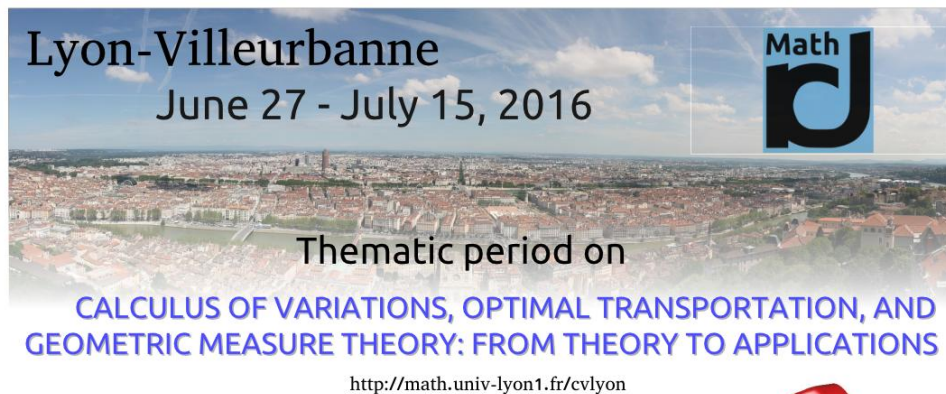


THEMATIC PERIOD

"Calculus of Variations, Optimal Transportation, and Geometric Measure Theory: from Theory to Applications"

Week 2 July 4 – 8, 2016
International Conference
Program



Lyon-Villeurbanne
June 27 - July 15, 2016

Math UJL

Thematic period on
CALCULUS OF VARIATIONS, OPTIMAL TRANSPORTATION, AND GEOMETRIC MEASURE THEORY: FROM THEORY TO APPLICATIONS

<http://math.univ-lyon1.fr/cvlyon>

Speakers

Week 2 : July 4 - July 8

Giovanni Alberti
Jean-François Aujol
Giovanni Bellettini
Virginie Bonnaille-Noël
Guy Bouchitté
Blaise Bourdin
Lia Bronsard
Michael Bronstein
Almut Burchard
Daniel Cremers
Qiang Du
Selim Esedoglu
Ilaria Fragalà
Adriana Garroni
Young-Heon Kim
Jacques-Olivier Lachaud
Francesco Maggi
Maks Ovsjanikov
Manuel Ritoré
Dejan Slepcev
Jeremy Tyson
Bozhidar Velichkov
Max Wardetsky
Stefan Wenger
Benedikt Wirth



Mini courses

Week 1 : June 27 - July 1

Dorin Bucur

Shape optimization of spectral functionals

Guido De Philippis

The selection principle: the use of regularity theory in proving quantitative inequalities

Filippo Santambrogio

Optimal transport, optimal curves, optimal flows

Week 3 : July 11 - July 15

Daniel Cremers

Variational Methods for Computer Vision

Jérôme Darbon

On Optimization Algorithms in Imaging Sciences and Hamilton-Jacobi equations

Quentin Mérigot

Computational optimal transport

Program at a Glance

Monday, July 4

12.30 *Registration*
1.15 *Opening*
1.30 - 2.30 Francesco Maggi
2.30 - 3.30 Lia Bronsard
3.30 - 4.00 *Coffee break*
4.00 - 5.00 Young-Heon Kim
5.00 - 6.00 Benedikt Wirth
6.15 - 6.45 PhD/Postdoc Seminar

Tuesday, July 5

9.00 - 10.00 Qiang Du
10.00 - 11.00 Blaise Bourdin
11.00 - 11.30 *Coffee break*
11.30 - 12.30 Selim Esedoğlu
12.30 - 2.00 *Lunch*
2.00 - 3.00 Stefan Wenger
3.00 - 4.00 Jeremy Tyson
4.00 - 4.30 *Coffee break*
4.30 - 5.30 Manuel Ritoré
5.45 - 6.15 PhD/Postdoc Seminar
7.30 *Dinner at Brasserie
des Confluences*

