In this Maple file, we compute the Casimir coefficients of the Lax matrix L associated to the Painlevé 6 equation in relation with the spectral curve

```
> restart:
  P1:=x-> P011/x+P111/(x-1)+Pt11/(x-t);
  P2:=x-> P022/x^2+P012/x+P122/(x-1)^2+P112/(x-1)+Pt22/(x-t)^2+
  Pt12/(x-t);
  SpectralCurve:=unapply( y^2-P1(x)*y+P2(x),y);
```

$$P1 := x \to \frac{P011}{x} + \frac{P111}{x-1} + \frac{Pt11}{x-t} \tag{1}$$

$$P2 := x \to \frac{P022}{x^2} + \frac{P012}{x} + \frac{P122}{(x-1)^2} + \frac{P112}{x-1} + \frac{Pt22}{(x-t)^2} + \frac{Pt12}{x-t}$$

$$SpectralCurve := y \to y^2 - \left(\frac{P011}{x} + \frac{P111}{x-1} + \frac{Pt11}{x-t} \right) y + \frac{P022}{x^2} + \frac{P012}{x} + \frac{P122}{(x-1)^2}$$

$$+ \frac{P112}{x-1} + \frac{Pt22}{(x-t)^2} + \frac{Pt12}{x-t}$$

```
> DiaginftySheet1:=-tinfty10/x+Unknown/x^2;
  DiaginftySheet2:=-tinfty20/x+Unknown2/x^2;
  Diag0Sheet1:=t010/x+Unknown3;
  Diag0Sheet2:=t020/x+Unknown4;
  Diag1Sheet1:=t110/(x-1)+Unknown5;
  Diag1Sheet2:=t120/(x-1)+Unknown6;
  DiagtSheet1:=tt10/(x-t)+Unknown5;
  DiagtSheet2:=tt20/(x-t)+Unknown6;
```

$$DiaginftySheet1 := -\frac{tinfty10}{x} + \frac{Unknown}{x^2} \tag{2}$$

$$DiaginftySheet2 := -\frac{tinfty20}{x} + \frac{Unknown2}{x^2}$$

$$Diag0Sheet1 := \frac{t010}{x} + Unknown3$$

$$Diag0Sheet2 := \frac{t020}{x} + Unknown4$$

$$Diag1Sheet1 := \frac{t110}{x-1} + Unknown5$$

$$Diag1Sheet2 := \frac{t120}{x-1} + Unknown6$$

$$DiagtSheet1 := \frac{tt10}{x-t} + Unknown5$$

$$DiagtSheet2 := \frac{tt20}{x-t} + Unknown6$$

Expression of P_1 in terms of the diagonal expansion in both sheets
Study at infinity

```
> series(DiaginftySheet1+DiaginftySheet2-P1(x),x=infinity);
  series(Diag0Sheet1+Diag0Sheet2-P1(x),x=0,5);
  series(Diag1Sheet1+Diag1Sheet2-P1(x),x=1,5);
```

```
series(DiagtSheet1+DiagtSheet2-P1(x),x=t,5);
P011:=t010+t020;
P111:=t110+t120;
Pt11:=tt10+tt20;
CoherenceEquation1:=residue(DiaginftySheet1+DiaginftySheet2-P1
(x),x=infinity);
```

$$\frac{-tinfty10-tinfty20-P011-P111-Pt11}{x}+\frac{-Pt11\,t-P111+Unknown+Unknown2}{x^2}$$

$$\quad (3)$$

$$+\frac{-Pt11\,t^2-P111}{x^3}+\frac{-Pt11\,t^3-P111}{x^4}+\frac{-Pt11\,t^4-P111}{x^5}+O\!\left(\frac{1}{x^6}\right)$$

$$\frac{t010+t020-P011}{x}+Unknown3+Unknown4+P111+\frac{Pt11}{t}+\left(P111+\frac{Pt11}{t^2}\right)x$$

$$+\left(P111+\frac{Pt11}{t^3}\right)x^2+\left(P111+\frac{Pt11}{t^4}\right)x^3+\left(P111+\frac{Pt11}{t^5}\right)x^4+O(x^5)$$

$$\frac{t110+t120-P111}{x-1}+Unknown5+Unknown6-P011-\frac{Pt11}{1-t}+\left(P011\right.$$

