In this Maple file, we conmpute the Lax pair in the oper gauge in the Painlevé 1 case. We also check that the formulas proposed in the article are correct in this case.

```
> restart:
with(LinearAlgebra):
tinfty10:=0;
Pinfty11:=-(2*tinfty14)/2;
Pinfty01:=-(2*tinfty12)/2;
Pinfty32 := -(1/4)*tinfty15^2;
Pinfty22 := -(1/2)*tinfty15*tinfty13+(1/4)*tinfty14^2;
Pinfty12 := -(1/2)*tinfty15*tinfty11+(1/2)*tinfty14*tinfty12-
(1/4)*tinfty13^2;
P1:=x-> Pinfty01+Pinfty11*x;
P2:=x-> Pinfty02+Pinfty12*x+Pinfty22*x^2+Pinfty32*x^3;

Unknownn:=-Unknown:
LUnknownn:=-LUnknown:
Unknownn2:=Unknown2:
LUnknownn2:=LUnknown2:

Ltinfty25:=-Ltinfty15;
Ltinfty23:=-Ltinfty13;
Ltinfty21:=-Ltinfty11;
Ltinfty20:=-Ltinfty10;
Ltinfty24:=Ltinfty14;
Ltinfty22:=Ltinfty12;
Ltinfty10:=0:
```

$$tinfty10 := 0$$
$$Pinfty11 := -tinfty14$$
$$Pinfty01 := -tinfty12$$
$$Pinfty32 := -\frac{tinfty15^2}{4}$$
$$Pinfty22 := -\frac{tinfty15\ tinfty13}{2} + \frac{tinfty14^2}{4}$$
$$Pinfty12 := -\frac{tinfty15\ tinfty11}{2} + \frac{tinfty14\ tinfty12}{2} - \frac{tinfty13^2}{4}$$
$$P1 := x \mapsto Pinfty01 + Pinfty11\, x$$
$$P2 := x \mapsto Pinfty02 + Pinfty12\, x + Pinfty22\, x^2 + Pinfty32\, x^3$$
$$Ltinfty25 := -Ltinfty15$$
$$Ltinfty23 := -Ltinfty13$$
$$Ltinfty21 := -Ltinfty11$$
$$Ltinfty20 := -Ltinfty10$$
$$Ltinfty24 := Ltinfty14 \tag{1}$$

