Quiz 8

Friday, April 13th.

NAME _____

1. Given the weighted voting system [4:1,2,3], list all winning coalitions and all minimal winning coalitions.
Answer. The winning coalitions are {B,C}, {A,C} and {A,B,C}.
The minimal winning coalitions are those which do not contain a smaller winning coalition, so in this case the minimal winning coalitions are {B,C} and {A,C}.

2. A weighted voting system has 6 members. How many distinct coalitions are there in which exactly two voters vote yes?
Answer. To make up such a coalition, one has to choose the two voters (out of six possibilities) that vote yes; thus there are C_2^6 such coalitions, and $C_2^6 = 6 \times 5/2 = 15$.

3. A sorority has an executive board consisting of a chair, a vice chair, and three other members. Its voting rules indicate that an issue can pass in two ways: if the chair, vice chair and one other member support the issue, or if one of the chair or vice chair and two other members support the issue. Find a weighted voting system that is equivalent to this.
Answer. Trying to make a weighted voting system that works, we first notice that the chair and vice chair actually have the same power (given the rule), so we give them the same weight (say, 5). The other three members also have the same weight, say 3 (two members must weigh more than of of the chairs). Then the quota would have to be smaller than or equal to 13 (for the two chairs plus one other member to win), smaller than or equal to 11 (for one of the chairs plus two members to win), and larger than 10 and 9 (so that the two chairs alone, or the three members alone, cannot win). Thus it looks like 11 works here; indeed, the weighted voting system [11:5,5,3,3,3] is equivalent to the system used by the sorority.

4.(a) Calculate the Banzhaf power index for the weighted voting system [9:8,4,2,1], and the Shapley-Shubik power index of voter A and D.
(b) Is there a dictator in this system? A voter with veto power? A dummy voter?
Answer. (a) The table below shows that the Banzhaf power index of this system is [14,2,2,2] (don't forget to multiply by 2 to account for the blocking coalitions!).

Winning coalition	Weight	Extra votes	Critical votes			
			A	B	C	D
{A,B,C,D}	15	6	1	0	0	0
{A,B,C}	14	5	1	0	0	0
{A,B,D}	13	4	1	0	0	0
{A,C,D}	11	2	1	0	0	0
{A,B}	12	3	1	1	0	0
{A,C}	10	1	1	0	1	0
{A,D}	9	0	1	0	0	1
			7	1	1	1

As for the Shapley-Shubik index of A, notice that A will be pivotal in each permutation in which he/she isn't in the first place (the sum of the other voters' weights is less than the quota); there are 18 (the number of total permutations, minus the six permutations in which A is first) of them. The voter D will be pivotal exactly if the sum of weights coming before him/her in the permutation is exactly equal to 8, which is possible only if D comes second in a permutation that begins with A. There are two such permutations (ADBC and ADCB), so overall D is pivotal in 2 permutations. Hence we get that the Shapley-Shubik power index of D is $2/24 = 1/12$, and the Shapley-Shubik power index of A is $18/24 = 3/4$.
This wasn't part of the question, but notice that actually the Shapley-Shubik power indices of B and C are equal to that of D (by a reasoning similar to the one above); so even though the weight of B is four times that of D, they actually have the same power in both models.
(b) There is no dictator since all winning coalitions have at least two voters; A has veto power since he/she belongs to all winning coalitions, and there is no dummy voter since no voter has a Banzhaf power index equal to 0.