$$\left.-\frac{Pt11}{(-1+t)(1-t)}\right)(x-1)+\left(-P011-\frac{Pt11}{(-1+t)^2(1-t)}\right)(x-1)^2+\left(P011\right.$$

$$\left.-\frac{Pt11}{(-1+t)^3(1-t)}\right)(x-1)^3+\left(-P011-\frac{Pt11}{(-1+t)^4(1-t)}\right)(x-1)^4+O(\,(x$$

$$-1)^5)$$

$$\frac{tt10+tt20-Pt11}{x-t}+Unknown5+Unknown6-\frac{P011}{t}-\frac{P111}{-1+t}+\left(\frac{P011}{t^2}\right.$$

$$\left.+\frac{P111}{(-1+t)^2}\right)(x-t)+\left(-\frac{P011}{t^3}-\frac{P111}{(-1+t)^3}\right)(x-t)^2+\left(\frac{P011}{t^4}\right.$$

$$\left.+\frac{P111}{(-1+t)^4}\right)(x-t)^3+\left(-\frac{P011}{t^5}-\frac{P111}{(-1+t)^5}\right)(x-t)^4+O(\,(x-t)^5)$$

$$P011:=t010+t020$$
$$P111:=t110+t120$$
$$Pt11:=tt10+tt20$$

$$CoherenceEquation1:=tinfty10+tinfty20+t010+t020+t110+t120+tt10+tt20$$

Study at 0

```
> factor(series(SpectralCurve(Diag0Sheet1),x=0));
  factor(series(SpectralCurve(Diag0Sheet2),x=0));
  EQ01:=residue(x^3*SpectralCurve(Diag0Sheet1),x=0);
  EQ02:=residue(x^3*SpectralCurve(Diag0Sheet2),x=0);
  EQ03:=residue(x^2*SpectralCurve(Diag0Sheet1),x=0);
  EQ04:=residue(x^2*SpectralCurve(Diag0Sheet2),x=0);
  EQ05:=residue(x*SpectralCurve(Diag0Sheet1),x=0);
  EQ06:=residue(x*SpectralCurve(Diag0Sheet2),x=0);
  EQ07:=residue(x^0*SpectralCurve(Diag0Sheet1),x=0):
  EQ08:=residue(x^0*SpectralCurve(Diag0Sheet2),x=0):
  P022:=solve(EQ05,P022);
  simplify(EQ05);
```

```
simplify(EQ06);
simplify(EQ07);
simplify(EQ08);
```

$$\frac{-t010\,t020 + P022}{x^2} + \frac{1}{x}\frac{1}{t}\,(t\,t010\,t110 + t\,t010\,t120 + t\,t010\,Unknown3 - t\,t020\,Unknown3 \qquad (4)$$

$$+ P012\,t + t010\,tt10 + t010\,tt20) - \frac{1}{t^2}\left(-t^2\,t010\,t110 - t^2\,t010\,t120\right.$$

$$- t^2\,t110\,Unknown3 - t^2\,t120\,Unknown3 - t^2\,Unknown3^2 + P112\,t^2 - P122\,t^2$$

$$- t\,tt10\,Unknown3 - t\,tt20\,Unknown3 + Pt12\,t - t010\,tt10 - t010\,tt20 - Pt22\Big)$$

$$- \frac{1}{t^3}\left(-t^3\,t010\,t110 - t^3\,t010\,t120 - t^3\,t110\,Unknown3 - t^3\,t120\,Unknown3 + P112\,t^3\right.$$

$$- 2\,P122\,t^3 - t\,tt10\,Unknown3 - t\,tt20\,Unknown3 + Pt12\,t - t010\,tt10 - t010\,tt20$$

$$- 2\,Pt22\Big)\,x - \frac{1}{t^4}\left(-t^4\,t010\,t110 - t^4\,t010\,t120 - t^4\,t110\,Unknown3\right.$$

$$- t^4\,t120\,Unknown3 + P112\,t^4 - 3\,P122\,t^4 - t\,tt10\,Unknown3 - t\,tt20\,Unknown3 + Pt12\,t$$

$$- t010\,tt10 - t010\,tt20 - 3\,Pt22\Big)\,x^2 - \frac{1}{t^5}\left(-t^5\,t010\,t110 - t^5\,t010\,t120\right.$$

