University of Illinois at Urbana-Champaign | Fall 2006
Math 444 | Group E13

Graded Homework VI

Due Friday, October 20.

1. Let A be a bounded subset of $\mathbb{R}$. Show that there exists a sequence (a_n) of elements of A such that $\lim(a_n) = \sup(A)$.

2. Define, for $n \in \mathbb{N}$, $u_n = 1 + \frac{1}{2!} + \dots \frac{1}{n!}$ and $v_n = u_n + \frac{1}{n.n!}$.
(a) Show that (u_n) is increasing, (v_n) is decreasing, $u_n \leq v_n$ and $\lim(u_n - v_n) = 0$.
(b) Prove that both sequences converge to the same limit (called e.).
(c) Show that, for any n, one has the inequality $u_n \leq e \leq u_n + \frac{1}{n.n!}$. Use this to show that e is irrational.

3. Let (u_n) be a sequence such that $\lim(u_n) = u \in \mathbb{R}$, and $\varphi \colon \mathbb{N} \to \mathbb{N}$ be a bijection (not necessarily increasing !). Show that $\lim(u_{\varphi(n)}) = u$.

4. Let (x_n) be a monotone sequence such that a subsequence of (x_n) is convergent. Show that (x_n) is convergent.

5. Show that the sequences (u_n) and (v_n) defined by $u_n = (-1)^n + \frac{2}{n}$ and $v_n = \cos(\pi n^2)$ are not convergent.