```
> restart:
  with(LinearAlgebra):

  rs:=7:
  list0:=seq(t[i],i=1..rs-1);
  MXs:=Matrix(rs-1,rs-1,0):
  for i from 1 to rs-1 do for j from 1 to rs-1 do if j<=i then MXs
  [i,j]:=t[rs-1-(i-j)]: fi: od: od:
  MXs;
  Diago:=Matrix(rs-1,rs-1,0):
  for i from 1 to rs-1 do Diago[i,i]:=rs-i: od:
  Diago;

  RHSMatrix:=Matrix(rs-1,rs-1,0):
  for i from 1 to rs-1 do for j from 1 to rs-1 do if j<=i then
  RHSMatrix[i,j]:=Q[rs-(i-j)](list0)/(rs-j): fi: od: od:
  RHSMatrix;

  deltaQMatrix:=Matrix(rs-1,rs-1,0):
  for i from 1 to rs-1 do for j from 1 to rs-1 do
  deltaQMatrix[i,j]:=1/(rs-i)*diff(Q[rs+1-i](list0),t[rs-j]):
  od: od:
  deltaQMatrix:

  EquationSystem:=[]:
  for i from 1 to rs-1 do for j from 1 to rs-1 do
  EquationSystem:=[op(EquationSystem),(Multiply(MXs,deltaQMatrix)-
  RHSMatrix)[i,j]=0]: od: od:
  EquationSystem:
  pdsolve({op(EquationSystem)});
```

$$list0 := t_1, t_2, t_3, t_4, t_5, t_6 \tag{1}$$

$$\begin{bmatrix} t_6 & 0 & 0 & 0 & 0 & 0 \\ t_5 & t_6 & 0 & 0 & 0 & 0 \\ t_4 & t_5 & t_6 & 0 & 0 & 0 \\ t_3 & t_4 & t_5 & t_6 & 0 & 0 \\ t_2 & t_3 & t_4 & t_5 & t_6 & 0 \\ t_1 & t_2 & t_3 & t_4 & t_5 & t_6 \end{bmatrix}$$

$$
\begin{bmatrix}
6 & 0 & 0 & 0 & 0 & 0 \\
0 & 5 & 0 & 0 & 0 & 0 \\
0 & 0 & 4 & 0 & 0 & 0 \\
0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

$$
\left[\left[\left[\frac{1}{6}\,Q_7(t_1, t_2, t_3, t_4, t_5, t_6),\ 0,\ 0,\ 0,\ 0,\ 0\right],\right.\right.
$$

$$
\left[\frac{1}{6}\,Q_6(t_1, t_2, t_3, t_4, t_5, t_6),\ \frac{1}{5}\,Q_7(t_1, t_2, t_3, t_4, t_5, t_6),\ 0,\ 0,\ 0,\ 0\right],
$$

$$
\left[\frac{1}{6}\,Q_5(t_1, t_2, t_3, t_4, t_5, t_6),\ \frac{1}{5}\,Q_6(t_1, t_2, t_3, t_4, t_5, t_6),\ \frac{1}{4}\,Q_7(t_1, t_2, t_3, t_4, t_5, t_6),\ 0,\ 0,\ 0\right],
$$

$$
\left[\frac{1}{6}\,Q_4(t_1, t_2, t_3, t_4, t_5, t_6),\ \frac{1}{5}\,Q_5(t_1, t_2, t_3, t_4, t_5, t_6),\ \frac{1}{4}\,Q_6(t_1, t_2, t_3, t_4, t_5, t_6),\ \frac{1}{3}\,Q_7(t_1, t_2,\right.
$$

$$
\left.t_3, t_4, t_5, t_6),\ 0,\ 0\right],
$$

$$
\left[\frac{1}{6}\,Q_3(t_1, t_2, t_3, t_4, t_5, t_6),\ \frac{1}{5}\,Q_4(t_1, t_2, t_3, t_4, t_5, t_6),\ \frac{1}{4}\,Q_5(t_1, t_2, t_3, t_4, t_5, t_6),\ \frac{1}{3}\,Q_6(t_1, t_2,\right.
$$

$$
\left.t_3, t_4, t_5, t_6),\ \frac{1}{2}\,Q_7(t_1, t_2, t_3, t_4, t_5, t_6),\ 0\right],
$$

$$
\left[\frac{1}{6}\,Q_2(t_1, t_2, t_3, t_4, t_5, t_6),\ \frac{1}{5}\,Q_3(t_1, t_2, t_3, t_4, t_5, t_6),\ \frac{1}{4}\,Q_4(t_1, t_2, t_3, t_4, t_5, t_6),\ \frac{1}{3}\,Q_5(t_1, t_2,\right.
$$

$$
\left.\left.\left.t_3, t_4, t_5, t_6),\ \frac{1}{2}\,Q_6(t_1, t_2, t_3, t_4, t_5, t_6),\ Q_7(t_1, t_2, t_3, t_4, t_5, t_6)\right]\right]\right]
$$

$$
\left\{Q_2(t_1, t_2, t_3, t_4, t_5, t_6) = \frac{1}{60000}\,\frac{1}{t_6^7}\left(60000\,\_C6\,t_6^{(43/6)} + 60000\,\_C1\,t_1\,t_6^7 + 30000\,\_C2\,t_2\,t_6^{(41/6)}\right.\right.
$$

$$
+\ 20000\,\_C3\,t_3\,t_6^{(20/3)} + 15000\,\_C4\,t_4\,t_6^{(13/2)} + 12000\,\_C5\,t_5\,t_6^{(19/3)} + \left(-4000\,\_C2\,t_3\,t_5\right.
$$

$$
\left.-\ 1875\,\_C2\,t_4^2\right)t_6^{(35/6)} - 6000\,\_C3\,t_4\,t_5\,t_6^{(17/3)} - 3600\,\_C4\,t_5^2\,t_6^{(11/2)} + 2100\,\_C2\,t_4\,t_5^2\,t_6^{(29/6)}
$$

$$
+\ 1280\,\_C3\,t_5^3\,t_6^{(14/3)} - 364\,\_C2\,t_5^4\,t_6^{(23/6)}\Big),\ Q_3(t_1, t_2, t_3, t_4, t_5, t_6) = \_C1\,t_2 + \frac{2}{3}\,\frac{\_C2\,t_3}{t_6^{(1/6)}}
$$

$$
-\ \frac{1}{10}\,\frac{t_4\,\_C2\,t_5}{t_6^{(7/6)}} + \frac{1}{2}\,\frac{t_4\,\_C3}{t_6^{(1/3)}} + \frac{7}{375}\,\frac{\_C2\,t_5^3}{t_6^{(13/6)}} - \frac{2}{25}\,\frac{\_C3\,t_5^2}{t_6^{(4/3)}} + \frac{2}{5}\,\frac{\_C4\,t_5}{\sqrt{t_6}} + \_C5
$$