Wednesday, July 6

9.00 - 10.00 Giovanni Bellettini
10.00 - 11.00 ~~Adriana Garroni (Cancelled)~~
10.00 - 11.00 Jacques-Olivier Lachaud
11.00 - 11.30 *Coffee break*
11.30 - 12.30 Guy Bouchitté
12.30 - 2.00 *Lunch*
2.00 - 3.00 Dejan Slepcevic
3.00 - 4.00 Maks Ovsjanikov
4.00 - 4.30 *Coffee break*
4.30 - 5.00 PhD/Postdoc Seminar

Thursday, July 7

9.00 - 10.00 Jean-François Aujol
10.00 - 11.00 Max Wardetsky
11.00 - 11.30 *Coffee break*
11.30 - 12.30 Iliara Fragalà
12.30 - 2.00 *Lunch*
2.00 - 3.00 Almut Burchard
3.00 - 4.00 Bozhidar Velichkov
4.00 - 4.30 *Coffee break*
4.30 - 5.30 Michael Bronstein
6.30 - 8.30 *Social event (guided tours)*

Friday, July 8

9.00 - 10.00 Giovanni Alberti
10.00 - 11.00 Virginie Bonnaillie-Noël
11.00 - 11.30 *Coffee break*
11.30 - 12.30 Daniel Cremers
12.30 - 2.00 *Lunch*
End of Conference

Location : Jordan Conference Hall, Braconnier Building, La Doua, Université Lyon1

Daily Program

Monday, July 4

1.30 - 2.30 **Francesco Maggi,**

Abdus Salam International Center for Theoretical Physics, Trieste, Italy,

Quantitative isoperimetric principles and applications to phase transitions

2.30 - 3.30 **Lia Bronsard,**

McMaster University, Canada,

Minimizers of the Landau-de Gennes energy around a spherical colloid particle

3.30 - 4.00 *Coffee break*

4.00 - 5.00 **Young-Heon Kim,**

University of British Columbia, Canada,

Optimal martingale transport in general dimensions

5.00 - 6.00 **Benedikt Wirth,**

Universität Münster, Germany,

Optimal design of transport networks

6.15 - 6.45 **PhD/Postdoc Seminar**

Alessandra Pluda (*Università di Pisa, Italy*)

Evolution by curvature of networks in the plane

Daily Program

Tuesday, July 5

9.00 - 10.00 **Qiang Du**,
Columbia University in the City of New York, USA,
Calculus of variations of some nonlocal problems

10.00 - 11.00 **Blaise Bourdin**,
Lousiana State University, USA,
Variational vs. phase field models of fracture

11.00 - 11.30 *Coffee break*

11.30 - 12.30 **Selim Esedoğlu**,
University of Michigan, USA,
Algorithms for anisotropic mean curvature flow of networks

2.00 - 3.00 **Stefan Wenger**,
University of Fribourg, Switzerland,
Area minimizing discs in metric spaces and applications

3.00 - 4.00 **Jeremy Tyson**,
University of Illinois at Urbana-Champaign, USA,
Densities of measures and the geometry of submanifolds in the Heisenberg group

4.00 - 4.30 *Coffee break*

4.30 - 5.30 **Manuel Ritoré**,
Universidad de Granada, Spain,
Isoperimetric inequalities in unbounded convex bodies

5.45 - 6.15 **PhD/Postdoc Seminar**
Harrison Pugh (*Stony Brook University, USA*)
The Elliptic Plateau Problem

7.30 *Dinner at Brasserie des Confluences*

Daily Program

Wednesday July 6

9.00 - 10.00 **Giovanni Bellettini**,
Università di Roma "Tor Vergata", Italy,
Constrained BV functions on covering spaces and Plateau's type problems

10.00 - 11.00 **Adriana Garroni (Cancelled)**,
Università di Roma "La Sapienza", Italy,
Line tension for dislocations and crystal plasticity

10.00 - 11.00 **Jacques-Olivier Lachaud**,
Université de Savoie, France,
Convergent geometric estimators with digital volume and surface integrals

