RESEARCH STATEMENT

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My research interests lie in applications of set theory, in particular combinatorial set theory, to operator algebras. This is an area that has grown rapidly in the last decade, beginning with work of Akemann and Phillips ([3]) showing that, assuming CH, the Calkin algebra $\mathcal{Q}(H) = \mathcal{B}(H)/\mathcal{K}(H)$ on a separable, infinite-dimensional Hilbert space H, has outer automorphisms. (Here $\mathcal{B}(H)$ is the C*-algebra of bounded linear operators on H, and $\mathcal{K}(H)$ the ideal of compact operators.) Farah showed in [2] that the opposite conclusion holds under the assumption of TA (Todorčević's Axiom, also known as the Open Coloring Axiom). This partially resolves some questions of Brown, Douglas, and Fillmore going back to the 70's; however there are still open problems to consider, namely:

(1) Is there an automorphism of $\mathcal{Q}(H)$ which is outer on a separable subalgebra of $\mathcal{Q}(H)$?
(2) Is there an automorphism φ of $\mathcal{Q}(H)$ such that $\varphi(a) = b$ for some $a, b \in \mathcal{Q}(H)$ which are not unitarily equivalent?
(3) Let S be the unilateral shift on H. Is there an automorphism of $\mathcal{Q}(H)$ sending the class of S to the class of S^*?

Each of these questions asks for an automorphism of $\mathcal{Q}(H)$ satisfying increasingly stronger "outerness" conditions. Farah's theorem shows that under TA, the answer to each is "no"; however, no known construction of an outer automorphism of $\mathcal{Q}(H)$, under any assumption, satisfies even (1).

One can also ask how much the above results of Akemann-Phillips and Farah extend to the case where H is nonseparable. Another theorem of Farah ([1]) shows that under TA + MA, all automorphisms of $\mathcal{Q}(H)$ are inner whenever the dimension of H is at most $\aleph_1$; and under PFA, the same holds when H has arbitrary dimension. The existence of outer automorphisms under L-like assumptions on V (CH, $\diamondsuit$, etc.) is so far an open problem when H is nonseparable. We do have the following result when $\dim(H) = \aleph_1$;

Theorem 1. *(Farah, M.) Let $\mathcal{K}$ be the ideal of compact operators on H, and $\mathcal{J}$ the ideal of operators with separable range.*

(i) Assume CH. Then $\mathcal{J}/\mathcal{K}$ has outer automorphisms.
(ii) Assume $2^{\aleph_1} = \aleph_2$. Then $\mathcal{B}/\mathcal{J}$ has outer automorphisms.

References

1. Ilijas Farah, *All automorphisms of all Calkin algebras*, Math. Res. Lett. **18** (2011), no. 3, 489–503. MR 2802582 (2012e:03104)
2. ______, *All automorphisms of the Calkin algebra are inner*, Ann. of Math. (2) **173** (2011), no. 2, 619–661. MR 2776359 (2012e:03103)
3. N. Christopher Phillips and Nik Weaver, *The Calkin algebra has outer automorphisms*, Duke Math. J. **139** (2007), no. 1, 185–202. MR 2322680 (2009a:46123)