$$
t_6^{(1/3)},\ Q_4(t_1, t_2, t_3, t_4, t_5, t_6) = \_C1\,t_3 + \frac{3}{4}\,\frac{\_C2\,t_4}{t_6^{(1/6)}} - \frac{3}{50}\,\frac{\_C2\,t_5^2}{t_6^{(7/6)}} + \frac{3}{5}\,\frac{\_C3\,t_5}{t_6^{(1/3)}}
$$

$$
+\ \_C4\,\sqrt{t_6},\ Q_5(t_1, t_2, t_3, t_4, t_5, t_6) = \_C1\,t_4 + \frac{4}{5}\,\frac{\_C2\,t_5}{t_6^{(1/6)}} + \_C3\,t_6^{(2/3)},\ Q_6(t_1, t_2, t_3, t_4, t_5, t_6)
$$

$$= \_C1\ t_5 + \_C2\ t_6^{(5/6)},\ Q_7\left(t_1, t_2, t_3, t_4, t_5, t_6\right) = \_C1\ t_6$$

```
> Q[2] :=unapply( (60000*C6*t[6]^(43/6)+60000*C1*t[1]*t[6]^7+30000*
  C2*t[2]*t[6]^(41/6)+20000*t[3]*C3*t[6]^(20/3)+15000*C4*t[4]*t[6]^
  (13/2)+12000*C5*t[5]*t[6]^(19/3)+(-4000*C2*t[3]*t[5]-1875*C2*t[4]
  ^2)*t[6]^(35/6)-6000*C3*t[5]*t[4]*t[6]^(17/3)-3600*C4*t[5]^2*t[6]
  ^(11/2)+2100*C2*t[5]^2*t[4]*t[6]^(29/6)+1280*C3*t[5]^3*t[6]^
  (14/3)-364*C2*t[5]^4*t[6]^(23/6))/(60000*t[6]^7),t[1], t[2], t
  [3], t[4], t[5], t[6]);
   Q[3]:=unapply( C1*t[2]+2*C2*t[3]/(3*t[6]^(1/6))-t[4]*C2*t[5]/
  (10*t[6]^(7/6))+t[4]*C3/(2*t[6]^(1/3))+7*C2*t[5]^3/(375*t[6]^
  (13/6))-2*C3*t[5]^2/(25*t[6]^(4/3))+2*C4*t[5]/(5*sqrt(t[6]))+C5*t
  [6]^(1/3),t[1], t[2], t[3], t[4], t[5], t[6]);
  Q[4]:=unapply( C1*t[3]+3*C2*t[4]/(4*t[6]^(1/6))-3*C2*t[5]^2/(50*t
  [6]^(7/6))+3*C3*t[5]/(5*t[6]^(1/3))+C4*sqrt(t[6]),t[1], t[2], t
  [3], t[4], t[5], t[6]);
  Q[5]:=unapply( C1*t[4]+4*C2*t[5]/(5*t[6]^(1/6))+C3*t[6]^(2/3),t
  [1], t[2], t[3], t[4], t[5], t[6]);
  Q[6]:= unapply( C1*t[5]+C2*t[6]^(5/6),t[1], t[2], t[3], t[4], t
  [5], t[6]);
  Q[7]:= unapply( C1*t[6], t[1], t[2], t[3], t[4], t[5], t[6]);
  simplify(EquationSystem);
```

$$Q_2 := (t\_1, t\_2, t\_3, t\_4, t\_5, t\_6) \rightarrow \frac{1}{60000}\ \frac{1}{t\_6^7}\ \left(60000\ C6\ t\_6^{43/6} + 60000\ C1\ t\_1\ t\_6^7 \right. \tag{2}$$
$$+ 30000\ C2\ t\_2\ t\_6^{41/6} + 20000\ t\_3\ C3\ t\_6^{20/3} + 15000\ C4\ t\_4\ t\_6^{13/2}$$
$$+ 12000\ C5\ t\_5\ t\_6^{19/3} + \left(-4000\ C2\ t\_3\ t\_5 - 1875\ C2\ t\_4^2\right)\ t\_6^{35/6}$$
$$- 6000\ C3\ t\_5\ t\_4\ t\_6^{17/3} - 3600\ C4\ t\_5^2\ t\_6^{11/2} + 2100\ C2\ t\_5^2\ t\_4\ t\_6^{29/6}$$
$$\left. + 1280\ C3\ t\_5^3\ t\_6^{14/3} - 364\ C2\ t\_5^4\ t\_6^{23/6}\right)$$

$$Q_3 := (t\_1, t\_2, t\_3, t\_4, t\_5, t\_6) \rightarrow C1\ t\_2 + \frac{2}{3}\ \frac{C2\ t\_3}{t\_6^{1/6}} - \frac{1}{10}\ \frac{t\_4\ C2\ t\_5}{t\_6^{7/6}} + \frac{1}{2}\ \frac{t\_4\ C3}{t\_6^{1/3}}$$
$$+ \frac{7}{375}\ \frac{C2\ t\_5^3}{t\_6^{13/6}} - \frac{2}{25}\ \frac{C3\ t\_5^2}{t\_6^{4/3}} + \frac{2}{5}\ \frac{C4\ t\_5}{\sqrt{t\_6}} + C5\ t\_6^{1/3}$$

$$Q_4 := (t\_1, t\_2, t\_3, t\_4, t\_5, t\_6) \rightarrow C1\ t\_3 + \frac{3}{4}\ \frac{C2\ t\_4}{t\_6^{1/6}} - \frac{3}{50}\ \frac{C2\ t\_5^2}{t\_6^{7/6}} + \frac{3}{5}\ \frac{C3\ t\_5}{t\_6^{1/3}}$$
$$+ C4\ \sqrt{t\_6}$$

$$Q_5 := (t\_1, t\_2, t\_3, t\_4, t\_5, t\_6) \rightarrow C1\ t\_4 + \frac{4}{5}\ \frac{C2\ t\_5}{t\_6^{1/6}} + C3\ t\_6^{2/3}$$

$$Q_6 := (t\_1, t\_2, t\_3, t\_4, t\_5, t\_6) \rightarrow C1\ t\_5 + C2\ t\_6^{5/6}$$