$$Ltinfty22 := Ltinfty12 \quad (1)$$

Study at infinity

```
> espilon:=1:
logPsi1Infty:=-1/5*tinfty15/h*lambda^(5/2)-1/4*tinfty14/h*
lambda^2-1/3*tinfty13/h*lambda^(3/2)-1/2*tinfty12/h*lambda^1-
tinfty11/h*lambda^(1/2)+1/4*epsilon*ln(lambda)+A10+
Unknown/lambda^(1/2)+ Unknown2/lambda;
logPsi2Infty:=1/5*tinfty15/h*lambda^(5/2)-1/4*tinfty14/h*
lambda^2+1/3*tinfty13/h*lambda^(3/2)-1/2*tinfty12/h*lambda^1+
tinfty11/h*lambda^(1/2)+1/4*epsilon*ln(lambda)+A20-
Unknown/lambda^(1/2)+ Unknown2/lambda;


GrosLlogpsi1Infty:=-1/5*Ltinfty15/h*lambda^(5/2)-1/4*Ltinfty14/h*
lambda^2-1/3*Ltinfty13/h*lambda^(3/2)-1/2*Ltinfty12/h*lambda^1-
Ltinfty11/h*lambda^(1/2)+LA10+ LUnknown/lambda^(1/2)+
LUnknown2/lambda   ;
GrosLlogpsi2Infty:=1/5*Ltinfty15/h*lambda^(5/2)-1/4*Ltinfty14/h*
lambda^2+1/3*Ltinfty13/h*lambda^(3/2)-1/2*Ltinfty12/h*lambda^1+
Ltinfty11/h*lambda^(1/2)+LA20- LUnknown/lambda^(1/2)+
LUnknown2/lambda;

GrosLpsi1Infty := exp(-1/5*tinfty15/h*lambda^(5/2)-1/4*
tinfty14/h*lambda^2-1/3*tinfty13/h*lambda^(3/2)-1/2*tinfty12/h*
lambda^1-tinfty11/h*lambda^(1/2)+1/4*epsilon*ln(lambda)+A10+
Unknown/lambda^(1/2)+ Unknown2/lambda)*(-1/5*Ltinfty15/h*lambda^
(5/2)-1/4*Ltinfty14/h*lambda^2-1/3*Ltinfty13/h*lambda^(3/2)-1/2*
Ltinfty12/h*lambda^1-Ltinfty11/h*lambda^(1/2)+LA10+
LUnknown/lambda^(1/2)+ LUnknown2/lambda);
GrosLpsi2Infty := exp(1/5*tinfty15/h*lambda^(5/2)-1/4*tinfty14/h*
lambda^2+1/3*tinfty13/h*lambda^(3/2)-1/2*tinfty12/h*lambda^1+
tinfty11/h*lambda^(1/2)+1/4*epsilon*ln(lambda)+A20-
Unknown/lambda^(1/2)+ Unknown2/lambda)*(1/5*Ltinfty15/h*lambda^
(5/2)-1/4*Ltinfty14/h*lambda^2+1/3*Ltinfty13/h*lambda^(3/2)-1/2*
Ltinfty12/h*lambda^1+Ltinfty11/h*lambda^(1/2)+LA20-
LUnknown/lambda^(1/2)+ LUnknown2/lambda);
psi1Infty:=exp(logPsi1Infty);
psi2Infty:=exp(logPsi2Infty);
dpsi1dlambdaInfty:=diff(psi1Infty,lambda):
dpsi2dlambdaInfty:=diff(psi2Infty,lambda):
d2psi1dlambda2Infty:=diff(psi1Infty,lambda$2):
d2psi2dlambda2Infty:=diff(psi2Infty,lambda$2):
```

```
WronskianLambdaInfty:=h*factor(psi1Infty*dpsi2dlambdaInfty-
psi2Infty*dpsi1dlambdaInfty):
WronskianLambdabisInfty:=h*simplify(factor( (diff(logPsi2Infty,
lambda)-diff(logPsi1Infty,lambda))*exp(logPsi1Infty+logPsi2Infty)
)):

WronskianTildeLambdaInfty:=h^3*factor(dpsi2dlambdaInfty*
d2psi1dlambda2Infty-dpsi1dlambdaInfty*d2psi2dlambda2Infty):
```

$$logPsi1Infty := -\frac{tinfty15\,\lambda^{5/2}}{5\,h} - \frac{tinfty14\,\lambda^{2}}{4\,h} - \frac{tinfty13\,\lambda^{3/2}}{3\,h} - \frac{tinfty12\,\lambda}{2\,h} - \frac{tinfty11\,\sqrt{\lambda}}{h} + \frac{\epsilon\ln(\lambda)}{4} + A10 + \frac{Unknown}{\sqrt{\lambda}} + \frac{Unknown2}{\lambda}$$

$$logPsi2Infty := \frac{tinfty15\,\lambda^{5/2}}{5\,h} - \frac{tinfty14\,\lambda^{2}}{4\,h} + \frac{tinfty13\,\lambda^{3/2}}{3\,h} - \frac{tinfty12\,\lambda}{2\,h} + \frac{tinfty11\,\sqrt{\lambda}}{h} + \frac{\epsilon\ln(\lambda)}{4} + A20 - \frac{Unknown}{\sqrt{\lambda}} + \frac{Unknown2}{\lambda}$$

$$GrosLlogpsi1Infty := -\frac{Ltinfty15\,\lambda^{5/2}}{5\,h} - \frac{Ltinfty14\,\lambda^{2}}{4\,h} - \frac{Ltinfty13\,\lambda^{3/2}}{3\,h} - \frac{Ltinfty12\,\lambda}{2\,h} - \frac{Ltinfty11\,\sqrt{\lambda}}{h} + LA10 + \frac{LUnknown}{\sqrt{\lambda}} + \frac{LUnknown2}{\lambda}$$

$$GrosLlogpsi2Infty := \frac{Ltinfty15\,\lambda^{5/2}}{5\,h} - \frac{Ltinfty14\,\lambda^{2}}{4\,h} + \frac{Ltinfty13\,\lambda^{3/2}}{3\,h} - \frac{Ltinfty12\,\lambda}{2\,h} + \frac{Ltinfty11\,\sqrt{\lambda}}{h} + LA20 - \frac{LUnknown}{\sqrt{\lambda}} + \frac{LUnknown2}{\lambda}$$

