University of Illinois at Urbana-Champaign | Fall 2006
Math 444 | Group E13

Graded Homework II
Due Friday, Sept. 15 .

1. Let $f\colon \mathbb{N} \to \mathbb{R}$ be defined by : $f(1) = 1$, $f(2) = 2$ and $f(n+1) = \begin{cases} 2f(\frac{n}{2}) + 1 & \text{if } n \text{ is even} \\ f(n-1) + 2 & \text{if } n \text{ is odd} \end{cases}$ for all $n \geq 2$.
Prove by induction that $f(n) = n$ for all n.

2. Let $f\colon E \to F$ be a function ; show that :
$\big(f \text{ is one-to-one}\big) \Leftrightarrow \big(\text{for all } A, A' \subset E,\ f(A) \cap f(A') = f(A \cap A')\big)$;
$\big(f \text{ is onto}\big) \Leftrightarrow \big(\text{ for all } B \subset F,\quad B = f(f^{-1}(B))\big)$.

3. Let $f\colon E \to F$ be a function. Given $A \subset E$, $B \subset F$, are the following asertions true in general ? You have to either prove the result or provide a counterexample, and explain your assertions in detail (using if necessary the definition of a finite set given in class).
(a) A is finite $\Rightarrow f(A)$ is finite .
(b) $f(A)$ is finite $\Rightarrow A$ is finite.
(c) B is finite $\Rightarrow f^{-1}(B)$ is finite.
(d) $f^{-1}(B)$ is finite $\Rightarrow B$ is finite .

4. Let $x, y \in \mathbb{R}$. Prove that $\max(x, y) = \dfrac{x + y + |x - y|}{2}$, and $\min(x, y) = \dfrac{x + y - |x - y|}{2}$.

5. Prove that, for all $a, b \in \mathbb{R}$, one has $|a - b| + |a + b| \geq |a| + |b|$.

6. Using only the axioms seen in class (or those in section 1.1 of the textbook), prove that, for all reals a, b, c, d, the following assertions are true :
- $(a + b) + (c + d) = (a + d) + (c + b)$.
- $-\dfrac{1}{ab} = \dfrac{1}{-ba}$ (assuming that $a, b \neq 0$).