Research statement

Cristóbal Rodríguez Porras

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I am a PhD student at the Equipe de Logique Mathématique of the University of Paris 7 – Diderot, working under Prof. Boban Veličković's supervision.

The Boolean Algebra $P(\omega)/Fin$, that is, the quotient of the algebra of subsets of ω by the ideal of finite sets, together with its dual Stone space ($\omega^* := \beta\omega \setminus \omega$), are very rich in interpretations and have applications in a wide range of Mathematical fields. For this reason, and because of the realization that they are sensible to variations in set-theoretical assumptions, they have been throughly studied under different extensions of ZFC (for a general survey see [5]).

In recent years, particular attention has been given to the consequences that the Open Coloring Axiom (as introduced by Prof. Stevo Todorčević in [4]) has in regards to this structure (e.g. see: [7, 6, 2, 1]). One of the main advantages of this axiom is the fact that it is a Ramsey-type statement, asserting the existence of homogeneous sets in certain kinds of partitions, thus being much easier to apply than abstract forcing axioms. Furthermore, the Open Coloring Axiom (OCA) has proven to be very useful as it has settled many interesting questions about sets of reals, and in particular about ω^*.

On the other hand, research done on $P(\omega)/Fin$ has served as inspiration for the search of analogous results in other areas, such as Banach space theory and the theory of C^*-algebras, where analogous quotient structures are found. Quite naturally, the characteristics of the OCA and its applications to the study of $P(\omega)/Fin$ have led to the study of its effects over such structures (see for example, [3]). It is here that my research efforts have been focused lately, specifically on the study of the the Banach space l_∞/c_0 under the Open Coloring Axiom.

References

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