$$- t^5\,t110\,Unknown3 - t^5\,t120\,Unknown3 + P112\,t^5 - 4\,P122\,t^5 - t\,tt10\,Unknown3$$

$$- t\,tt20\,Unknown3 + Pt12\,t - t010\,tt10 - t010\,tt20 - 4\,Pt22\Big)\,x^3 - \frac{1}{t^6}\left(-t^6\,t010\,t110\right.$$

$$- t^6\,t010\,t120 - t^6\,t110\,Unknown3 - t^6\,t120\,Unknown3 + P112\,t^6 - 5\,P122\,t^6$$

$$- t\,tt10\,Unknown3 - t\,tt20\,Unknown3 + Pt12\,t - t010\,tt10 - t010\,tt20 - 5\,Pt22\Big)\,x^4$$

$$+ O(x^5)$$

$$\frac{-t010\,t020 + P022}{x^2} + \frac{1}{x}\frac{1}{t}\,(-t\,t010\,Unknown4 + t\,t020\,t110 + t\,t020\,t120$$

$$+ t\,t020\,Unknown4 + P012\,t + t020\,tt10 + t020\,tt20) - \frac{1}{t^2}\left(-t^2\,t020\,t110\right.$$

$$- t^2\,t020\,t120 - t^2\,t110\,Unknown4 - t^2\,t120\,Unknown4 - t^2\,Unknown4^2 + P112\,t^2$$

$$- P122\,t^2 - t\,tt10\,Unknown4 - t\,tt20\,Unknown4 + Pt12\,t - t020\,tt10 - t020\,tt20 - Pt22\Big)$$

$$- \frac{1}{t^3}\left(-t^3\,t020\,t110 - t^3\,t020\,t120 - t^3\,t110\,Unknown4 - t^3\,t120\,Unknown4 + P112\,t^3\right.$$

$$- 2\,P122\,t^3 - t\,tt10\,Unknown4 - t\,tt20\,Unknown4 + Pt12\,t - t020\,tt10 - t020\,tt20$$

$$- 2\,Pt22\Big)\,x - \frac{1}{t^4}\left(-t^4\,t020\,t110 - t^4\,t020\,t120 - t^4\,t110\,Unknown4\right.$$

$$- t^4\,t120\,Unknown4 + P112\,t^4 - 3\,P122\,t^4 - t\,tt10\,Unknown4 - t\,tt20\,Unknown4 + Pt12\,t$$

$$- t020\,tt10 - t020\,tt20 - 3\,Pt22\Big)\,x^2 - \frac{1}{t^5}\left(-t^5\,t020\,t110 - t^5\,t020\,t120\right.$$

$$- t^5\,t110\,Unknown4 - t^5\,t120\,Unknown4 + P112\,t^5 - 4\,P122\,t^5 - t\,tt10\,Unknown4$$

$$- t\,tt20\,Unknown4 + Pt12\,t - t020\,tt10 - t020\,tt20 - 4\,Pt22\Big)\,x^3 - \frac{1}{t^6}\left(-t^6\,t020\,t110\right.$$

$$-t^6\,t020\,t120 - t^6\,t110\,Unknown4 - t^6\,t120\,Unknown4 + P112\,t^6 - 5\,P122\,t^6$$
$$- t\,tt10\,Unknown4 - t\,tt20\,Unknown4 + Pt12\,t - t020\,tt10 - t020\,tt20 - 5\,Pt22\big)\,x^4$$
$$+ O(x^5)$$

$$EQ01 := 0$$
$$EQ02 := 0$$
$$EQ03 := 0$$
$$EQ04 := 0$$
$$EQ05 := \frac{-t^2\,t010\,t020 + P022\,t^2}{t^2}$$
$$EQ06 := \frac{-t^2\,t010\,t020 + P022\,t^2}{t^2}$$
$$P022 := t010\,t020$$
$$0$$
$$0$$
$$\frac{\big((t110 + t120 + Unknown3)\,t010 - t020\,Unknown3 + P012\big)\,t + t010\,(tt10 + tt20)}{t}$$
$$\frac{\big((t110 + t120 + Unknown4)\,t020 - t010\,Unknown4 + P012\big)\,t + t020\,(tt10 + tt20)}{t}$$

