# Research Statement

Carsten Szardenings

At a broad level my interests lie in the interplay between set theory and other branches of mathematics, e. g. algebra, topology, ergodic theory. Since I have been a PhD student, my focus has lain on descriptive set theory and its applications and I am in particular working on Borel/analytic equivalence relations on Polish spaces and related topics.

An equivalence relation $E$ is called analytic respectively Borel if it is analytic respectively Borel as a subset of the plane. For any two such equivalence relations $E \subset X^2$, $F \subset Y^2$ one can ask whether there exists a Borel map $f : X \to Y$ such that for all $x, x' \in X$ $xEx'$ iff $f(x)Ff(x')$. If the answer is yes we write $E \leq_B F$. Many classifications problems can be realized as analytic/Borel equivalence classes over a suitable Polish space; then $\leq_B$ indicates the relative complexity of two classification problems in the following sense: Given a set of complete invariants for the $\leq_B$-larger problem it is easy to obtain a set of complete invariants for the smaller one. I am interested the structure of the partial order $\leq_B$; furthermore I have taken interest in pregeometries, a generalization of equivalence relations, and the structure of $\leq_B$ in that wider class. A nice introduction to the topic including the above statements is given in [1].

Another point of interest for me are full groups. If we fix an equivalence relation $E$ whoose classes are countable, then it is known due to J. Feldmann and C.C. Moore that there exists $G$, a countable Group, acting Borel on $X$ such that for all $x, y \in X$ $xEy$ iff there exists $g \in G$ with $g \cdot x = y$. Let $E$ furthermore live on a standard measure space $(X, \mu)$ (e. g. $[0, 1]$ with Lebesgue measure) then $E$ is called *measure preserving* if for all $\mu$-measurable subsets $A \subset X$ and $g \in G$ $\mu(A) = \mu(gA)$. It is called *ergodic* if additionally for all $\mu$-measurable $A \subset X$ $(\forall g \in G\, gA = A)$ implies $(\mu(A) = 0 \vee \mu(A) = 1)$. Let $Aut(X, \mu)$ denote the set of measure preserving ($\mu$-preserving) transformations of $X$ and let $E$ be ergodic. In this setup we can define the full group $[E]$ of $E$ by

$$[E] = \{T \in Aut(X, \mu) \,|\, \text{T(x)Ex for } \mu\text{-almost all } x \in X\}.$$

With a suitable topology, $[E]$ turns out to be Polish and the algebraic properties the full group capture the properties of $E$ by Dye's renconstruction theorem (see also [2] 3 and 4). Because of this, I would like to know more about algebraic properties that characterize whether a given group can be realized as a full group, and the relation between algebraic properties of a full group and structural properties of the corresponding equivalence relation.

---


[1] A.S. Kechris, *New Directions In Descriptive Set Theory*, Bulletin of Symbolic Logic (5) (2) (1999), pages 161-173

[2] A.S. Kechris, *Global Aspects Of Ergodic Group Actions*, AMS Mathematical Surveys and Monographs, vol. 160 (2010)