11.00 - 11.30 *Coffee break*

11.30 - 12.30 **Guy Bouchitté**,
Université du Sud-Toulon-Var, France,
A duality theory for non-convex problems in the Calculus of Variations

2.00 - 3.00 **Dejan Slepcev**,
Carnegie Mellon University, USA,
Variational problems on graphs and their continuum limits

3.00 - 4.00 **Maks Ovsjanikov**,
École Polytechnique, France,
Functional Characterization of Shapes and their Relations

4.00 - 4.30 *Coffee break*

4.30 - 5.00 **PhD/Postdoc Seminar**
Johannes Persch (*TU Kaiserslautern, Germany*)
Parallel Douglas Rachford Algorithm for Restoring Images with Values
in Symmetric Hadamard Manifolds

Daily Program

Thursday July 7

9.00 - 10.00 **Jean-François Aujol**,
Université de Bordeaux, France,
Image colorization by a variational approach

10.00 - 11.00 **Max Wardetsky**,
Georg-August-Universität Göttingen, Germany,
Variational Convergence of Minimal Surfaces

11.00 - 11.30 *Coffee break*

11.30 - 12.30 **Ilaria Fragalà**,
Politecnico di Milano, Italy,
Boundary value problems for the infinity Laplacian: regularity and
geometric results

2.00 - 3.00 **Almut Burchard**,
University of Toronto, Canada,
Extremals of the Polya-Szego inequality

3.00 - 4.00 **Bozhidar Velichkov**,
Université Grenoble Alpes, France,
Lipschitz regularity for quasi-minimizers and applications to some shape
optimization problems

4.00 - 4.30 *Coffee break*

4.30 - 5.30 **Michael Bronstein**,
Università della Svizzera Italiana, Switzerland,
Partial functional maps

6.30 - 8.30 *Social event: Guided tours*

Daily Program

Friday July 8

- 9.00 - 10.00 **Giovanni Alberti**,
Università di Pisa, Italy,
Structure of the boundary of integral currents and Frobenius theorem
- 10.00 - 11.00 **Virginie Bonnaille-Noël**,
École Normale Supérieure de Paris, France,
Minimal k-partition for the p-norm of the eigenvalues
- 11.00 - 11.30 *Coffee break*
- 11.30 - 12.30 **Daniel Cremers**,
Technische Universität München, Germany,
Sublabel Accurate Relaxation of Nonconvex Energies
- 12.30 - 2.00 *Lunch and End of Conference*
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Plenary Talks – Abstracts

Giovanni Alberti (*Università di Pisa, Italy*)

Structure of the boundary of integral currents and Frobenius theorem

Abstract

According to Frobenius theorem, a non-involutive distribution of k -dimensional planes in \mathbb{R}^n cannot be tangent to any (regular) k -dimensional surface S . In recent years, this result has been extended to weaker notions of surface, and in this talk I will show that it holds also if S is an integral current, or even a normal current of certain type (and yet it does not hold for all normal and rectifiable currents). This extension relies on a seemingly harmless property of the boundary of integral currents, which generalizes an obvious property of smooth surfaces, namely that the tangent space to the boundary is contained in the tangent space to the surface. Note that these results are strictly related to the problem of decomposing a normal currents in terms of rectifiable (or integral) ones. These results are part of an ongoing project with A. Massaccesi (University of Zurich) and E. Stepanov (Steklov Institute of Mathematics, St. Petersburg).

Jean-François Aujol (*Université de Bordeaux, France*)

Image colorization by a variational approach

Abstract

This work provides a new method to colorize gray-scale images. While the reverse operation is only a matter of standard, the colorization process is an ill-posed problem that requires some priors. In the literature two classes of approach exist. The first class includes manual methods that needs the user to manually add colors on the image to colorize. The second class includes exemplar-based approaches where a color image, with a similar semantic content, is provided as input to the method. These two types of priors have their own advantages and drawbacks. In this work, a new variational framework for exemplar-based colorization is proposed. A non-local approach is used to find relevant color in the source image in order to suggest colors on the gray-scale image. The spatial coherency of the result as well as the final color selection is provided by a non-convex variational framework based on a total variation. An efficient primal-dual algorithm is provided and a proof of its convergence is proposed. In this work, we also extend the proposed exemplar-based approach to combine both exemplar-based and manual methods. It provides a single framework that unifies advantages of both approaches. Finally, experiments and comparisons with state-of-the-art methods illustrate the efficiency of our method.