Studty at 1

```
> series(SpectralCurve(Diag1Sheet1),x=1,2);
  series(SpectralCurve(Diag1Sheet2),x=1,2);
  EQ11:=residue((x-1)^(2)*SpectralCurve(Diag1Sheet1),x=1);
  EQ12:=residue((x-1)^(2)*SpectralCurve(Diag1Sheet2),x=1);
  EQ13:=residue((x-1)^(1)*SpectralCurve(Diag1Sheet1),x=1);
  EQ14:=residue((x-1)^(1)*SpectralCurve(Diag1Sheet2),x=1);
  EQ15:=residue((x-1)^(0)*SpectralCurve(Diag1Sheet1),x=1):
  EQ16:=residue((x-1)^(0)*SpectralCurve(Diag1Sheet2),x=1):
  EQ17:=residue((x-1)^(-1)*SpectralCurve(Diag1Sheet1),x=1):
  EQ18:=residue((x-1)^(-1)*SpectralCurve(Diag1Sheet2),x=1):
  P122:=solve(EQ13,P122);
  simplify(EQ13);
  simplify(EQ14);
  simplify(EQ15);
  simplify(EQ16);
```

$$\frac{t110^2 - (t110 + t120)\,t110 + P122}{(x-1)^2}$$
$$+ \frac{1}{x-1}\Bigg(2\,t110\,Unknown5 - (t110 + t120)\,Unknown5 - \bigg(t010 + t020$$
$$+ \frac{tt10 + tt20}{1-t}\bigg)\,t110 + P112\bigg) + Unknown5^2 - \bigg(t010 + t020 + \frac{tt10 + tt20}{1-t}\bigg)\,Unknown5$$
$$- \bigg(-t010 - t020 + \frac{tt10 + tt20}{(-1+t)\,(1-t)}\bigg)\,t110 + t010\,t020 + P012 + \frac{Pt22}{(-1+t)^2} + \frac{Pt12}{1-t}$$
$$+ O(x-1)$$

(5)

$$\frac{t120^2 - (t110 + t120)\, t120 + P122}{(x-1)^2}$$

$$+ \frac{1}{x-1}\left(2\,t120\,\text{Unknown6} - (t110 + t120)\,\text{Unknown6} - \left(t010 + t020\right.\right.$$

$$\left.+ \frac{tt10 + tt20}{1-t}\right) t120 + P112\Big) + \text{Unknown6}^2 - \left(t010 + t020 + \frac{tt10 + tt20}{1-t}\right)\text{Unknown6}$$

$$- \left(-t010 - t020 + \frac{tt10 + tt20}{(-1+t)(1-t)}\right) t120 + t010\,t020 + P012 + \frac{Pt22}{(-1+t)^2} + \frac{Pt12}{1-t}$$

$$+ O(x-1)$$

$$EQ11 := 0$$
$$EQ12 := 0$$

$$EQ13 := \frac{-t^2\,t110\,t120 + P122\,t^2 + 2\,t\,t110\,t120 - 2\,P122\,t - t110\,t120 + P122}{t^2 - 2\,t + 1}$$

$$EQ14 := \frac{-t^2\,t110\,t120 + P122\,t^2 + 2\,t\,t110\,t120 - 2\,P122\,t - t110\,t120 + P122}{t^2 - 2\,t + 1}$$

$$P122 := t110\,t120$$
$$0$$
$$0$$

$$\frac{1}{-1+t}\left(\left(\left(-t010 - t020 + \text{Unknown5}\right) t + t010 + t020 + tt10 + tt20 - \text{Unknown5}\right) t110 + \left(\right.\right.$$
$$\left.-t120\,\text{Unknown5} + P112\right)(-1+t))$$

$$\frac{1}{-1+t}\left(\left(\left(-t010 - t020 + \text{Unknown6}\right) t + t010 + t020 + tt10 + tt20 - \text{Unknown6}\right) t120 + \left(\right.\right.$$
$$\left.-t110\,\text{Unknown6} + P112\right)(-1+t))$$