$$Q_7 := (t\_1, t\_2, t\_3, t\_4, t\_5, t\_6) \rightarrow C1\ t\_6$$

$$[0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0]$$

```
> FMatrix:=Matrix(rs-1,rs-1,0):
  for i from 1 to rs-1 do for j from 1 to rs-1 do if j<=i then
  FMatrix[i,j]:=f[i,j]: fi: od: od:
  FMatrix;

  CMatrix:=Matrix(rs-1,1,0):
  CMatrix[1,1]:=C1:
  CMatrix[2,1]:=C2:
  CMatrix[3,1]:=C3:
  CMatrix[4,1]:=C4:
  CMatrix[5,1]:=C5:
  CMatrix[6,1]:=C6:
  CMatrix;

  QMatrix:=Matrix(rs-1,1,0):
  for i from 1 to rs-1 do QMatrix[i,1]:=Q[rs+1-i](t[1], t[2], t[3],
  t[4], t[5], t[6]): od:
  QMatrix:
```

$$
\begin{bmatrix}
f_{1,1} & 0 & 0 & 0 & 0 & 0 \\
f_{2,1} & f_{2,2} & 0 & 0 & 0 & 0 \\
f_{3,1} & f_{3,2} & f_{3,3} & 0 & 0 & 0 \\
f_{4,1} & f_{4,2} & f_{4,3} & f_{4,4} & 0 & 0 \\
f_{5,1} & f_{5,2} & f_{5,3} & f_{5,4} & f_{5,5} & 0 \\
f_{6,1} & f_{6,2} & f_{6,3} & f_{6,4} & f_{6,5} & f_{6,6}
\end{bmatrix}
$$

$$
\begin{bmatrix}
C1 \\
C2 \\
C3 \\
C4 \\
C5 \\
C6
\end{bmatrix}
$$

(3)

```
> for i from 1 to rs-1 do f[i,1]:=t[rs-i]: od:
  for j from 1 to rs-1 do f[j,j]:=t[rs-1]^((rs-j)/(rs-1)): od:
  for j from 1 to rs-2 do f[j+1,j]:= (rs-j-1)/(rs-2)*t[rs-1]^((1-j)
  /(rs-1))*t[rs-2]: od:
  f[4,2]:=(3*(25*t[4]*t[6]-2*t[5]^2))/(100*t[6]^(7/6));
```

```
f[5,3]:=(25*t[4]*t[6]-4*t[5]^2)/(50*t[6]^(4/3));
f[6,4]:=(25*t[4]*t[6]-6*t[5]^2)/(100*t[6]^(3/2));
f[5,2]:=(500*t[3]*t[6]^2-75*t[4]*t[5]*t[6]+14*t[5]^3)/(750*t[6]^
(13/6));
f[6,3]:=(250*t[3]*t[6]^2-75*t[4]*t[5]*t[6]+16*t[5]^3)/(750*t[6]^
(7/3));
f[6,2]:=(30000*t[2]*t[6]^3-4000*t[3]*t[5]*t[6]^2-1875*t[4]^2*t[6]
^2+2100*t[4]*t[5]^2*t[6]-364*t[5]^4)/(60000*t[6]^(19/6));
simplify(Multiply(FMatrix,CMatrix)-QMatrix);
```

$$f_{4,2} := \frac{3}{100}\,\frac{25\,t_4\,t_6 - 2\,t_5^2}{t_6^{(7/6)}} \tag{4}$$

$$f_{5,3} := \frac{1}{50}\,\frac{25\,t_4\,t_6 - 4\,t_5^2}{t_6^{(4/3)}}$$

$$f_{6,4} := \frac{1}{100}\,\frac{25\,t_4\,t_6 - 6\,t_5^2}{t_6^{(3/2)}}$$

$$f_{5,2} := \frac{1}{750}\,\frac{500\,t_3\,t_6^2 - 75\,t_4\,t_5\,t_6 + 14\,t_5^3}{t_6^{(13/6)}}$$

$$f_{6,3} := \frac{1}{750}\,\frac{250\,t_3\,t_6^2 - 75\,t_4\,t_5\,t_6 + 16\,t_5^3}{t_6^{(7/3)}}$$

$$f_{6,2} := \frac{1}{60000}\,\frac{30000\,t_2\,t_6^3 - 4000\,t_3\,t_5\,t_6^2 - 1875\,t_4^2\,t_6^2 + 2100\,t_4\,t_5^2\,t_6 - 364\,t_5^4}{t_6^{(19/6)}}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

```
> restart:
  with(LinearAlgebra):
> rinfty:=8:
  list0:=seq(t[i],i=1..rinfty-3);
  Minfty:=Matrix(rinfty-4,rinfty-4,0):
  for i from 1 to rinfty-4 do Minfty[i,i]:=1: od:
  for i from 1 to rinfty-5 do Minfty[i+1,i]:=0: od:
  for i from 1 to rinfty-4 do for j from 1 to rinfty-4 do if j<=i-2
  then Minfty[i,j]:=t[rinfty-(i-j+1)]:  fi: od: od:
  Minfty;
```

```
DiagoInfty:=Matrix(rinfty-4,rinfty-4,0):
for i from 1 to rinfty-4 do DiagoInfty[i,i]:= 1/(rinfty-3-i): od:

DiagoInfty;
DiagoInfty2:=Matrix(rinfty-4,rinfty-4,0):
for i from 1 to rinfty-4 do DiagoInfty2[i,i]:= 1/(rinfty-2-i):
od:
DiagoInfty2;

RHSMatrix:=Matrix(rinfty-4,rinfty-4,0):
for i from 1 to rinfty-4 do RHSMatrix[i,i]:=1: od:
for i from 1 to rinfty-4 do for j from 1 to rinfty-4 do if j<=i-1
then RHSMatrix[i,j]:=Qinf[rinfty-3-(i-j)](list0): fi: od: od:
RHSMatrix;

deltaQinfMatrix:=Matrix(rinfty-4,rinfty-4,0):
for i from 1 to rinfty-4 do for j from 1 to rinfty-4 do
deltaQinfMatrix[i,j]:=diff(Qinf[rinfty-4-i](list0),t[rinfty-2-j])
:
od: od:
deltaQinfMatrix;
```

$$list0 := t_1, t_2, t_3, t_4, t_5 \tag{5}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t_5 & 0 & 1 & 0 \\ t_4 & t_5 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{1}{4} & 0 & 0 & 0 \\ 0 & \dfrac{1}{3} & 0 & 0 \\ 0 & 0 & \dfrac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
\begin{bmatrix}
\dfrac{1}{5} & 0 & 0 & 0 \\[2mm]
0 & \dfrac{1}{4} & 0 & 0 \\[2mm]
0 & 0 & \dfrac{1}{3} & 0 \\[2mm]
0 & 0 & 0 & \dfrac{1}{2}
\end{bmatrix}
$$