Giovanni Bellettini (*Università di Roma "Tor Vergata", Italy*)

Constrained BV functions on covering spaces and Plateau's type problems

Abstract

We link covering spaces with the theory of functions of bounded variation, in order to study Plateau's type problem without fixing a priori the topology of solutions. We solve the minimization problem in the class of BV functions defined on a covering space of the complement of an $(n-2)$ -dimensional manifold S without boundary, and satisfying a suitable constraint on the fibers. The model avoids all issues concerning the presence of the boundary S .

Virginie Bonnaille-Noël (*École Normale Supérieure de Paris, France*)

Minimal k -partition for the p -norm of the eigenvalues

Abstract

In this talk, we are interested in the analysis of minimal partitions and their relations with nodal domains of eigenfunctions for suitable operators. For any simply connected domain $\Omega \subset \mathbb{R}^2$, the optimization problem considered here consists in minimizing among the k -partitions of Ω the energy

$$\Lambda(\mathcal{D}) = \max\{\lambda(D_j), 1 \leq j \leq k\},$$

where $\mathcal{D} = (D_1, \dots, D_k)$ is a k -partition of Ω and $\lambda(D_j)$ denotes the first eigenvalue of the Dirichlet-Laplacian on D_j .

For this problem, we present some theoretical and numerical results and exhibit candidates to be minimal.

A generalization to p -minimal k -partition where the ℓ^∞ norm defining $\Lambda(\mathcal{D})$ is replaced by the ℓ^p norm is also considered from a numerical point of view.

Guy Bouchitté (*Université du Sud-Toulon-Var, France*)

A duality theory for non-convex problems in the Calculus of Variations

Abstract

In this talk I will present a duality theory for problems of the kind

$$J(\Omega, \mu) := \inf \left\{ \int_{\Omega} (f(\nabla u) + g(u)) dx - \int_{\Omega} u d\mu, u = 0 \text{ on } \partial\Omega \right\}$$

where g, γ are possibly non convex functions with suitable growth conditions and f is a convex intergrand on \mathbb{R}^d . Our aim is to characterize global minimizers of such a problem and study the stability of the minimal value with respect to small variations of the domain Ω or of the source term μ .

Our duality scheme is formulated in $\Omega \times \mathbb{R}$ on which the dual problem consists in maximimizing a transport flow under suitable convex constraints. Applications with numerical issues will be given for a class of free boundary problems. Joint work with Ilaria Fragala (Politecnico di Milano- Italy) and Minh Phan (Toulon)

Blaise Bourdin (*Lousiana State University, USA*)

Variational vs. phase field models of fracture

Abstract

Twenty years after its inception, the variational approach to brittle fracture, and its "phase-field" implementation have become widely accepted. In this talk, I will first recall the construction of the variational models of fracture and their approximation by Gamma-convergence, the well-known Ambrosio-Tortorelli functionals. I will discuss the strength and flaws of these approaches. Then, I will focus on recent results dealing with the analysis of Ambrosio-Tortorelli functionals when the regularization parameter is held constant. I will show how such models can be used to properly handle propagation and nucleation of cracks, even in situations where global minimization and Gamma-convergence lead to unphysical evolutions. I will illustrate the properties of these models with validation and verification numerical simulations in multiples situations.

Lia Bronsard (*McMaster University, Canada*)

*Minimizers of the Landau-de Gennes energy
around a spherical colloid particle*

Abstract

We consider energy minimizing configurations of a nematic liquid crystal around a spherical colloid particle, in the context of the Landau-de Gennes model. The nematic is assumed to occupy the exterior of a ball, and satisfy homeotropic weak anchoring at the surface of the colloid and approach a uniform uniaxial state far from the colloid. We study the minimizers in two different limiting regimes: for balls which are small compared to the characteristic length scale, and for large balls. The relationship between the radius and the anchoring strength is also relevant. For small balls we obtain a limiting quadrupolar configuration, with a “Saturn ring” defect for relatively strong anchoring, corresponding to an exchange of eigenvalues of the Q-tensor. In the limit of very large balls we obtain an axisymmetric minimizer of the Oseen–Frank energy, and a dipole configuration with exactly one point defect is obtained.