$$GrosLpsi1Infty := e^{-\frac{tinfty15\,\lambda^{5/2}}{5\,h} - \frac{tinfty14\,\lambda^{2}}{4\,h} - \frac{tinfty13\,\lambda^{3/2}}{3\,h} - \frac{tinfty12\,\lambda}{2\,h} - \frac{tinfty11\,\sqrt{\lambda}}{h} + \frac{\epsilon\ln(\lambda)}{4} + A10 + \frac{Unknown}{\sqrt{\lambda}} + \frac{Unknown2}{\lambda}} \left(-\frac{Ltinfty15\,\lambda^{5/2}}{5\,h} - \frac{Ltinfty14\,\lambda^{2}}{4\,h} - \frac{Ltinfty13\,\lambda^{3/2}}{3\,h} - \frac{Ltinfty12\,\lambda}{2\,h} - \frac{Ltinfty11\,\sqrt{\lambda}}{h} + LA10 + \frac{LUnknown}{\sqrt{\lambda}} + \frac{LUnknown2}{\lambda}\right)$$

$$GrosLpsi2Infty := e^{\frac{tinfty15\,\lambda^{5/2}}{5\,h} - \frac{tinfty14\,\lambda^{2}}{4\,h} + \frac{tinfty13\,\lambda^{3/2}}{3\,h} - \frac{tinfty12\,\lambda}{2\,h} + \frac{tinfty11\,\sqrt{\lambda}}{h} + \frac{\epsilon\ln(\lambda)}{4} + A20 - \frac{Unknown}{\sqrt{\lambda}}}$$

$$+\frac{Unknown2}{\lambda}\left(\frac{Ltinfty15\,\lambda^{5/2}}{5\,h}-\frac{Ltinfty14\,\lambda^{2}}{4\,h}+\frac{Ltinfty13\,\lambda^{3/2}}{3\,h}-\frac{Ltinfty12\,\lambda}{2\,h}+\frac{Ltinfty11\,\sqrt{\lambda}}{h}+LA20-\frac{LUnknown}{\sqrt{\lambda}}+\frac{LUnknown2}{\lambda}\right)$$

$$psi1Infty := \mathrm{e}^{-\frac{tinfty15\,\lambda^{5/2}}{5\,h}-\frac{tinfty14\,\lambda^{2}}{4\,h}-\frac{tinfty13\,\lambda^{3/2}}{3\,h}-\frac{tinfty12\,\lambda}{2\,h}-\frac{tinfty11\,\sqrt{\lambda}}{h}+\frac{\epsilon\ln(\lambda)}{4}+A10+\frac{Unknown}{\sqrt{\lambda}}+\frac{Unknown2}{\lambda}}$$

$$psi2Infty := \mathrm{e}^{\frac{tinfty15\,\lambda^{5/2}}{5\,h}-\frac{tinfty14\,\lambda^{2}}{4\,h}+\frac{tinfty13\,\lambda^{3/2}}{3\,h}-\frac{tinfty12\,\lambda}{2\,h}+\frac{tinfty11\,\sqrt{\lambda}}{h}+\frac{\epsilon\ln(\lambda)}{4}+A20-\frac{Unknown}{\sqrt{\lambda}}+\frac{Unknown2}{\lambda}} \tag{2}$$

```
> L21Infty:=simplify
  (WronskianTildeLambdaInfty/WronskianLambdabisInfty):
  L21InftyOrdrelambda5:=factor(-residue(L21Infty/lambda^6,lambda=
  infinity));
  L21InftyOrdrelambda4:=factor(-residue(L21Infty/lambda^5,lambda=
  infinity));
  L21InftyOrdrelambda3:=factor(-residue(L21Infty/lambda^4,lambda=
  infinity));
  L21InftyOrdrelambda2:=factor(-residue(L21Infty/lambda^3,lambda=
  infinity));
  L21InftyOrdrelambda1:=factor(-residue(L21Infty/lambda^2,lambda=
  infinity));
  L21InftyOrdrelambda0:=factor(-residue(L21Infty/lambda^1,lambda=
  infinity));
```

