UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN — Fall 2006
Math 380 — Group G1

Graded Homework X .
Due Friday, November 17.

1. Compute the surface integral $\iint_S x^2y^2z\,d\sigma$, where S is the portion of the cone of equation $x^2+y^2=z^2$ where $0 \le z \le 1$.

2. Compute the surface integral $\iint_S xz\,d\sigma$, where S is the surface parameterized by $\begin{cases} x = r\cos(\theta) \\ y = r\sin(\theta) \\ z = \theta \end{cases}$ $0 \le r \le R$, $0 \le \theta \le \pi$.

3. Compute the surface integral $\iint_S (x+y^2+z^3)d\sigma$, where S is the boundary of the cube given by the inequalities $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$.

4. Let H be the portion of hyperboloid parameterized by $\begin{cases} x = u\cos(v) - \sin(v) \\ y = u\sin(v) + \cos(v) \\ z = u \end{cases}$, $0 \le u \le 1$, $0 \le v \le 2\pi$.

(a) Show that the surface area of H is equal to $2\pi \int_0^1 \sqrt{2u^2+1}\,du$.

(b) Define $\mathrm{sh}(t) = \dfrac{e^t - e^{-t}}{2}$, $\mathrm{ch}(t) = \dfrac{e^t + e^{-t}}{2}$. Show that $1+\mathrm{sh}^2(t) = \mathrm{ch}^2(t)$. Use this to compute the area of H.