Michael Bronstein (*Università della Svizzera Italiana, Switzerland*)

Partial functional maps

Abstract

In this talk, I will show some recent results on computing functional correspondence between deformable shapes. Our main result is a perturbation analysis of the Laplacian operator, giving a bound on the change in its eigenvectors as a result of part removal. Based on this observation, I will show a generalization of functional maps capable of dealing with partial correspondence. Our method achieves state-of-the-art results in some of the most challenging correspondence settings.

Almut Burchard (*University of Toronto, Canada*)

Extremals of the Polya-Szego inequality

Abstract

The Polya-Szego inequality states that the p-norms of the gradient generally decrease under symmetric decreasing rearrangement. It is known that there are non-trivial cases of equality, even when $p > 1$. I will use Ryff’s polar factorization to describe these equality cases. (Based on joint work with A. Ferone).

Daniel Cremers (*Technische Universität München, Germany*)
Sublabel Accurate Relaxation of Nonconvex Energies

Abstract

Functional lifting and convex relaxation has become a popular framework to tackle a multitude of non-convex optimization problems in computer vision, image analysis and beyond. The key idea is to circumvent the non-convexity of the original problem by embedding it in a higher dimension. While this technique leads to optimal solutions for a certain class of non-convex problems it comes at the price of a substantial increase in memory and runtime. In this presentation, I will show how the embedding can be performed more efficiently. To this end, I will show that the existing lifting techniques of Ishikawa (in the discrete setting) and Pock et al (in the continuous setting) amount to piecewise linear approximations of the non-convex data term. Instead, we propose a piecewise convex approximation of the data term which is the tightest possible in a local sense and which can be optimized efficiently using provably convergent primal-dual algorithms.

Qiang Du (*Columbia University in the City of New York, USA*)
Calculus of variations of some nonlocal problems

Abstract

Recent development of nonlocal vector calculus and nonlocal calculus of variations provides a systematic mathematical framework for the analysis of nonlocal continuum models given in the form of partial-integral equations. In this lecture, we consider a few nonlocal models from various applications. These include examples such as the nonlocal Allen-Cahn equations and nonlocal phase field crystal models as well as nonlocal mechanics models with heterogeneous localization. A particular focus is on connections with their local limits given by traditional local PDEs.

Selim Esedođlu (*University of Michigan, USA*)

Algorithms for anisotropic mean curvature flow of networks

Abstract

Threshold dynamics is an algorithm for moving an interface (e.g. a surface in 3D) by mean curvature motion. It was proposed by Merriman, Bence, and Osher in 1989, and also extended to networks of surfaces in the same paper. This dynamics arises as gradient flow for the sum of the areas of the surfaces in the network, and plays a prominent role in materials science applications where it describes the motion of grain boundaries in polycrystals (such as most metals) under heat treatment. It also arises in computer vision, in certain versions of the Mumford-Shah model for image segmentation, and machine learning (clustering on graphs).

Further extension of the algorithm to weighted mean curvature flow of networks, where the surface tension of each interface in the network may be different and may depend on the direction of the normal, is of great interest for applications, but has remained elusive. In fact, even the simpler case where the tension of each surface in the network is isotropic but may be weighted by a different constant hadn't been addressed until recently. We describe how to extend threshold dynamics, first to unequal but constant (isotropic) surface tensions, and then to unequal and anisotropic (normal dependent) surface tensions. Based on joint works with Felix Otto, Matt Elsey, and Matt Jacobs.