$$L21InftyOrdrelambda5 := 0$$

$$L21InftyOrdrelambda4 := 0$$

$$L21InftyOrdrelambda3 := \frac{tinfty15^2}{4}$$

$$L21InftyOrdrelambda2 := \frac{tinfty15\,tinfty13}{2}-\frac{tinfty14^2}{4}$$

$$L21InftyOrdrelambda1 := \frac{tinfty15\,tinfty11}{2}-\frac{tinfty14\,tinfty12}{2}+\frac{tinfty13^2}{4}$$

$$L21InftyOrdrelambda0 := \frac{1}{2}\,Unknown\,h\,tinfty15+\frac{1}{4}\,\epsilon\,h\,tinfty14+\frac{1}{4}\,h\,tinfty14 \tag{3}$$

$$+\frac{1}{2}\, tinfty11\, tinfty13 - \frac{1}{4}\, tinfty12^2$$

```
> factor(simplify(L21InftyOrdrelambda4*
  lambda^4+L21InftyOrdrelambda3*lambda^3+L21InftyOrdrelambda2*
  lambda^2+L21InftyOrdrelambda1*lambda- (-P2(lambda)+Pinfty02)));
```

$$0 \tag{4}$$

We conclude that L_{2,1}is of the form -P2(lambda)+O(1) at infinity. Let us now study L_{2,2}

```
> L22Infty:=factor(h*simplify(diff(WronskianLambdabisInfty,lambda)
  /WronskianLambdabisInfty)):
  L22InftyOrdrelambda5:=factor(-residue(L22Infty/lambda^6,lambda=
  infinity));
  L22InftyOrdrelambda4:=factor(-residue(L22Infty/lambda^5,lambda=
  infinity));
  L22InftyOrdrelambda3:=factor(-residue(L22Infty/lambda^4,lambda=
  infinity));
  L22InftyOrdrelambda2:=factor(-residue(L22Infty/lambda^3,lambda=
  infinity));
  L22InftyOrdrelambda1:=factor(-residue(L22Infty/lambda^2,lambda=
  infinity));
  L22InftyOrdrelambda0:=factor(-residue(L22Infty/lambda^1,lambda=
  infinity));
  L22InftyOrdrelambdaMoins1:=factor(-residue(L22Infty/lambda^0,
  lambda=infinity));
  L22InftyOrdrelambdaMoins2:=factor(-residue(L22Infty/lambda^(-1),
  lambda=infinity));
```

$$L22InftyOrdrelambda5 := 0$$
$$L22InftyOrdrelambda4 := 0$$
$$L22InftyOrdrelambda3 := 0$$
$$L22InftyOrdrelambda2 := 0$$
$$L22InftyOrdrelambda1 := -tinfty14$$
$$L22InftyOrdrelambda0 := -tinfty12$$
$$L22InftyOrdrelambdaMoins1 := \frac{(\epsilon+3)\, h}{2}$$
$$L22InftyOrdrelambdaMoins2 := -\frac{h\,(2\, tinfty15\, Unknown2 + tinfty13)}{tinfty15} \tag{5}$$

We conclude that L_{2,2} behaves at infinity like -tinfty14*lambda -tinfty12+2*h/lambda +O(1/lambda^2) =P1(lambda) +2*h/lambda+O(1/lambda^2)

We end with the explicit formulas for L_{2,2} and L_{2,1}:
L_{2,2}= P_1(lambda) +2*h/(lambda-q) and **L_{2,1}= -P_2(lambda)+Pinfty02+C -p*h/(lambda-q)**

```
> L21Form:=-P2(lambda)+Pinfty02- p*h/(lambda-q);
  L22Form:=P1(lambda)   +2*h/(lambda-q);
```

$$L21Form := -\left(-\frac{tinfty15\, tinfty11}{2} + \frac{tinfty14\, tinfty12}{2} - \frac{tinfty13^2}{4}\right)\lambda - \left(-\frac{tinfty15\, tinfty13}{2} + \frac{tinfty14^2}{4}\right)\lambda^2 + \frac{tinfty15^2\,\lambda^3}{4} - \frac{p\,h}{\lambda - q}$$

$$L22Form := -tinfty14\,\lambda - tinfty12 + \frac{2\,h}{\lambda - q} \quad \textbf{(6)}$$