$$
\begin{bmatrix}
1 & 0 & 0 & 0 \\
Qinf_4(t_1, t_2, t_3, t_4, t_5) & 1 & 0 & 0 \\
Qinf_3(t_1, t_2, t_3, t_4, t_5) & Qinf_4(t_1, t_2, t_3, t_4, t_5) & 1 & 0 \\
Qinf_2(t_1, t_2, t_3, t_4, t_5) & Qinf_3(t_1, t_2, t_3, t_4, t_5) & Qinf_4(t_1, t_2, t_3, t_4, t_5) & 1
\end{bmatrix}
$$

$$
\Bigg[\Bigg[\frac{\partial}{\partial t_5} Qinf_3(t_1, t_2, t_3, t_4, t_5),\ \frac{\partial}{\partial t_4} Qinf_3(t_1, t_2, t_3, t_4, t_5),\ \frac{\partial}{\partial t_3} Qinf_3(t_1, t_2, t_3, t_4, t_5),
$$
$$
\frac{\partial}{\partial t_2} Qinf_3(t_1, t_2, t_3, t_4, t_5) \Bigg],
$$
$$
\Bigg[\frac{\partial}{\partial t_5} Qinf_2(t_1, t_2, t_3, t_4, t_5),\ \frac{\partial}{\partial t_4} Qinf_2(t_1, t_2, t_3, t_4, t_5),\ \frac{\partial}{\partial t_3} Qinf_2(t_1, t_2, t_3, t_4, t_5),
$$
$$
\frac{\partial}{\partial t_2} Qinf_2(t_1, t_2, t_3, t_4, t_5) \Bigg],
$$
$$
\Bigg[\frac{\partial}{\partial t_5} Qinf_1(t_1, t_2, t_3, t_4, t_5),\ \frac{\partial}{\partial t_4} Qinf_1(t_1, t_2, t_3, t_4, t_5),\ \frac{\partial}{\partial t_3} Qinf_1(t_1, t_2, t_3, t_4, t_5),
$$
$$
\frac{\partial}{\partial t_2} Qinf_1(t_1, t_2, t_3, t_4, t_5) \Bigg],
$$
$$
\Bigg[\frac{\partial}{\partial t_5} Qinf_0(t_1, t_2, t_3, t_4, t_5),\ \frac{\partial}{\partial t_4} Qinf_0(t_1, t_2, t_3, t_4, t_5),\ \frac{\partial}{\partial t_3} Qinf_0(t_1, t_2, t_3, t_4, t_5),
$$
$$
\frac{\partial}{\partial t_2} Qinf_0(t_1, t_2, t_3, t_4, t_5) \Bigg]\Bigg]
$$

```
> SystemEquationInf:=[]:
  for i from 1 to rinfty-4 do for j from 1 to rinfty-4 do
  SystemEquationInf:=[op(SystemEquationInf),(Multiply(Multiply
  (Minfty,DiagoInfty),deltaQinfMatrix)-Multiply
  (RHSMatrix,DiagoInfty2))[i,j]=0]: od: od:
  SystemEquationInf:
  pdsolve({op(SystemEquationInf)});
```

$$
\Bigg\{ Qinf_0(t_1, t_2, t_3, t_4, t_5) = \frac{1}{180}\left(-12\, t_5^2 + 100\, t_3\right) \_F2(t_1) - \frac{1}{20}\, t_4 t_5 + \frac{1}{4}\, t_4\, \_F1(t_1) \tag{6}
$$
$$
+ \frac{1}{5}\, \_F3(t_1)\, t_5 + \frac{1}{2}\, t_2 + \_F5(t_1),\ Qinf_1(t_1, t_2, t_3, t_4, t_5) = \frac{2}{3}\, t_3 + \frac{5}{6}\, \_F2(t_1)\, t_4 - \frac{1}{25}\, t_5^2
$$

$$+ \frac{2}{5} \_F1(t_1)\, t_5 + \_F4(t_1),\ Qinf_2(t_1, t_2, t_3, t_4, t_5) = \frac{3}{4}\, t_4 + \_F2(t_1)\, t_5 + \_F3(t_1),$$

$$Qinf_3(t_1, t_2, t_3, t_4, t_5) = \frac{4}{5}\, t_5 + \_F1(t_1),\ Qinf_4(t_1, t_2, t_3, t_4, t_5) = \frac{5}{3}\, \_F2(t_1) \Big\}$$

```
> F2(t[1]):=3/5*U4;
  Qinf[0](t[1], t[2], t[3], t[4], t[5]) := (1/180)*(-12*t[5]^2+100*
  t[3])*F2(t[1])-(1/20)*t[4]*t[5]+(1/4)*t[4]*F1(t[1])+(1/5)*F3(t[1]
  )*t[5]+(1/2)*t[2]+F5(t[1]);
  Qinf[1](t[1], t[2], t[3], t[4], t[5]) := 2*t[3]*(1/3)+5*F2(t[1])*
  t[4]*(1/6)-(1/25)*t[5]^2+2*F1(t[1])*t[5]*(1/5)+F4(t[1]);
  Qinf[2](t[1], t[2], t[3], t[4], t[5]) := 3*t[4]*(1/4)+F2(t[1])*t
  [5]+F3(t[1]);
  Qinf[3](t[1], t[2], t[3], t[4], t[5]) := 4*t[5]*(1/5)+F1(t[1]);
  Qinf[4](t[1], t[2], t[3], t[4], t[5]) := 5*F2(t[1])*(1/3);
  simplify(SystemEquationInf);
```

$$F2(t_1) := \frac{3}{5}\, U4 \tag{7}$$

$$Qinf_0(t_1, t_2, t_3, t_4, t_5) := \frac{1}{300}\left(-12\, t_5^2 + 100\, t_3\right) U4 - \frac{1}{20}\, t_4\, t_5 + \frac{1}{4}\, t_4\, F1(t_1) + \frac{1}{5}\, F3(t_1)\, t_5$$

$$+ \frac{1}{2}\, t_2 + F5(t_1)$$

$$Qinf_1(t_1, t_2, t_3, t_4, t_5) := \frac{2}{3}\, t_3 + \frac{1}{2}\, U4\, t_4 - \frac{1}{25}\, t_5^2 + \frac{2}{5}\, F1(t_1)\, t_5 + F4(t_1)$$

$$Qinf_2(t_1, t_2, t_3, t_4, t_5) := \frac{3}{4}\, t_4 + \frac{3}{5}\, U4\, t_5 + F3(t_1)$$