Ilaria Fragalà (*Politecnico di Milano, Italy*)

*Boundary value problems for the infinity Laplacian:
regularity and geometric results*

Abstract

We discuss regularity and geometric results for boundary value problems where the operator is the infinity Laplacian, or its normalized version. In particular, we focus our attention on the homogeneous Dirichlet problem with constant source term, and on a related Serrin-type overdetermined problem. The talk is based on some recent joint works with Graziano Crasta, University of Roma "La Sapienza".

Adriana Garroni (*Università di Roma "La Sapienza", Italy*)
Line tension for dislocations and crystal plasticity

Abstract

Dislocations are line defects in the crystalline structure of metals. They are considered the main mechanism for plastic deformation and their interaction at the microscopic scale are relevant to understand complex phenomena as, yielding and hardening.

I will present variational model of energies concentrated on lines and its derivation, in terms of Gamma-convergence. Depending on the energy scaling one can obtain a line tension energy associated to dislocations in 3D or a gradient theory for plasticity. The rigorous analysis shows that in some cases the effective models may reveal the presence of relaxation and formation of microstructure.

Young-Heon Kim (*University of British Columbia, Canada*)
Optimal martingale transport in general dimensions

Abstract

We discuss the optimal solutions to a transport problem where mass has to move under martingale constraint; this constraint forces the transport to split the mass. This problem was originated from mathematical finance, e.g. option pricing. There have been intensive studies on the one-dimensional case, but, rarely in higher dimensions. We present structural results in general dimensions. This is a joint work with Nassif Ghoussoub and Tongseok Lim.

Jacques-Olivier Lachaud (*Université de Savoie, France*)
*Convergent geometric estimators
with digital volume and surface integrals*

Abstract

This talk presents several methods to estimate geometric quantities on subsets of the digital space \mathbb{Z}^d . We take an interest both on global geometric quantities like volume and area, and on local geometric quantities like normal and curvatures. All presented methods have the common property to be multigrid convergent, i.e. the estimated quantities tend to their Euclidean counterpart on finer and finer digitizations of (smooth enough) Euclidean shapes. Furthermore, all methods rely on digital integrals, which approach either volume integrals or surface integrals along shape boundary. With such tools, we achieve multigrid convergent estimators of volume, moments and area in \mathbb{Z}^d , of normals, curvature and curvature tensor in \mathbb{Z}^2 and \mathbb{Z}^3 , and of covariance measure and normals in \mathbb{Z}^d even with Hausdorff noise.

Francesco Maggi (*ICTP, Trieste, Italy*)

Quantitative isoperimetric principles and applications to phase transitions

Abstract

We introduce some new sharp stability theorems for Almgren's isoperimetric principle and for the Euclidean concentration inequality that are motivated by the study of critical points of the Gauss free energy and of near-minimizers of the Gates-Penrose-Lebowitz free energy. The talk is based on joint works with Eric Carlen (Rutgers U), Giulio Ciraolo (U Palermo), Alessio Figalli (UT Austin), Brian Krummel (UT Austin) and Connor Mooney (UT Austin).

Maks Ovsjanikov (*École Polytechnique, France*)

Functional Characterization of Shapes and their Relations

Abstract

In recent years, several works have proposed to look at basic constructs in geometry processing from a "functional" point of view, by representing them as linear operators acting on real-valued functions defined on the shapes. In this talk I will describe what these representations entail for mappings or correspondences, tangent vector fields and shape distortions. Finally, I will describe how surfaces themselves can be represented and manipulated in a coordinate-free fashion via a functional characterization of the first and second fundamental forms.

Manuel Ritoré (*Universidad de Granada, Spain*)

Isoperimetric inequalities in unbounded convex bodies

Abstract

In this talk I shall consider the problem of minimizing the relative perimeter under a volume constraint in an unbounded convex body C in Euclidean space, without assuming any further regularity on the boundary of C . Motivated by an example of an unbounded convex body with null isoperimetric profile, we introduce the concept of unbounded convex body with uniform geometry and the related notion of asymptotic cylinder. I shall describe how to prove existence of isoperimetric regions in a generalized sense, the strict concavity of the isoperimetric profile and the connectedness of generalized isoperimetric regions. We also focus on the cases of small as well as of large volumes; in particular we show existence of isoperimetric regions with sufficiently large volumes, for special classes of unbounded convex bodies. Time permits, I shall address some questions about isoperimetric rigidity and analyze the asymptotic behavior of the isoperimetric profile in connection with the notion of isoperimetric dimension.

