Research Statement

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My research interest center around combinatorial and topological properties of filters and ultrafilters on the natural numbers, in particular the rich interplay between definability, combinatorics and forcing.

Topological properties. I am interested in the topological properties of ultrafilters as points in ω^* and of points in other stone spaces as well. The results in this area go back to W. Rudin showing the existence of P-points under CH, K. Kunen showing the existence of weak P-points in ZFC and S. Shelah showing that P-points need not exist. A sample result that I was able to obtain is the following:

Theorem. *There is an ultrafilter $p \in \omega^*$ which is an accumulation point of a countable set without isolated points and any two countable sets whose accumulation point it is have nonempty intersection.*

The space ω^* has the interesting property that, although it is compact, it has no convergent sequences. However the following is surprising.

Theorem (Banakh, T., Mykhaylyuk, V., Zdomskyy, L.)**.** *There is an injective sequence $\langle x_n : n < \omega \rangle$ of ultrafilters and a meager filter $\mathcal{F}$ such that $\mathcal{F} -$* $\lim\langle x_n : n < \omega$ *exists.*

It turns out, that one cannot replace meager by definable:

Proposition. *An injective sequence $\langle x_n : n < \omega \rangle$ of ultrafilters never converges w.r.t. an* analytic *filter!*

In fact, more is true:

Proposition. *If an injective sequence $\langle x_n : n < \omega \rangle$ converges w.r.t. a filter, then this filter is Rudin-Keisler above an ultrafilter.*

The following question might be interesting:

Question. *Given a filter which is RK-above an ultrafilter, can we find an injective sequence which converges w.r.t. this filter?*

Ideals and the Katětov order.

Question. *Given a tall ideal $\mathcal{I}$, is there a "nice" forcing extension where $\mathcal{I}$ fails to be tall?*

This question is linked with the old question of M. E. Rudin whether it is consistent that $\aleph_1 = \mathfrak{d} < \mathfrak{a}$. C. Laflamme proved that any tall F_σ-ideal can be destroyed by an ω^ω-bounding forcing. He also proved that, assuming CH, there is an ideal generated by a MAD family, which is not contained in any F_σ-ideal. This result was recently slightly improved by H. Minami. However, the question whether such a family exists in ZFC, is still open. The following is also still open:

Question. *Does every tall Borel ideal contain a tall* F_σ*-ideal?*

Hrušák and Zapletal linked the destructibility of ideals to the Katětov order by proving the following theorem:

Theorem (Hrušák, Zapletal). *Assume* $P_{\mathcal{I}}$ *is proper and has the continuous reading of names; then there is a condition forcing that an ideal* $\mathcal{J}$ *is destroyed if and only if the ideal* $\mathcal{J}$ *is Katětov below* $\mathrm{tr}(\mathcal{I})$ *restricted to some positive set.*

Some work on the structure of the Katětov order has been carried out by D. Meza in his PhD thesis. He proved, e.g., that in ZFC there are increasing and decreasing chains of length $\mathfrak{b}$ in the Katětov order. This leaves the question, whether such chains of length $\mathfrak{c}$ exist. He also showed the existence of Katětov minimal tall Borel ideals among those ideals $\mathcal{I}$ such that the quotient $\mathcal{P}(\omega)/\mathcal{I}$ is proper. It would be very interesting to extend this result to all tall Borel ideals, however this is still open. A similar question, which is also still open, is the following.

Question (Hrušák). *Is there a Katětov maximal MAD family?*