University of Illinois at Urbana-Champaign | Fall 2006
Math 444 | Group E13

Graded Homework III

Due Monday, September 25.

1. Compute, if they exist, $\sup(A)$ and $\inf(A)$ in the following cases. In each case, state whether A admits a maximal element, and do the same for minimal elements.
$A = \{\frac{1}{n} : n \in \mathbb{N}\}$; $A = \{x \in \mathbb{Q} : x^2 < 2\}$; $A = \{(-1)^n + \frac{1}{n} : n \in \mathbb{N}\}$.

2. Let $A = \{x^2 + y^2 : x, y \in \mathbb{R} \text{ and } xy = 1\}$. Prove that A is bounded below, but not bounded above. Compute $\inf(A)$.

3. Let $A, B \subset \mathbb{R}$ be bounded subsets of $\mathbb{R}$. We define $A + B = \{a + b : a \in A, b \in B\}$.
Show that $\sup(A)$, $\sup(B)$, $\sup(A + B)$ exist and that $\sup(A + B) = \sup(A) + \sup(B)$.

4. Let $A \subset \mathbb{R}$ be a bounded set containing at least two elements, and $x \in A$.
(a) Prove that $\sup(A \setminus \{x\})$ exists (recall that $A \setminus \{x\} = \{a \in A : a \neq x\}$).
(b) Prove that if $\sup(A \setminus \{x\}) < \sup(A)$ then $x = \sup(A)$.
(c) Prove that if $x < \sup(A \setminus \{x\})$ then $\sup(A \setminus \{x\}) = \sup(A)$.

5 Let A, B be bounded subsets of $\mathbb{R}$. Prove that $A \cup B$ is also bounded and that $\sup(A \cup B) = \max(\sup(A), \sup(B))$, $\inf(A \cup B) = \min(\inf(A), \inf(B))$.