*(DGform(s22))+
    (simplify(diff(Hams22,X1)))*(DGform(X1))+
    (simplify(diff(Hams22,Q)))*(DGform(Q))+
    (simplify(diff(Hams22,P)))*(DGform(P)):

    dHamX1:=
    (simplify(diff(HamX1,s11)))*(DGform(s11))+
```

```
(simplify(diff(HamX1,s21)))*(DGform(s21))+
(simplify(diff(HamX1,s12)))*(DGform(s12))+
(simplify(diff(HamX1,s22)))*(DGform(s22))+
(simplify(diff(HamX1,X1)))*(DGform(X1))+
(simplify(diff(HamX1,Q)))*(DGform(Q))+
(simplify(diff(HamX1,P)))*(DGform(P)):
```

```
B > Omega:=((dHams11)&wedge(DGform(s11))):
    Omega:=((dHams21)&wedge(DGform(s21))) &plus(Omega):
    Omega:=((dHams12)&wedge(DGform(s12))) &plus(Omega):
    Omega:=((dHams22)&wedge(DGform(s22))) &plus(Omega):
    Omega:=((dHamX1)&wedge(DGform(X1))) &plus(Omega):
    Omega:=(h&mult(DGform(Q))&wedge (DGform(P))) &plus(Omega):
    Omega:=simplify(Omega):
B > DifferenceFundamentalForm:= (Omega)&minus(Omega2):
B > DifferenceFundamentalForm:=simplify
    (DifferenceFundamentalForm):
B > DifferenceFundamentalForm;
```

_DG([["form", B, 2], [[[1, 2], 0]]]) **(2.4)**