$$Qinf_3(t_1, t_2, t_3, t_4, t_5) := \frac{4}{5}\, t_5 + F1(t_1)$$

$$Qinf_4(t_1, t_2, t_3, t_4, t_5) := U4$$

$$[0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0, 0=0]$$

```
> GMatrix:=Matrix(rinfty-2,rinfty-2,0):
  for i from 1 to rinfty-2 do for j from 1 to rinfty-2 do if j<=i
  then GMatrix[i,j]:=g[i,j]: fi: od: od:
  GMatrix;

  UMatrix:=Matrix(rinfty-2,1,0):
  UMatrix[1,1]:=1:
  UMatrix[2,1]:=U4:
  UMatrix[3,1]:=U3:
  UMatrix[4,1]:=U2:
  UMatrix[5,1]:=U1:
  UMatrix[6,1]:=U0:
  UMatrix;
```

```
QinfMatrix:=Matrix(rinfty-2,1,0):
QinfMatrix[1,1]:=1:
for i from 2 to rinfty-2 do QinfMatrix[i,1]:=Qinf[rinfty-2-i]
(list0): od:
QinfMatrix;
```

$$\begin{bmatrix} g_{1,1} & 0 & 0 & 0 & 0 & 0 \\ g_{2,1} & g_{2,2} & 0 & 0 & 0 & 0 \\ g_{3,1} & g_{3,2} & g_{3,3} & 0 & 0 & 0 \\ g_{4,1} & g_{4,2} & g_{4,3} & g_{4,4} & 0 & 0 \\ g_{5,1} & g_{5,2} & g_{5,3} & g_{5,4} & g_{5,5} & 0 \\ g_{6,1} & g_{6,2} & g_{6,3} & g_{6,4} & g_{6,5} & g_{6,6} \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} 1 \\ U4 \\ U3 \\ U2 \\ U1 \\ U0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ U4 \\ \dfrac{4}{5}\, t_5 + F1(t_1) \\ \dfrac{3}{4}\, t_4 + \dfrac{3}{5}\, U4\, t_5 + F3(t_1) \\ \dfrac{2}{3}\, t_3 + \dfrac{1}{2}\, U4\, t_4 - \dfrac{1}{25}\, t_5^2 + \dfrac{2}{5}\, F1(t_1)\, t_5 + F4(t_1) \\ \dfrac{1}{300}\left(-12\, t_5^2 + 100\, t_3 \right) U4 - \dfrac{1}{20}\, t_4\, t_5 + \dfrac{1}{4}\, t_4\, F1(t_1) + \dfrac{1}{5}\, F3(t_1)\, t_5 + \dfrac{1}{2}\, t_2 + F5(t_1) \end{bmatrix}$$

```
> F1(t[1]):=U3;
  F2(t[1]):=0;
  F3(t[1]):=U2;
  F4(t[1]):=U1;
  F5(t[1]):=U0;
  for i from 1 to rinfty-2 do g[i,i]:=1: od:
  for i from 1 to rinfty-3 do g[i+1,i]:=0: od:
  g[3,1]:=4/5*t[5];
  g[4,2]:=3/5*t[5];
```

```
g[5,3]:=2/5*t[5];
g[6,4]:=1/5*t[5];

g[4,1]:=3/4*t[4];
g[5,2]:=(1/2)*t[4];
g[6,3]:=(1/4)*t[4];

g[5,1]:=2*t[3]*(1/3)-(1/25)*t[5]^2;

g[6,2]:=-(1/25)*t[5]^2+(1/3)*t[3];
g[6,1]:=-(1/20)*t[4]*t[5]+(1/2)*t[2];

simplify(Multiply(GMatrix,UMatrix)-QinfMatrix);
```

$$F1(t_1) := U3$$ **(9)**
$$F2(t_1) := 0$$
$$F3(t_1) := U2$$
$$F4(t_1) := U1$$
$$F5(t_1) := U0$$

$$g_{3,1} := \frac{4}{5}\,t_5$$

$$g_{4,2} := \frac{3}{5}\,t_5$$

$$g_{5,3} := \frac{2}{5}\,t_5$$

$$g_{6,4} := \frac{1}{5}\,t_5$$

$$g_{4,1} := \frac{3}{4}\,t_4$$

$$g_{5,2} := \frac{1}{2}\,t_4$$

$$g_{6,3} := \frac{1}{4}\,t_4$$

$$g_{5,1} := \frac{2}{3}\,t_3 - \frac{1}{25}\,t_5^2$$

$$g_{6,2} := -\frac{1}{25}\,t_5^2 + \frac{1}{3}\,t_3$$

$$g_{6,1} := -\frac{1}{20}\,t_4\,t_5 + \frac{1}{2}\,t_2$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

> `simplify(GMatrix);`

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \dfrac{4}{5}\,t_5 & 0 & 1 & 0 & 0 & 0 \\ \dfrac{3}{4}\,t_4 & \dfrac{3}{5}\,t_5 & 0 & 1 & 0 & 0 \\ \dfrac{2}{3}\,t_3 - \dfrac{1}{25}\,t_5^2 & \dfrac{1}{2}\,t_4 & \dfrac{2}{5}\,t_5 & 0 & 1 & 0 \\ -\dfrac{1}{20}\,t_4\,t_5 + \dfrac{1}{2}\,t_2 & -\dfrac{1}{25}\,t_5^2 + \dfrac{1}{3}\,t_3 & \dfrac{1}{4}\,t_4 & \dfrac{1}{5}\,t_5 & 0 & 1 \end{bmatrix}$$

(10)

> ```
restart:
with(LinearAlgebra):
rinfty:=10:
list0:=seq(t[i],i=1..rinfty-3);
Minfty:=Matrix(rinfty-3,rinfty-3,0):
for i from 1 to rinfty-3 do Minfty[i,i]:=1: od:
for i from 1 to rinfty-4 do Minfty[i+1,i]:=0: od:
for i from 1 to rinfty-3 do for j from 1 to rinfty-3 do if j<=i-2
then Minfty[i,j]:=t[rinfty-(i-j+1)]: fi: od: od:
Minfty;
DiagoInfty:=Matrix(rinfty-3,rinfty-3,0):
for i from 1 to rinfty-3 do DiagoInfty[i,i]:= 1/(rinfty-2-i): od:

DiagoInfty;
DiagoInfty2:=Matrix(rinfty-3,rinfty-3,0):
for i from 1 to rinfty-3 do DiagoInfty2[i,i]:= 1/(rinfty-2-i):
od:
DiagoInfty2;

RHSMatrix:=Matrix(rinfty-3,rinfty-3,0):
for i from 1 to rinfty-3 do RHSMatrix[i,i]:=-1: od:
for i from 1 to rinfty-3 do for j from 1 to rinfty-3 do if j<=i-2
```

