University of Illinois at Urbana-Champaign — Spring 2007
Math 181 — Group F1

Quiz 7

Friday, April 6th.

NAME ____

1. (a) A committee has four voters, with the system [30: 20, 17, 10, 5]. Which voters are critical in the winning coalition $\{A,C,D\}$?
Answer. The coalition has weight 35, so it has 5 extra votes; thus A and C are the critical voters here.
(b) Same question with the system [8:5,2,2,2,2,2] and the same coalition.
Answer. This time the coalition has weight 9, so it has 1 extra vote, thus all voters are critical.

2. In the following weighted voting systems, say which voters (if any) have veto power.
(a) [30: 20, 17, 10, 5].
Answer. A voter has veto power exactly when the sum of the weights of the other voters is less than the quota. Thus in this case no one has veto power.
(b) [38: 20, 15, 12, 5].
Answer. For the same reason, we see that in this case both A and B have veto power while C and D don't.

3. True of False?
(a) In a weighted voting system, all blocking coalitions would become winning coalitions if each voter switched his/her vote from NO to YES.
Answer. This is **FALSE**: for instance, if the system requires unanimity, any one voter voting NO is (on his own) a blocking coalition, but switching her vote to YES does not turn her into a winning coalition.
(b) In a weighted voting system, the number of winning coalitions in which a voter is critical is equal to the number of blocking coalitions in which that voter is critical.
Answer. This is **TRUE** (we saw it in class).
(c) In a weighted voting system, any voter with veto power is a dictator.
Answer. This is not true, as shown for instance by the voting system in 2(b) above: two voters have veto power, and there is no dictator. (d) The Banzhaf power index of a dummy voter is always 0.
Answer. This is **TRUE**: by definition, the vote of a dummy voter never matters, so she cannot be critical in any coalition, hence her Banzhaf power index must be 0.

4. List all the winning and blocking coalitions for the weighted voting system [52: 45, 43, 7, 5].
Answer.

Winning coalitions	Blocking coalitions
{A,B,C,D}	{A,B,C,D}
{A,B,C}	{A,B,C}
{A,B,D}	{A,B,D}
{A,C,D}	{A,C,D}
{A,B}	{A,B}
{A,C}	{A,C}
{B,C,D}	{B,C,D}
	{A,D}
	{B,C}

Notice that adding the number of winning and blocking coalitions one obtains 16, the total number of coalitions– This is because a coalition is blocking exactly if its complement is not winning!

5. Compute the Banzhaf and Shapley-Shubik power index of each voter in the weighted voting system [52:45,43,7,5].
Answer. From the list in exercise 4, we see that A is critical in 5 winning coalitions, B and C in 3 both, and D in 1. Thus the Banzhaf power index of this voting system is [10,6,6,2].
To compute the Shapley-Shubik power index of this system, we make a list of all the 24 permutations of the voters, and find for each permutation who its pivotal voter is:

A B C D	B A C D	C A B D	D A B C
A B D C	B A D C	C A D B	D A C B
A C B D	B C A D	C B A D	D B A C
A C D B	B C D A	C B D A	D B C A
A D B C	B D A C	C D A B	D C A B
A D C B	B D C A	C D B A	D C B A

From this list, we see that A is pivotal in 10 permutations, B in 6, C in 6 and D in 2. Thus the Shapley-Shubik power index of this weighted voting system is [5/12,1/4,1/4,1/12].
(Notice that in this case both power index schemes give the same result, but this is just a coincidence and is false in general).