Study at lambda=t

```
> series(SpectralCurve(DiagtSheet1),x=t,3);
  series(SpectralCurve(DiagtSheet2),x=t,3);
  EQt1:=residue((x-t)^(2)*SpectralCurve(DiagtSheet1),x=t);
  EQt2:=residue((x-t)^(2)*SpectralCurve(DiagtSheet2),x=t);
  EQt3:=residue((x-t)^(1)*SpectralCurve(DiagtSheet1),x=t);
  EQt4:=residue((x-t)^(1)*SpectralCurve(DiagtSheet2),x=t);
  EQt5:=residue((x-t)^(0)*SpectralCurve(DiagtSheet1),x=t):
  EQt6:=residue((x-t)^(0)*SpectralCurve(DiagtSheet2),x=t):
  EQt7:=residue((x-t)^(-1)*SpectralCurve(DiagtSheet1),x=t):
  EQt8:=residue((x-t)^(-1)*SpectralCurve(DiagtSheet2),x=t):
  Pt22:=factor(solve(EQt3,Pt22));
  simplify(EQt3);
  simplify(EQt4);
  simplify(EQt5);
  simplify(EQt6);
```

$$\frac{tt10^2 - (tt10 + tt20)\, tt10 + Pt22}{(x-t)^2}$$

$$+ \frac{1}{x-t}\left(2\,tt10\,\text{Unknown5} - (tt10 + tt20)\,\text{Unknown5} - \left(\frac{t010 + t020}{t}\right.\right.$$

(6)

$$+ \frac{t110 + t120}{-1 + t} \Bigg) tt10 + Pt12 \Bigg) + Unknown5^2 - \Bigg(\frac{t010 + t020}{t}$$

$$+ \frac{t110 + t120}{-1 + t} \Bigg) Unknown5 - \Bigg(-\frac{t010 + t020}{t^2} - \frac{t110 + t120}{(-1 + t)^2} \Bigg) tt10 + \frac{t010\, t020}{t^2}$$

$$+ \frac{P012}{t} + \frac{t110\, t120}{(-1 + t)^2} + \frac{P112}{-1 + t} + \Bigg(-\Bigg(-\frac{t010 + t020}{t^2} - \frac{t110 + t120}{(-1 + t)^2} \Bigg) Unknown5$$

$$- \Bigg(\frac{t010 + t020}{t^3} + \frac{t110 + t120}{(-1 + t)^3} \Bigg) tt10 - \frac{2\, t010\, t020}{t^3} - \frac{P012}{t^2} - \frac{2\, t110\, t120}{(-1 + t)^3}$$

$$- \frac{P112}{(-1 + t)^2} \Bigg) (x - t) + O\big((x - t)^2 \big)$$

$$\frac{tt20^2 - (tt10 + tt20)\, tt20 + Pt22}{(x - t)^2}$$

$$+ \frac{1}{x - t} \Bigg(2\, tt20\, Unknown6 - (tt10 + tt20)\, Unknown6 - \Bigg(\frac{t010 + t020}{t}$$

$$+ \frac{t110 + t120}{-1 + t} \Bigg) tt20 + Pt12 \Bigg) + Unknown6^2 - \Bigg(\frac{t010 + t020}{t}$$

$$+ \frac{t110 + t120}{-1 + t} \Bigg) Unknown6 - \Bigg(-\frac{t010 + t020}{t^2} - \frac{t110 + t120}{(-1 + t)^2} \Bigg) tt20 + \frac{t010\, t020}{t^2}$$

$$+ \frac{P012}{t} + \frac{t110\, t120}{(-1 + t)^2} + \frac{P112}{-1 + t} + \Bigg(-\Bigg(-\frac{t010 + t020}{t^2} - \frac{t110 + t120}{(-1 + t)^2} \Bigg) Unknown6$$

$$- \Bigg(\frac{t010 + t020}{t^3} + \frac{t110 + t120}{(-1 + t)^3} \Bigg) tt20 - \frac{2\, t010\, t020}{t^3} - \frac{P012}{t^2} - \frac{2\, t110\, t120}{(-1 + t)^3}$$

$$- \frac{P112}{(-1 + t)^2} \Bigg) (x - t) + O\big((x - t)^2 \big)$$

$$EQt1 := 0$$
$$EQt2 := 0$$

$$EQt3 := \frac{-t^4\, tt10\, tt20 + Pt22\, t^4 + 2\, t^3\, tt10\, tt20 - 2\, Pt22\, t^3 - t^2\, tt10\, tt20 + Pt22\, t^2}{t^4 - 2\, t^3 + t^2}$$