```
then RHSMatrix[i,j]:=R[rinfty-2-(i-j)](list0): fi: od: od:
RHSMatrix;

deltaRinfMatrix:=Matrix(rinfty-3,rinfty-3,0):
for i from 1 to rinfty-3 do for j from 1 to rinfty-3 do
deltaRinfMatrix[i,j]:=diff(R[rinfty-3-i](list0),t[rinfty-2-j]):
od: od:
deltaRinfMatrix;
```

$$list0 := t_1, t_2, t_3, t_4, t_5, t_6, t_7 \tag{11}$$

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
t_7 & 0 & 1 & 0 & 0 & 0 & 0 \\
t_6 & t_7 & 0 & 1 & 0 & 0 & 0 \\
t_5 & t_6 & t_7 & 0 & 1 & 0 & 0 \\
t_4 & t_5 & t_6 & t_7 & 0 & 1 & 0 \\
t_3 & t_4 & t_5 & t_6 & t_7 & 0 & 1
\end{bmatrix}
$$

$$
\begin{bmatrix}
\frac{1}{7} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{5} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

$$\begin{bmatrix} \dfrac{1}{7} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \dfrac{1}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{1}{5} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\big[\,\big[\,-1,\,0,\,0,\,0,\,0,\,0,\,0\,\big],$

$\quad\big[\,0,\,-1,\,0,\,0,\,0,\,0,\,0\,\big],$

$\quad\big[\,R_6(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\,0,\,-1,\,0,\,0,\,0,\,0\,\big],$

$\quad\big[\,R_5(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\,R_6(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\,0,\,-1,\,0,\,0,\,0\,\big],$

$\quad\big[\,R_4(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\,R_5(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\,R_6(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\,0,\,-1,\,0,\,0\,\big],$

$\quad\big[\,R_3(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\,R_4(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\,R_5(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\,R_6(t_1,t_2,t_3,t_4,t_5,$
$\quad t_6,t_7),\,0,\,-1,\,0\,\big],$

$\quad\big[\,R_2(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\,R_3(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\,R_4(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\,R_5(t_1,t_2,t_3,t_4,t_5,$
$\quad t_6,t_7),\,R_6(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\,0,\,-1\,\big]\,\big]$

$\Big[\,\Big[\,\dfrac{\partial}{\partial t_7}R_6(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\;\dfrac{\partial}{\partial t_6}R_6(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\;\dfrac{\partial}{\partial t_5}R_6(t_1,t_2,t_3,t_4,t_5,t_6,t_7),$

$\quad\dfrac{\partial}{\partial t_4}R_6(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\;\dfrac{\partial}{\partial t_3}R_6(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\;\dfrac{\partial}{\partial t_2}R_6(t_1,t_2,t_3,t_4,t_5,t_6,t_7),$

$\quad\dfrac{\partial}{\partial t_1}R_6(t_1,t_2,t_3,t_4,t_5,t_6,t_7)\,\Big],$

$\quad\Big[\,\dfrac{\partial}{\partial t_7}R_5(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\;\dfrac{\partial}{\partial t_6}R_5(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\;\dfrac{\partial}{\partial t_5}R_5(t_1,t_2,t_3,t_4,t_5,t_6,t_7),$

$\quad\dfrac{\partial}{\partial t_4}R_5(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\;\dfrac{\partial}{\partial t_3}R_5(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\;\dfrac{\partial}{\partial t_2}R_5(t_1,t_2,t_3,t_4,t_5,t_6,t_7),$

$\quad\dfrac{\partial}{\partial t_1}R_5(t_1,t_2,t_3,t_4,t_5,t_6,t_7)\,\Big],$

$\quad\Big[\,\dfrac{\partial}{\partial t_7}R_4(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\;\dfrac{\partial}{\partial t_6}R_4(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\;\dfrac{\partial}{\partial t_5}R_4(t_1,t_2,t_3,t_4,t_5,t_6,t_7),$

$\quad\dfrac{\partial}{\partial t_4}R_4(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\;\dfrac{\partial}{\partial t_3}R_4(t_1,t_2,t_3,t_4,t_5,t_6,t_7),\;\dfrac{\partial}{\partial t_2}R_4(t_1,t_2,t_3,t_4,t_5,t_6,t_7),$

$\quad\dfrac{\partial}{\partial t_1}R_4(t_1,t_2,t_3,t_4,t_5,t_6,t_7)\,\Big],$

$$\left[\frac{\partial}{\partial t_7} R_3(t_1, t_2, t_3, t_4, t_5, t_6, t_7), \ \frac{\partial}{\partial t_6} R_3(t_1, t_2, t_3, t_4, t_5, t_6, t_7), \ \frac{\partial}{\partial t_5} R_3(t_1, t_2, t_3, t_4, t_5, t_6, t_7), \right.$$

$$\frac{\partial}{\partial t_4} R_3(t_1, t_2, t_3, t_4, t_5, t_6, t_7), \ \frac{\partial}{\partial t_3} R_3(t_1, t_2, t_3, t_4, t_5, t_6, t_7), \ \frac{\partial}{\partial t_2} R_3(t_1, t_2, t_3, t_4, t_5, t_6, t_7),$$

$$\left. \frac{\partial}{\partial t_1} R_3(t_1, t_2, t_3, t_4, t_5, t_6, t_7) \right],$$

$$\left[\frac{\partial}{\partial t_7} R_2(t_1, t_2, t_3, t_4, t_5, t_6, t_7), \ \frac{\partial}{\partial t_6} R_2(t_1, t_2, t_3, t_4, t_5, t_6, t_7), \ \frac{\partial}{\partial t_5} R_2(t_1, t_2, t_3, t_4, t_5, t_6, t_7), \right.$$

$$\frac{\partial}{\partial t_4} R_2(t_1, t_2, t_3, t_4, t_5, t_6, t_7), \ \frac{\partial}{\partial t_3} R_2(t_1, t_2, t_3, t_4, t_5, t_6, t_7), \ \frac{\partial}{\partial t_2} R_2(t_1, t_2, t_3, t_4, t_5, t_6, t_7),$$

$$\left. \frac{\partial}{\partial t_1} R_2(t_1, t_2, t_3, t_4, t_5, t_6, t_7) \right],$$

$$\left[\frac{\partial}{\partial t_7} R_1(t_1, t_2, t_3, t_4, t_5, t_6, t_7), \ \frac{\partial}{\partial t_6} R_1(t_1, t_2, t_3, t_4, t_5, t_6, t_7), \ \frac{\partial}{\partial t_5} R_1(t_1, t_2, t_3, t_4, t_5, t_6, t_7), \right.$$

$$\frac{\partial}{\partial t_4} R_1(t_1, t_2, t_3, t_4, t_5, t_6, t_7), \ \frac{\partial}{\partial t_3} R_1(t_1, t_2, t_3, t_4, t_5, t_6, t_7), \ \frac{\partial}{\partial t_2} R_1(t_1, t_2, t_3, t_4, t_5, t_6, t_7),$$

$$\left. \frac{\partial}{\partial t_1} R_1(t_1, t_2, t_3, t_4, t_5, t_6, t_7) \right],$$

$$\left[\frac{\partial}{\partial t_7} R_0(t_1, t_2, t_3, t_4, t_5, t_6, t_7), \ \frac{\partial}{\partial t_6} R_0(t_1, t_2, t_3, t_4, t_5, t_6, t_7), \ \frac{\partial}{\partial t_5} R_0(t_1, t_2, t_3, t_4, t_5, t_6, t_7), \right.$$

$$\frac{\partial}{\partial t_4} R_0(t_1, t_2, t_3, t_4, t_5, t_6, t_7), \ \frac{\partial}{\partial t_3} R_0(t_1, t_2, t_3, t_4, t_5, t_6, t_7), \ \frac{\partial}{\partial t_2} R_0(t_1, t_2, t_3, t_4, t_5, t_6, t_7),$$

$$\left. \left. \frac{\partial}{\partial t_1} R_0(t_1, t_2, t_3, t_4, t_5, t_6, t_7) \right] \right]$$