Dejan Slepcev (*Carnegie Mellon University, USA*)

Variational problems on graphs and their continuum limits

Abstract

We will discuss variational problems arising in machine learning and their limits as the number of data points goes to infinity. Consider point clouds obtained as random samples of an underlying "ground-truth" measure. Graph representing the point cloud is obtained by assigning weights to edges based on the distance between the points. Many machine learning tasks, such as clustering and classification, can be posed as minimizing functionals on such graphs. We consider functionals involving graph cuts and their limits. The question is considered in the setting of Gamma convergence. The Gamma limit, and associated compactness property, are considered with respect to a topology which uses optimal transportation to suitably compare functions defined on graphs with functions defined with respect to the continuum ground-truth measure. Taking the Gamma limit relies on connecting the graph cuts with the nonlocal continuum perimeter. The talk is primarily based on joint work with Nicolas Garcia Trillos, as well as on works with Xavier Bresson, Moritz Gerlach, Matthias Hein, Thomas Laurent, and James von Brecht.

Jeremy Tyson (*University of Illinois at Urbana-Champaign, USA*)

*Densities of measures and the geometry of submanifolds
in the Heisenberg group*

Abstract

The Marstrand and Preiss density theorems are fundamental results in the geometric theory of measures and rectifiability in Euclidean space. In this talk I will describe an ongoing measure-theoretic study of densities in the Heisenberg group \mathbb{H}^n equipped with a sub-Riemannian metric. We establish Marstrand's Density Theorem for the Korányi (gauge) metric d_H on \mathbb{H}^n via an analysis of uniform and uniformly distributed measures. Uniform measures feature prominently in the classical proof of the Preiss Density Theorem. In Euclidean space, uniform measures are classified in low dimension and low codimension; the full classification remains an open problem. I will discuss an ongoing project aimed at classifying uniform measures in (\mathbb{H}^1, d_H) . Following an approach of Kowalski and Preiss, we analyze the structure of uniform measures via an asymptotic formula for the volume of small extrinsic balls on smooth submanifolds. Our formulas, which involve several intrinsic notions of sub-Riemannian curvature for submanifolds, have other applications to uniform rectifiability. I will give a brief overview of relevant aspects of the geometry of submanifolds in sub-Riemannian spaces. This talk is based on joint work with Vasilis Chousionis and Valentino Magnani.

Bozhidar Velichkov (*Université Grenoble Alpes, France*)

*Lipschitz regularity for quasi-minimizers
and applications to some shape optimization problems*

Abstract

We consider the variational shape optimization problem of the minimization of the sum of the first k Dirichlet eigenvalues of a variable set Ω under the volume constraint $|\Omega| = 1$,

$$\min \left\{ \sum_{j=1}^k \lambda_j(\Omega) : \Omega \subset \mathbb{R}^d, |\Omega| = 1 \right\}.$$

We will prove that the free boundary of the optimal set is $C^{1,\alpha}$ regular up to a set of zero $(d-1)$ -Hausdorff measure. Since the optimal set is a solution of a free boundary problem of Alt-Caffarelli type involving vector valued functions, we will dedicate most of our attention to the study of the free boundary $\partial\{|U| > 0\}$ of the local minimizers $U : \mathbb{R}^d \rightarrow \mathbb{R}^k$ of the functional

$$H_{loc}^1(\mathbb{R}^d; \mathbb{R}^k) \ni U \mapsto \int |\nabla U|^2 dx + |\{|U| > 0\}|.$$

Max Wardetsky (*Georg-August-Universität Göttingen, Germany*)

Variational Convergence of Minimal Surfaces

Abstract

While discrete minimal surfaces are perhaps one of the most widely studied examples of discrete surfaces in Discrete Differential Geometry, their convergence to smooth minimal surfaces has only been proven for special cases, such as for disk-like and cylinder-like topologies. Using tools from variational analysis, I will present a convergence result for triangulated area-minimizing surfaces that deals with the general case of arbitrary topology.

