University of Illinois at Urbana-Champaign — Fall 2006
Math 380 — Group G1

Graded Homework II

Due Friday, September 15.

1. Let $z = z(u, v)$ where $u = u(s, t)$ and $v = v(s, t)$. Give an expression of the differential dz in terms of du and dv, then in terms of ds and dt, using the first order derivatives $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$.
Obtain from this formula the values of $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ in the following cases :
- $z = ue^v$, $u = t^2 + s$, $v = s - t$;
- $z = \cos(uv)$, $u = ts$, $v = \sin(t + s)$.

2. The temperature on the surface of a heated disk of radius a is given by the formula $T(r, \theta) = T_0 + T_1(1 - \dfrac{r^2}{a^2})$ (where T_0, T_1 are constants and r, θ are polar coordinates). You are standing on this disk, at the point $(c, 0)$ (where $0 < c < a$) and start moving parallel to the y axis at constant speed v_0. What is the rate of change of temperature that you feel at a given time t (assuming that you haven't yet fallen from the disk) ?
Explain the value obtained at $t = 0$.

3. Let $z(x, y) = f(xy)$, where $f : \mathbb{R} \to \mathbb{R}$ is a function two times continuously differentiable. Give formulas for the first and second-order partial derivatives of z ; check that in that case both mixed derivatives $\dfrac{\partial^2 z}{\partial x \partial y}$ and $\dfrac{\partial^2 z}{\partial y \partial x}$ are equal.

4. Compute a normal vector to the surface S at the point P, and an equation for the tangent plane to S at P, in the following cases :
- S is defined by the equation $z = xy - x + y + 2$, and $P = (0, 2, 4)$;
- S is defined by the equation $z = \sin(xy)$ and $P = (-\sqrt{2}, \sqrt{2}, 0)$;
- S is defined by the equation $x^2 + 4y^2 + xyz = 0$ and $P = (1, -1, 5)$.

5. You are on a mountain of equation $z = 24 - x^2 - 2y^2$, at the point $P = (3, 2, 7)$, and want to go down as quickly as possible. In which direction (in 3-dimensional space) should you turn at first ?