```
> SystemEquationR:=[]:
  for i from 1 to rinfty-3 do for j from 1 to rinfty-3 do
  SystemEquationR:=[op(SystemEquationR),(Multiply(Multiply(Minfty,
  DiagoInfty),deltaRinfMatrix)-Multiply(RHSMatrix,DiagoInfty2))[i,
  j]=0]: od: od:
  SystemEquationR:
  pdsolve({op(SystemEquationR)});
```

$$\left\{ R_0(t_1, t_2, t_3, t_4, t_5, t_6, t_7) = \frac{11}{1029} \_C2 \, t_7^3 - \frac{2}{49} \_C4 \, t_7^2 + \frac{1}{61740} \left(-4410 \_C1 \, t_6 - 3528 \_C2 \, t_5 \right. \right. \tag{12}$$

$$+ 8820 \_C6 \right) t_7 + \frac{1}{61740} \left(-1715 \, t_6^2 + 20580 \, t_3 \right) \_C2 + \frac{1}{4} \_C1 \, t_4 + \frac{1}{5} \, t_5 \_C4$$

$$+ \frac{1}{6} \, t_6 \_C3 + \_C7 - t_1, \ R_1(t_1, t_2, t_3, t_4, t_5, t_6, t_7) = -\frac{3}{49} \_C1 \, t_7^2 + \frac{1}{1470} \left(-140 \_C2 \, t_6 \right.$$

$$+ 420 \_C3 \right) t_7 + \frac{2}{5} \_C1 \, t_5 + \frac{1}{2} \_C2 \, t_4 + \frac{1}{3} \, t_6 \_C4 + \_C5 - t_2, \ R_2(t_1, t_2, t_3, t_4, t_5, t_6, t_7)$$

$$= \frac{1}{490} \left(-30 \, t_7^2 + 294 \, t_5 \right) \_C2 + \frac{1}{2} \_C1 \, t_6 + \frac{3}{7} \_C4 \, t_7 + \_C6 - t_3, \ R_3(t_1, t_2, t_3, t_4, t_5, t_6,$$

$$\left. t_7) = -t_4 + \frac{2}{3} \_C2 \, t_6 + \frac{4}{7} \_C1 \, t_7 + \_C3, \ R_4(t_1, t_2, t_3, t_4, t_5, t_6, t_7) = -t_5 + \frac{5}{7} \_C2 \, t_7 \right.$$

```
> R[0](t[1], t[2], t[3], t[4], t[5], t[6], t[7]) := 11*V6*t[7]^3*
  (1/1029)-2*V4*t[7]^2*(1/49)+(1/61740)*(-4410*V5*t[6]-3528*V6*t[5]
  +8820*V2)*t[7]+(1/61740)*(-1715*t[6]^2+20580*t[3])*V6+(1/4)*V5*t
  [4]+(1/5)*t[5]*V4+(1/6)*t[6]*V3+V0-t[1];
  R[1](t[1], t[2], t[3], t[4], t[5], t[6], t[7]) := -3*V5*t[7]^2*
  (1/49)+(1/1470)*(-140*V6*t[6]+420*V3)*t[7]+2*V5*t[5]*(1/5)+(1/2)*
  V6*t[4]+(1/3)*t[6]*V4+V1-t[2];
  R[2](t[1], t[2], t[3], t[4], t[5], t[6], t[7]) := (1/490)*(-30*t
  [7]^2+294*t[5])*V6+(1/2)*V5*t[6]+3*V4*t[7]*(1/7)+V2-t[3];
  R[3](t[1], t[2], t[3], t[4], t[5], t[6], t[7]) := -t[4]+2*V6*t[6]
  *(1/3)+4*V5*t[7]*(1/7)+V3;
  R[4](t[1], t[2], t[3], t[4], t[5], t[6], t[7]) := -t[5]+5*V6*t[7]
  *(1/7)+V4;
  R[5](t[1], t[2], t[3], t[4], t[5], t[6], t[7]) := -t[6]+V5;
  R[6](t[1], t[2], t[3], t[4], t[5], t[6], t[7]) := -t[7]+V6;
  simplify(SystemEquationR);
```

$$R_0\left(t_1, t_2, t_3, t_4, t_5, t_6, t_7\right) := \frac{11}{1029} V6\, t_7^3 - \frac{2}{49} V4\, t_7^2 + \frac{1}{61740}\left(-4410\, V5\, t_6 - 3528\, V6\, t_5\right.$$
$$\left. + 8820\, V2\right) t_7 + \frac{1}{61740}\left(-1715\, t_6^2 + 20580\, t_3\right) V6 + \frac{1}{4} V5\, t_4 + \frac{1}{5} t_5\, V4 + \frac{1}{6} t_6\, V3$$
$$+ V0 - t_1 \tag{13}$$

$$R_1\left(t_1, t_2, t_3, t_4, t_5, t_6, t_7\right) := -\frac{3}{49} V5\, t_7^2 + \frac{1}{1470}\left(-140\, V6\, t_6 + 420\, V3\right) t_7 + \frac{2}{5} V5\, t_5$$
$$+ \frac{1}{2} V6\, t_4 + \frac{1}{3} t_6\, V4 + V1 - t_2$$

$$R_2\left(t_1, t_2, t_3, t_4, t_5, t_6, t_7\right) := \frac{1}{490}\left(-30\, t_7^2 + 294\, t_5\right) V6 + \frac{1}{2} V5\, t_6 + \frac{3}{7} V4\, t_7 + V2 - t_3$$

$$R_3\left(t_1, t_2, t_3, t_4, t_5, t_6, t_7\right) := -t_4 + \frac{2}{3} V6\, t_6 + \frac{4}{7} V5\, t_7 + V3$$

$$R_4\left(t_1, t_2, t_3, t_4, t_5, t_6, t_7\right) := -t_5 + \frac{5}{7} V6\, t_7 + V4$$

$$R_5\left(t_1, t_2, t_3, t_4, t_5, t_6, t_7\right) := -t_6 + V5$$

$$R_6\left(t_1, t_2, t_3, t_4, t_5, t_6, t_7\right) := -t_7 + V6$$

$[0 = 0, 0 = 0]$

