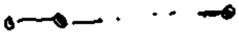
Forum du 9 avril

Classification of simple Lie algebras.

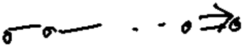
Dynkin diagrams

A_n $\underline{\mathrm{sl}}_{n+1}$

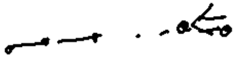


n vertices

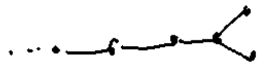
B_n $\mathrm{so}(2n+1)$



C_n $\mathrm{sp}(2n)$



D_n $\mathrm{so}(2n)$



$+$ 5 exceptional

$so(4) \cong sl(2) \oplus sl(2)$

M_2 = 2×2 matrices

Action of $SL(2) \times SL(2)$

$\det : M_2 \longrightarrow \mathbb{C}$

det is a quadratique form

$SL(2) \times SL(2) \longrightarrow SO(4)$

$$\underbrace{SL(2) \times SL(2)}_{\dim : 6} / \pm 1 \cong \underbrace{SO(4)}_{\dim : 4 \times 3/2 = 6}$$

$SO(3) \cong SL(2)/\pm 1.$

$SL(2)$ acts over $\underline{sl}(2)$

$\underline{sl}(2) = \{ A \in M_2 \mid \operatorname{tr} A = 0 \}$

$SL(2)$ by conjugaison

$\operatorname{tr} B^{-1} A B = \operatorname{tr} A$

sl(2) has a quadratic form. Killing form

$$A, B \longmapsto \operatorname{tr}(AB)$$

~~$A = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$~~

$$A = \begin{pmatrix} \lambda & 0 \\ 0 & -\lambda \end{pmatrix}$$

$$\operatorname{tr} A^2 = 2\lambda^2 = -2 \det A$$

SL(2) preserves the quadratic form on $\underline{sl}(2)$

$$\underline{sl}(2) \cong \mathbb{C}^3$$

$$SL(2)/\pm 1 \xrightarrow{\sim} SO(3)$$

$$\underline{sl}(2) \cong \underline{so}(3)$$

$\mathfrak{sl}(2)$ acting on $\mathbb{C}[x_1, \dots x_n]$

$$\left\{\begin{array}{l} \Delta = \text{Laplace operator} \\ x_1^2 + \dots + x_n^2 = \text{multiplication by} \\ h = [\Delta,\ x_1^2 + \dots + x_n^2] \end{array}\right.$$

→ closed by bracket it is (e, f, h)