Conputation for the auxiliary matrix A in the oper gauge

The deformation operator is \mathcal{L}=\hbar (alpha15\partial_{t_{\infty^{(1)},5} +alpha14\partial_{t_{\infty^{(1)},4}+e*\partial_{t_{\infty^{(1)},3} +f*\partial_{t_{\infty^{(1)},2}+g*\partial_{t_{\infty^{(1)},1})

```
> WronskianGrosLInfty:=factor(psi1Infty*GrosLpsi2Infty-psi2Infty*
  GrosLpsi1Infty):
  A12Infty:=factor(simplify
  (WronskianGrosLInfty/WronskianLambdaInfty)):
  Y1Infty:=h*factor(dpsi1dlambdaInfty/psi1Infty):
  Y2Infty:=h*factor(dpsi2dlambdaInfty/psi2Infty):
  Z1Infty:=factor(GrosLpsi1Infty/psi1Infty):
  Z2Infty:=factor(GrosLpsi2Infty/psi2Infty):
  A12bisInfty:=factor(simplify((Z2Infty-Z1Infty)/(Y2Infty-Y1Infty))
  ):
  A11Infty:=factor(simplify( (Y2Infty*Z1Infty-Y1Infty*Z2Infty)/
  (Y2Infty-Y1Infty) )):
> factor(simplify(A12bisInfty-A12Infty));
```

$$0 \quad \textbf{(7)}$$

```
> Ltinfty15:=h*alpha15:
  Ltinfty14:=h*alpha14:
  Ltinfty13:=h*alpha13:
  Ltinfty12:=h*alpha12:
  Ltinfty11:=h*alpha11:
  Ltinfty10:=0:
  Ltinfty20:=0:
  LA20:=LA10:
```

```
> A12InftyLambda3:=factor(-residue(A12Infty/lambda^4,lambda=
  infinity));
  A12InftyLambda2:=factor(-residue(A12Infty/lambda^3,lambda=
  infinity));
  A12InftyLambda1:=factor(-residue(A12Infty/lambda^2,lambda=
  infinity));
  A12InftyLambda0:=factor(-residue(A12Infty/lambda^1,lambda=
  infinity));
```

```
A12InftyLambdaMoins1:=factor(-residue(A12Infty/lambda^0,lambda=
infinity));
A12InftyLambdaMoins2:=factor(-residue(A12Infty/lambda^(-1),
lambda=infinity));
```

$$A12InftyLambda3 := 0$$
$$A12InftyLambda2 := 0$$
$$A12InftyLambda1 := \frac{2\ \alpha l5}{5\ tinfty15}$$
$$A12InftyLambda0 := \frac{2\ (5\ \alpha l3\ tinfty15 - 3\ \alpha l5\ tinfty13)}{15\ tinfty15^2}$$
$$A12InftyLambdaMoins1 := \frac{2\ (15\ \alpha l1\ tinfty15^2 - 5\ \alpha l3\ tinfty13\ tinfty15 - 3\ \alpha l5\ tinfty11\ tinfty15 + 3\ \alpha l5\ tinfty13^2)}{15\ tinfty15^3}$$
$$A12InftyLambdaMoins2 := -\frac{1}{15\ tinfty15^4}\Big(2\ \big(3\ \alpha l5\ Unknown\ h\ tinfty15^2 + 15\ \alpha l1\ tinfty13\ tinfty15^2 + 5\ \alpha l3\ tinfty11\ tinfty15^2 - 5\ \alpha l3\ tinfty13^2\ tinfty15 - 6\ \alpha l5\ tinfty11\ tinfty13\ tinfty15 + 3\ \alpha l5\ tinfty13^3 + 15\ LUnknown\ tinfty15^3\big)\Big) \quad \textbf{(8)}$$

We get that **A_{1,2}=2*alpha15/5/tinfty15*lambda+nu +mu/(lambda-q)**

with nu= $-\frac{2\ (3\ \alpha\ tinfty13 - 5\ alpha13\ tinfty15)}{15\ tinfty15^2}$ **and**

mu= $-\frac{2\ (3\ \alpha\ tinfty11\ tinfty15 - 3\ \alpha\ tinfty13^2 + 5\ alpha13\ tinfty13\ tinfty15 - 15\ alpha11\ tinfty15^2)}{15\ tinfty15^3}$

```
> A12Form:=2*alpha15/5/tinfty15*lambda+nu+ mu/(lambda-q);
simplify(-residue(A12Form/lambda^2,lambda=infinity)-
A12InftyLambda1);
solve({factor(-residue(A12Form/lambda,lambda=infinity))=
A12InftyLambda0,factor(-residue(A12Form,lambda=infinity))=factor
(A12InftyLambdaMoins1)},{mu,nu});
```