$$EQt4 := \frac{-t^4\, tt10\, tt20 + Pt22\, t^4 + 2\, t^3\, tt10\, tt20 - 2\, Pt22\, t^3 - t^2\, tt10\, tt20 + Pt22\, t^2}{t^4 - 2\, t^3 + t^2}$$

$$Pt22 := tt10\, tt20$$
$$0$$
$$0$$

$$\frac{1}{t\, (-1 + t)} \Big((tt10\, Unknown5 - tt20\, Unknown5 + Pt12)\, t^2 + ((-t010 - t020 - t110 - t120$$

$$- Unknown5)\, tt10 + tt20\, Unknown5 - Pt12)\, t + tt10\, (t010 + t020) \Big)$$

$$\frac{1}{t\, (-1 + t)} \Big((-tt10\, Unknown6 + tt20\, Unknown6 + Pt12)\, t^2 + ((-t010 - t020 - t110 - t120$$

$$- Unknown6)\, tt20 + tt10\, Unknown6 - Pt12)\, t + tt20\, (t010 + t020) \Big)$$

Study at infinity

```
> series(SpectralCurve(DiaginftySheet1),x=infinity,3);
  series(SpectralCurve(DiaginftySheet2),x=infinity,3);
  EQinftDiag1Sheet1:=residue(x^(-2)*SpectralCurve(DiaginftySheet1),
  x=infinity);
  EQinfty2:=residue(x^(-2)*SpectralCurve(DiaginftySheet2),x=
```

```
infinity);
EQinfty3:=residue(x^(-1)*SpectralCurve(DiaginftySheet1),x=
infinity);
EQinfty4:=residue(x^(-1)*SpectralCurve(DiaginftySheet2),x=
infinity);
EQinfty5:=residue(x^(0)*SpectralCurve(DiaginftySheet1),x=
infinity);
EQinfty6:=residue(x^(0)*SpectralCurve(DiaginftySheet2),x=
infinity);
EQinfty7:=residue(x^(1)*SpectralCurve(DiaginftySheet1),x=
infinity);
EQinfty8:=residue(x^(1)*SpectralCurve(DiaginftySheet2),x=
infinity);
EQinfty9:=residue(x^(2)*SpectralCurve(DiaginftySheet1),x=
infinity);
EQinftDiag1Sheet10:=residue(x^(2)*SpectralCurve(DiaginftySheet2),
x=infinity);
```

$$\frac{P012 + P112 + Pt12}{x} + \frac{1}{x^2}\left(tinfty10^2 + (t010 + t020 + t110 + t120 + tt10 + tt20)\, tinfty10 \right. \tag{7}$$

$$\left. + t010\, t020 + t110\, t120 + P112 + tt10\, tt20 + Pt12\, t\right) + \mathrm{O}\!\left(\frac{1}{x^3}\right)$$

$$\frac{P012 + P112 + Pt12}{x} + \frac{1}{x^2}\left(tinfty20^2 + (t010 + t020 + t110 + t120 + tt10 + tt20)\, tinfty20 \right.$$

$$\left. + t010\, t020 + t110\, t120 + P112 + tt10\, tt20 + Pt12\, t\right) + \mathrm{O}\!\left(\frac{1}{x^3}\right)$$

$$EQinftDiag1Sheet1 := 0$$
$$EQinfty2 := 0$$
$$EQinfty3 := 0$$
$$EQinfty4 := 0$$
$$EQinfty5 := -P012 - P112 - Pt12$$
$$EQinfty6 := -P012 - P112 - Pt12$$

$$EQinfty7 := -tinfty10^2 - (t010 + t020 + t110 + t120 + tt10 + tt20)\, tinfty10 - t010\, t020$$
$$\quad - t110\, t120 - P112 - tt10\, tt20 - Pt12\, t$$

$$EQinfty8 := -tinfty20^2 - (t010 + t020 + t110 + t120 + tt10 + tt20)\, tinfty20 - t010\, t020$$
$$\quad - t110\, t120 - P112 - tt10\, tt20 - Pt12\, t$$

