University of Illinois at Urbana-Champaign — Fall 2006
Math 380 — Group G1

Graded Homework VII .
Due Friday, October 27.

1. (a) Compute $\int_\Gamma x ds$, where Γ is the arc of the parabola $y = x^2 + 1$ joining $(0,1)$ and $(1,2)$ oriented counterclockwise.
(b) Compute $\int_\Gamma (x^2 + y^2 + z^2) ds$, where Γ is the triangle (in $\mathbb{R}^3$) with edges $(a,0,0)$, $(0,a,0)$ and $(0,0,a)$ (oriented in that order).

2. Compute $\int_\Gamma (x^2 + y^2)dx + (x^2 - y^2)dy$, where Γ is the boundary of the domain $D = \{(x,y) \in \mathbb{R}^2 \colon x \geq 0,\ y \geq 0,\ x + y \leq 1\}$, oriented clockwise.

3. Let V be the vector field of coordinates $V(x,y,z) = (x+z, y, x)$. Compute the circulation of V along the path with parametric equation $x(t) = \cos(t)$, $y(t) = \sin(t)$, $z(t) = t$, where $0 \leq t \leq 4\pi$.

4. Consider a particle moving in a force field of equation $F(x,y,z) = (x - y - z, x^2 + y, z - y)$. Compute the work of that force field in the following cases :
(a) The particle moves on a straight line from $(0,0,0)$ to $(1,2,4)$;
(b) The particle moves first on a straight line from $(0,0,0)$ to $(1,2,2)$, then on a straight line from $(1,2,2)$ to $(1,2,4)$.

5. Compute $I = \int_\Gamma x^2(y+1)dx + xy(2a - y)dy$ using two different methods (remember Green's theorem), where Γ is the boundary of the upper-half of the disk of center $(0,0)$ and radius $a > 0$, oriented counterclockwise.
For one of these methods, it might be useful to use trigonometric formulae such as $\sin(x)\cos(x) = \dfrac{\sin(2x)}{2}$ and $\sin^2(2x) = \dfrac{1 - \cos(4x)}{2}$. (do you know how to recover these equalities ?)