$$A12Form := \frac{2\ \alpha l5\ \lambda}{5\ tinfty15} + \nu + \frac{\mu}{\lambda - q}$$
$$0$$
$$\Big\{\mu = \frac{1}{15\ tinfty15^3}\big(2\ (15\ \alpha l1\ tinfty15^2 - 5\ \alpha l3\ tinfty13\ tinfty15 - 3\ \alpha l5\ tinfty11\ tinfty15 + 3\ \alpha l5\ tinfty13^2)\big), \nu = \frac{2\ (5\ \alpha l3\ tinfty15 - 3\ \alpha l5\ tinfty13)}{15\ tinfty15^2}\Big\} \quad \textbf{(9)}$$

```
> mu := -(2*(3*alpha15*tinfty11*tinfty15-3*alpha15*tinfty13^2+5*
alpha13*tinfty13*tinfty15-15*alpha11*tinfty15^2))/(15*tinfty15^3)
```

```
;
mubis := -2*(1/5*alpha15*tinfty11/tinfty15-1/5*alpha15*
tinfty13^2/tinfty15^2 +1/3*alpha13*tinfty13/tinfty15-alpha11)
/tinfty15;
factor(simplify(mu-mubis));
nu := -(2*(3*alpha15*tinfty13-5*alpha13*tinfty15))/(15*
tinfty15^2);
```

$$\mu := -\frac{1}{15\ tinfty15^3}\left(2\left(-15\ \alpha 11\ tinfty15^2 + 5\ \alpha 13\ tinfty13\ tinfty15 + 3\ \alpha 15\ tinfty11\ tinfty15 - 3\ \alpha 15\ tinfty13^2\right)\right)$$

$$mubis := -\frac{2\left(\frac{\alpha 15\ tinfty11}{5\ tinfty15} - \frac{\alpha 15\ tinfty13^2}{5\ tinfty15^2} + \frac{\alpha 13\ tinfty13}{3\ tinfty15} - \alpha 11\right)}{tinfty15}$$

$$0$$

$$\nu := -\frac{2\left(-5\ \alpha 13\ tinfty15 + 3\ \alpha 15\ tinfty13\right)}{15\ tinfty15^2} \quad \textbf{(10)}$$

These are theoretical results to compute the nu_i's, i.e. the behavior of A_{1,2} at lambda->\infty

```
> Minfty:=Matrix(3,3,0):
Minfty[1,1]:=tinfty15:
Minfty[2,2]:=tinfty15:
Minfty[3,3]:=tinfty15:
Minfty[2,1]:=tinfty13:
Minfty[3,2]:=tinfty13:
Minfty[3,1]:=tinfty11:
Minfty;
NuVector:=Matrix(3,1,0):
NuVector[1,1]:=nuMoins1:
NuVector[2,1]:=nu0:
NuVector[3,1]:=nu1:
NuVector;
RHSVector:=Matrix(3,1,0):
RHSVector[1,1]:=2*alpha1515/5:
RHSVector[2,1]:=2*alpha1513/3:
RHSVector[3,1]:=2*alpha1511/1:
RHSVector;
NuVectorSol:=Multiply(Minfty^(-1),RHSVector);
alpha1515:=alpha15:
alpha1514:=alpha14:
alpha1513:=alpha13:
alpha1512:=alpha12:
alpha1511:=alpha11:
```

```
mu1:=mu:
NuVector[1,1]=NuVectorSol[1,1];
NuVector[2,1]=NuVectorSol[2,1];
NuVector[3,1]=NuVectorSol[3,1];

nuMoins1 := NuVectorSol[1,1];
nu0 := NuVectorSol[2,1];
nu1:=NuVectorSol[3,1];
```

$$\begin{bmatrix} tinfty15 & 0 & 0 \\ tinfty13 & tinfty15 & 0 \\ tinfty11 & tinfty13 & tinfty15 \end{bmatrix}$$

$$\begin{bmatrix} nuMoins1 \\ \nu 0 \\ \nu 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{2\ \alpha 1515}{5} \\ \frac{2\ \alpha 1513}{3} \\ 2\ \alpha 1511 \end{bmatrix}$$