$$EQinfty9 := 2\, tinfty10\, Unknown + (t010 + t020 + t110 + t120 + tt10 + tt20)\, Unknown$$
$$\quad - (t110 + t120 - (-tt10 - tt20)\, t)\, tinfty10 - 2\, t110\, t120 - P112 - 2\, t\, tt10\, tt20 - Pt12\, t^2$$

$$EQinftDiag1Sheet10 := 2\, tinfty20\, Unknown2 + (t010 + t020 + t110 + t120 + tt10$$
$$\quad + tt20)\, Unknown2 - (t110 + t120 - (-tt10 - tt20)\, t)\, tinfty20 - 2\, t110\, t120 - P112$$
$$\quad - 2\, t\, tt10\, tt20 - Pt12\, t^2$$

```
> CoherenceEquation2:=simplify(-EQinfty5);
  CoherenceEquation1:=CoherenceEquation1;
```

```
factor(simplify(EQinfty7-EQinfty8+(tinfty10-tinfty20)*
CoherenceEquation1));
```
$$CoherenceEquation2 := P012 + P112 + Pt12 \tag{8}$$
$$CoherenceEquation1 := tinfty10 + tinfty20 + t010 + t020 + t110 + t120 + tt10 + tt20$$
$$0$$

```
> CoherenceEquation3:=Pt12*t+t010*t020+t110*t120+tt10*tt20-
  tinfty10*tinfty20+P112;
  CoherenceEquation2 := P012+P112+Pt12;
  CoherenceEquation1 := tinfty10+tinfty20+t010+t020+t110+t120+tt10+
  tt20;
  solve({CoherenceEquation1,CoherenceEquation2},{P012,P112});
```

$$CoherenceEquation3 := Pt12\,t + t010\,t020 + t110\,t120 + tt10\,tt20 - tinfty10\,tinfty20 + P112 \tag{9}$$
$$CoherenceEquation2 := P012 + P112 + Pt12$$
$$CoherenceEquation1 := tinfty10 + tinfty20 + t010 + t020 + t110 + t120 + tt10 + tt20$$

Summary of coefficients and additional coherence relations

```
> P022:=P022;
  P122:=P122;
  Pt22:=Pt22;
  P011:=P011;
  P111:=P111;
  Pt11:=Pt11;
  CoherenceEquation1 := tinftDiag1Sheet10+tt10+tt20+t010+t020+
  tinfty20+t110+t120;
  CoherenceEquation2 := Pt12+P012+P112;
  CoherenceEquation3 := -tinftDiag1Sheet10*tinfty20+tt20*tt10+Pt12*
  t+t010*t020+P112+t120*t110;
  CoherenceEquation3-(P112+t*Pt12+P022+P122+Pt22-tinftDiag1Sheet10*
  tinfty20);
  CoherenceEquation1- (P011+P111+Pt11+ tinftDiag1Sheet10+tinfty20);
  P2:=unapply( P2(lambda),lambda);
  P1:=unapply( P1(lambda),lambda);
```
$$P022 := t010\,t020 \tag{10}$$
$$P122 := t110\,t120$$
$$Pt22 := tt10\,tt20$$
$$P011 := t010 + t020$$
$$P111 := t110 + t120$$
$$Pt11 := tt10 + tt20$$
$$CoherenceEquation1 := tinftDiag1Sheet10 + tt10 + tt20 + t010 + t020 + tinfty20 + t110 + t120$$
$$CoherenceEquation2 := P012 + P112 + Pt12$$
$$CoherenceEquation3 := Pt12\,t + t010\,t020 + t110\,t120 + tt10\,tt20 - tinfty20\,tinftDiag1Sheet10 + P112$$
$$0$$
$$0$$

$$P2 := \lambda \rightarrow \frac{t010\,t020}{\lambda^2} + \frac{P012}{\lambda} + \frac{t110\,t120}{(\lambda-1)^2} + \frac{P112}{\lambda-1} + \frac{tt10\,tt20}{(\lambda-t)^2} + \frac{Pt12}{\lambda-t}$$

$$P1 := \lambda \rightarrow \frac{t010+t020}{\lambda} + \frac{t110+t120}{\lambda-1} + \frac{tt10+tt20}{\lambda-t}$$

We have 3 undetermined coefficients: P012, P112 and Pt12 but they are related by 2 coherence equations: sum of monodromies is null and

P112+t*Pt12+P022+P122+Pt22-tinfty10*tinfty20=0