First. Luca's question.

V vector space of dim n

Duality between

SV and SV^* ?

Lemma For any k

$$(S^k V)^* = S^k V^*$$

V^* basis $x_1, \cdots, x_n$

$$SV^* \simeq \mathbb{C}[x_1, \ldots, x_n]$$

Dual basis of V is

$$\frac{\partial}{\partial x_1} \cdots \frac{\partial}{\partial x_n} .$$

$\varphi \in S^k V^*$ $\quad k \in \mathbb{Z}_{\geq 0}$

$\Theta \in S^k V$

$\Theta \in \mathbb{C}[\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n}]$

$\langle \Theta | \varphi \rangle = \Theta \cdot \varphi \in \mathbb{C}$

$\Theta\varphi$ is polynomial of degree 0, $\Theta \cdot \varphi$ is a scalar.

Basis of $S^k V^*$

$x^\alpha = x_1^{\alpha_1} \cdots x_n^{\alpha_n}$ with

$\alpha_1 + \cdots + \alpha_n = k$

Basis of $S^k V$ is

$\partial^\alpha = (\frac{\partial}{\partial x_1})^{\alpha_1} \cdots (\frac{\partial}{\partial x_n})^{\alpha_n}$

$\langle \partial^\alpha | x^\beta \rangle = 0$ if $\alpha \neq \beta$

$$\langle \partial^{\alpha} | x^{\alpha} \rangle = \alpha_1! \dots \alpha_n!$$

Correct ground field has char. 0.

EXO $(\Lambda V)^* = \Lambda V^*$

even on a field of ch. p.

EXO2 $(S^k V)^*$ over $\overline{\mathbb{F}}_p = K$

Duality over $\overline{\mathbb{F}}_p$ is between $K[x_1, \dots, x_n]$ and the span of $\frac{\cancel{\partial}^{\alpha_1}}{\partial x_1}$

$$\frac{1}{\alpha_1!} \cdots \frac{1}{\alpha_n!} \left(\frac{\partial}{\partial x_1}^{\alpha_1} \cdots \frac{\partial}{\partial x_n}^{\alpha_n} \right)$$

$n=1$ duality

$$K[x] \longleftrightarrow K\left[\frac{d}{dx}, \left(\frac{d}{dx}\right)^{(p)}, \left(\frac{d}{dx}\right)^{(p^2)} \dots \right]$$

$K = \mathbb{C}$

$SV \longleftrightarrow SV^*$

$I \subset SV^*$ is an homogenous (graded) ideal, then $I^\perp$ is a subcoalgebra in SV.

coalgebra = cogèbre.

S^+V = all polynomial with trivial 0-component

[Same as $(SV)^+$

Duality between

Harmonic polynomial in SV^*

and the algebra

$SV / ((S^+V)^G \cdot SV)$

Harmonic polynomial

= dual of $(S^+V^G)\, SV$.

Hello!