$$NuVectorSol := \begin{bmatrix} \frac{2\ \alpha 1515}{5\ tinfty15} \\ -\frac{2\ tinfty13\ \alpha 1515}{5\ tinfty15^2} + \frac{2\ \alpha 1513}{3\ tinfty15} \\ -\frac{2\ (tinfty15\ tinfty11 - tinfty13^2)\ \alpha 1515}{5\ tinfty15^3} - \frac{2\ tinfty13\ \alpha 1513}{3\ tinfty15^2} + \frac{2\ \alpha 1511}{tinfty15} \end{bmatrix}$$

$$nuMoins1 = \frac{2\ \alpha 15}{5\ tinfty15}$$

$$\nu 0 = -\frac{2\ tinfty13\ \alpha 15}{5\ tinfty15^2} + \frac{2\ \alpha 13}{3\ tinfty15}$$

$$\nu 1 = -\frac{2\ (tinfty15\ tinfty11 - tinfty13^2)\ \alpha 15}{5\ tinfty15^3} - \frac{2\ tinfty13\ \alpha 13}{3\ tinfty15^2} + \frac{2\ \alpha 11}{tinfty15}$$

$$nuMoins1 := \frac{2\ \alpha 15}{5\ tinfty15}$$

$$\nu 0 := -\frac{2\, tinfty13\, \alpha l5}{5\, tinfty15^2} + \frac{2\, \alpha l3}{3\, tinfty15}$$

$$\nu 1 := -\frac{2\,(tinfty15\, tinfty11 - tinfty13^2)\, \alpha l5}{5\, tinfty15^3} - \frac{2\, tinfty13\, \alpha l3}{3\, tinfty15^2} + \frac{2\, \alpha l1}{tinfty15} \quad \textbf{(11)}$$

We now check that the formula for the nu_{i}'s is correct.

```
> simplify(-residue(A12Form/lambda^2,lambda=infinity)-nuMoins1);
  simplify(-residue(A12Form/lambda,lambda=infinity)-nu0);
  simplify(-residue(A12Form*lambda^0,lambda=infinity)-nu1);
```

$$0$$
$$0$$
$$0 \quad \textbf{(12)}$$

```
> NuMuVector:=Matrix(3,1,0):
  NuMuVector[1,1]:=nuMoins1:
  NuMuVector[2,1]:=nu0:
  NuMuVector[3,1]:=mu1:
  R:=Matrix(3,3,0):
  R[1,1]:=1:
  R[2,2]:=1:
  R[3,3]:=1:
  NuMuVectorTheo:=Multiply(Multiply(R^(-1),Minfty^(-1)),RHSVector);
  simplify(NuMuVector-NuMuVectorTheo);
```

$$NuMuVectorTheo :=$$

$$\left[\begin{array}{c} \frac{2\, \alpha l5}{5\, tinfty15} \\ -\frac{2\, tinfty13\, \alpha l5}{5\, tinfty15^2} + \frac{2\, \alpha l3}{3\, tinfty15} \\ -\frac{2\,(tinfty15\, tinfty11 - tinfty13^2)\, \alpha l5}{5\, tinfty15^3} - \frac{2\, tinfty13\, \alpha l3}{3\, tinfty15^2} + \frac{2\, \alpha l1}{tinfty15} \end{array}\right]$$

$$\left[\begin{array}{c} 0 \\ 0 \\ 0 \end{array}\right] \quad \textbf{(13)}$$

We have checked that Proposition dealing with A_{1,2} is correct.

```
> A11InftyLambda4:=factor(-residue(A11Infty/lambda^5,lambda=
  infinity));
  A11InftyLambda3:=factor(-residue(A11Infty/lambda^4,lambda=
  infinity));
  A11InftyLambda2:=factor(-residue(A11Infty/lambda^3,lambda=
  infinity));
  A11InftyLambda1:=factor(-residue(A11Infty/lambda^2,lambda=
  infinity));
```

```
A11InftyLambda0:=factor(-residue(A11Infty/lambda^1,lambda=
infinity));
A11InftyLambdaMoins1:=factor(-residue(A11Infty/lambda^0,lambda=
infinity)):
```

