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Research statement

Giorgio Venturi

I am a Phd student in philosophy at the Scuola Normale Superiore of Pisa, under the supervision of Professor Gabriele Lolli, and a Phd student in mathematics at the Université Paris 7, under the supervision of Professor Boban Veličković. I am attending my third year of Phd and the general subject of my thesis is Forcing Axioms. My work has two complementary sides: the philosophical part is about the means to justify new axioms in set theory, while the mathematical side is concerned with their demonstrative power.

More precisely, on the philosophical side, I worked on the general concept of axiom in Hilbert's thought ([8]), trying to elucidate the roots of the problem of the justification of the axioms of modern mathematics. Subsequently I focused on the analysis of Hilbert's versions of the Axiom of Completeness, comparing their intuitive content with their formal role in modern axiomatics ([7]). In this context the concepts of intuitions and naturalness combine with formal mathematical practice. The goal of philosophical part of my thesis is to inquire the concept of completeness and to see how Forcing Axioms conform to it.

On the mathematical side, I studied how to use Forcing Axioms in the context of forcings with models as side conditions. This work produced a paper with Professor Veličković ([5]), where we simplify the proof of the existence a five element basis for the class of the uncountable linear orders, under the same hypotheses of [1]. Last year I worked on a generalization of the method of forcing with models as side conditions, built on ideas from a talk by Professor Itay Neeman, given last year in Oberwolfach ([2]) and related to the new consistency proof of PFA given in [3]. This work has produced a paper ([6]) where the forcing with generalized side conditions is used to give uniform and simple proofs of:

- how to force a club on ω_2 with finite conditions,
- the existence of a chain of length ω_2 on $(\omega_1^{\omega_1}, <_{\text{fin}})$,
- the existence of a thin very tall superatomic Boolean algebra,

- the existence of an ω_2 -Souslin tree.

Modulo the side conditions, all these forcings have finite conditions and they preserve ω_1 and ω_2 .

The aim of my work is to use the method of forcing with generalized side conditions in order to obtain new results. Indeed the applications in [6] show that this method gives a general framework for forcing Σ_1 sentences over $H(\aleph_3)$.

References

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