Stefan Wenger (*University of Fribourg, Switzerland*)
Area minimizing discs in metric spaces and applications

Abstract

I will first discuss a solution of the classical problem of Plateau in the setting of metric spaces. I will then outline some applications of this solution, in particular to the problem of finding good parametrizations of metric planes, that is, metric spaces homeomorphic to the Euclidean plane with some additional metric properties. Based on joint work with Alexander Lytchak.

Benedikt Wirth (*Universität Münster, Germany*)
Optimal design of transport networks

Abstract

Several applications in biology and engineering are concerned with the optimal transportation of substances from source locations to sink locations. As opposed to classical optimal transport, models for transport networks take into account that it is more efficient to transport material in bulk. The resulting optimal transport networks typically have a branching structure. We discuss different model formulations and their equivalence as well as the geometry of (almost) optimal networks, which can be analyzed by proving energy scaling laws in the regime of small preference for bulk transport.

PhD/Postdoc Seminar – Abstracts

MONDAY, JULY 4: **Alessandra Pluda** (*Università di Pisa, Italy*)

Evolution by curvature of networks in the plane

Abstract

I will consider the motion by curvature of networks of curves in the plane, and discuss in particular on the existence of solutions (in a strong sense) and singularity formation, presenting some new results and recent developments obtained in collaboration with Carlo Mantegazza, Matteo Novaga and Felix Schulze.

TUESDAY, JULY 5: **Harrison Pugh** (*Stony Brook University, USA*)

The Elliptic Plateau Problem

Abstract

We show how the techniques of Reifenberg can be generalized to prove the existence of minimizers for the axiomatic elliptic Plateau problem in an ambient manifold. We will discuss two crucial ingredients; an isoperimetric inequality for arbitrary codimension, and a Federer-Fleming projection-type theorem modified to remove unrectifiable pieces from a candidate minimizer.

WEDNESDAY, JULY 6: **Johannes Persch** (*TU Kaiserslautern, Germany*)

Parallel Douglas Rachford Algorithm for Restoring Images with Values in Symmetric Hadamard Manifolds

Abstract

The talk addresses a generalization of the Douglas-Rachford algorithm to symmetric Hadamard manifolds. It can be used to minimize an anisotropic TV functional for images having values on these manifolds. We derive an parallel DR algorithm, that can be evaluated fast. Convergence of the algorithm to a fixed point is proofed for spaces with constant curvature. Several numerical examples show its beneficial performance when compared with the cyclic proximal point algorithm or half-quadratic minimization.

Participants

- Alberti Giovanni (University of Pisa, Italy)
- Aujol Jean-Francois (Université de Bordeaux, France)
- Bellettini Giovanni (Università di Roma "Tor Vergata", Italy)
- Bonafini Mauro (University of Trento, Italy)
- Bonnaillie-Noël Virginie (CNRS, France)
- Bonneel Nicolas (CNRS, France)
- Bouchitté Guy (Université du Sud-Toulon-Var, France)
- Bourdin Blaise (Louisiana State University, USA)
- Bowles Malcolm (University of British Columbia, Canada)
- Bretin elie (INSA Lyon, France)
- Bronsard Lia (McMaster University, Canada)
- Bronstein Michael (USI Lugano / Intel, Switzerland)
- Buet Blanche (Université Paris Sud, France)
- Burchard Almut (University of Toronto, Canada)
- Caissard Thomas (LIRIS, France)
- Carioni Marcello (Max Planck Institute ? Leipzig, Germany)
- Cavallotto Edoardo (Université Paris Saclay, France)
- Coeurjolly David (CNRS/LIRIS, France)
- Coursin Thomas (IUT Paris Sud, France)
- Cremers Daniel (TU Munich, Germany)
- Dayrens François (ICJ/Lyon 1, France)
- Dekeyser Justin (Université catholique de Louvain, Belgium)
- Del Nin Giacomo (University of Pisa, Italy)
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