$$A11InftyLambda4 := 0$$
$$A11InftyLambda3 := 0$$
$$A11InftyLambda2 := -\frac{5\,\alpha l4\,tinfty15 - 4\,\alpha l5\,tinfty14}{20\,tinfty15}$$
$$A11InftyLambda1 := -\frac{1}{30\,tinfty15^2}\left(15\,\alpha l2\,tinfty15^2 - 10\,\alpha l3\,tinfty14\,tinfty15 - 6\,\alpha l5\,tinfty12\,tinfty15 + 6\,\alpha l5\,tinfty13\,tinfty14\right)$$
$$A11InftyLambda0 := \frac{1}{30\,tinfty15^3}\left(-3\,\alpha l5\,\epsilon\,h\,tinfty15^2 + 30\,LA10\,tinfty15^3 + 30\,\alpha l1\,tinfty14\,tinfty15^2 + 10\,\alpha l3\,tinfty12\,tinfty15^2 - 10\,\alpha l3\,tinfty13\,tinfty14\,tinfty15 - 6\,\alpha l5\,tinfty11\,tinfty14\,tinfty15 - 6\,\alpha l5\,tinfty12\,tinfty13\,tinfty15 + 6\,\alpha l5\,tinfty13^2\,tinfty14\right) \tag{14}$$

We conclude that A_{1,1}=(4*alpha15*tinfty14-5*alpha14*tinfty15)/(20*tinfty15)*lambda^2+c1*lambda+c0+ rho/(lambda-q)

```
> A11Form:=(4*alpha15*tinfty14-5*alpha14*tinfty15)/(20*tinfty15)*
lambda^2+c1*lambda+c0+ rho/(lambda-q);

simplify(-residue(A11Form/lambda^4,lambda=infinity)-
A11InftyLambda3);
solve({factor(-residue(A11Form/lambda^3,lambda=infinity))=
A11InftyLambda2,factor(-residue(A11Form/lambda^2,lambda=infinity)
)=A11InftyLambda1},{c1});
```

$$A11Form := \frac{\left(-5\,\alpha l4\,tinfty15 + 4\,\alpha l5\,tinfty14\right)\lambda^2}{20\,tinfty15} + c1\,\lambda + c0 + \frac{\rho}{\lambda - q}$$
$$0$$
$$\left\{c1 = -\frac{1}{30\,tinfty15^2}\left(15\,\alpha l2\,tinfty15^2 - 10\,\alpha l3\,tinfty14\,tinfty15 - 6\,\alpha l5\,tinfty12\,tinfty15 + 6\,\alpha l5\,tinfty13\,tinfty14\right)\right\} \tag{15}$$

```
> c1 := (6*alpha15*tinfty12*tinfty15-6*alpha15*tinfty13*
tinfty14+10*alpha13*tinfty14*tinfty15-15*alpha12*tinfty15^2)/(30*
tinfty15^2);
```

$$c1 := \frac{1}{30\,tinfty15^2}\left(-15\,\alpha l2\,tinfty15^2 + 10\,\alpha l3\,tinfty14\,tinfty15 + 6\,\alpha l5\,tinfty12\,tinfty15 \right. \tag{16}$$

$$- 6\, \alpha 15\, tinfty13\, tinfty14)$$

```
> c2:=(4*alpha15*tinfty14-5*alpha14*tinfty15)/(20*tinfty15);
```

$$c2 := \frac{-5\, \alpha 14\, tinfty15 + 4\, \alpha 15\, tinfty14}{20\, tinfty15} \tag{17}$$

```
> c2theo:=1/tinfty15*(alpha1515/5*tinfty14-alpha1514/4*tinfty15);
  c1theo:=1/tinfty15*(-tinfty13*c2theo  + alpha1515/5*tinfty12-
  alpha1514/4*tinfty13+alpha1513/3*tinfty14-alpha1512/2*tinfty15);
  simplify(c2-c2theo);
  simplify(c1-c1theo);
```

$$c2theo := \frac{\frac{\alpha 15\, tinfty14}{5} - \frac{\alpha 14\, tinfty15}{4}}{tinfty15}$$

$$c1theo := \frac{1}{tinfty15} \left(- \frac{tinfty13 \left(\frac{\alpha 15\, tinfty14}{5} - \frac{\alpha 14\, tinfty15}{4} \right)}{tinfty15} + \frac{\alpha 15\, tinfty12}{5} - \frac{\alpha 14\, tinfty13}{4} + \frac{\alpha 13\, tinfty14}{3} - \frac{\alpha 12\, tinfty15}{2} \right)$$

$$0$$

$$0 \tag{18}$$

We have checked that the formula for A_{1,1} is also correct.