```
> GMatrix:=Matrix(rinfty-3,rinfty-3,0):
  for i from 1 to rinfty-3 do for j from 1 to rinfty-3 do if j<=i
  then GMatrix[i,j]:=g[i,j]: fi: od: od:
  GMatrix;

  VMatrix:=Matrix(rinfty-3,1,0):
```

```
VMatrix[1,1]:=V6:
VMatrix[2,1]:=V5:
VMatrix[3,1]:=V4:
VMatrix[4,1]:=V3:
VMatrix[5,1]:=V2:
VMatrix[6,1]:=V1:
VMatrix[7,1]:=V0:
VMatrix;

RMatrix:=Matrix(rinfty-3,1,0):
for i from 1 to rinfty-3 do RMatrix[i,1]:=R[rinfty-3-i](list0):
od:
RMatrix:

TMatrix:=Matrix(rinfty-3,1,0):
for k from 1 to rinfty-3 do TMatrix[k]:=t[rinfty-2-k]: od:
TMatrix;
```

$$
\begin{bmatrix}
g_{1,1} & 0 & 0 & 0 & 0 & 0 & 0 \\
g_{2,1} & g_{2,2} & 0 & 0 & 0 & 0 & 0 \\
g_{3,1} & g_{3,2} & g_{3,3} & 0 & 0 & 0 & 0 \\
g_{4,1} & g_{4,2} & g_{4,3} & g_{4,4} & 0 & 0 & 0 \\
g_{5,1} & g_{5,2} & g_{5,3} & g_{5,4} & g_{5,5} & 0 & 0 \\
g_{6,1} & g_{6,2} & g_{6,3} & g_{6,4} & g_{6,5} & g_{6,6} & 0 \\
g_{7,1} & g_{7,2} & g_{7,3} & g_{7,4} & g_{7,5} & g_{7,6} & g_{7,7}
\end{bmatrix}
\qquad (14)
$$

$$
\begin{bmatrix}
V6 \\
V5 \\
V4 \\
V3 \\
V2 \\
V1 \\
V0
\end{bmatrix}
$$

$$\begin{bmatrix} t_7 \\ t_6 \\ t_5 \\ t_4 \\ t_3 \\ t_2 \\ t_1 \end{bmatrix}$$

```
> for i from 1 to rinfty-3 do g[i,i]:=1: od:
  for i from 1 to rinfty-4 do g[i+1,i]:=0: od:
  for i from 1 to rinfty-5 do g[i+2,i]:=(rinfty-4-i)/(rinfty-3)*t
  [rinfty-3]: od:
  for i from 1 to rinfty-6 do g[i+3,i]:=(rinfty-5-i)/(rinfty-4)*t
  [rinfty-4]: od:
  g[5,1]:=-3*t[7]^2*(1/49)+3*t[5]*(1/5);
  g[6,2]:=-3*t[7]^2*(1/49)+2*t[5]*(1/5);
  g[6,1]:=-2*t[6]*t[7]*(1/21)+(1/2)*t[4];
  g[7,3]:=-2*t[7]^2*(1/49)+(1/5)*t[5];
  g[7,2]:=-(1/14)*t[6]*t[7]+(1/4)*t[4];
  g[7,1]:=11/1029*(t[7]^3)-(2/35*t[5])*t[7]-(1/36)*t[6]^2+(1/3)*t
  [3];
  simplify(Multiply(GMatrix,VMatrix)-TMatrix-RMatrix);
```

$$g_{5,1} := -\frac{3}{49}\,t_7^2 + \frac{3}{5}\,t_5 \tag{15}$$

$$g_{6,2} := -\frac{3}{49}\,t_7^2 + \frac{2}{5}\,t_5$$

$$g_{6,1} := -\frac{2}{21}\,t_6\,t_7 + \frac{1}{2}\,t_4$$

$$g_{7,3} := -\frac{2}{49}\,t_7^2 + \frac{1}{5}\,t_5$$

$$g_{7,2} := \frac{1}{4}\,t_4 - \frac{1}{14}\,t_6\,t_7$$

$$g_{7,1} := \frac{11}{1029}\,t_7^3 - \frac{2}{35}\,t_5\,t_7 - \frac{1}{36}\,t_6^2 + \frac{1}{3}\,t_3$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

> simplify(GMatrix);

$$\left[\left[1, 0, 0, 0, 0, 0, 0\right],\right. \tag{16}$$

$$\left[0, 1, 0, 0, 0, 0, 0\right],$$

$$\left[\frac{5}{7}\, t_7, 0, 1, 0, 0, 0, 0\right],$$

$$\left[\frac{2}{3}\, t_6, \frac{4}{7}\, t_7, 0, 1, 0, 0, 0\right],$$

$$\left[-\frac{3}{49}\, t_7^2 + \frac{3}{5}\, t_5, \frac{1}{2}\, t_6, \frac{3}{7}\, t_7, 0, 1, 0, 0\right],$$

$$\left[-\frac{2}{21}\, t_6\, t_7 + \frac{1}{2}\, t_4, -\frac{3}{49}\, t_7^2 + \frac{2}{5}\, t_5, \frac{1}{3}\, t_6, \frac{2}{7}\, t_7, 0, 1, 0\right],$$

$$\left[\frac{11}{1029}\, t_7^3 - \frac{2}{35}\, t_5\, t_7 - \frac{1}{36}\, t_6^2 + \frac{1}{3}\, t_3, \frac{1}{4}\, t_4 - \frac{1}{14}\, t_6\, t_7, -\frac{2}{49}\, t_7^2 + \frac{1}{5}\, t_5, \frac{1}{6}\, t_6, \frac{1}{7}\, t_7, 0, \right.$$

$$\left.\left. 1\right]\right]$$