Research Statement

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I'm a first year PhD student at UPMC under the supervision of Dominique Lecomte. My main area of research is descriptive set theory; more specifically Borel chromatic numbers and certain separation results that can arise in this context.

Remember that the chromatic number of a graph is the smallest number of "colors" we can assign to each vertex so that no adjacent vertex have the same color. If we restrict ourselves to certain definability conditions, namely analytic graphs in polish spaces and a "color"-function that is Borel, we obtain what we call the Borel chromatic number. Kechris, Solecki and Todorcevic found in [1] the existance of a minimum graph, $\mathbb{G}_0$, in $2^\omega \times 2^\omega$ with non-countable Borel chromatic number, i.e. this graph can be continuously mapped in any other graph with a non-countable chromatic number, respecting the graph relation.

On the other side, we have separation problems: given two disjoint analytic sets in the plane, A_0, A_1 $(X \times Y)$, when can they be separated by a countable union of Borel rectangles (i.e. $A \times B$ with A and B Borel). Lecomte proved in [2] that this cannot be done when we can find continuous functions $f : X \mapsto 2^\omega$, $g : X \mapsto 2^\omega$ such that $(f(\alpha), g(\alpha)) \in A_0$ and if $(\alpha, \beta) \in \mathbb{G}_0$ then $(f(\alpha), g(\beta)) \in A_1$. Thus, again, the case for the separation between $\Delta(2^\omega)$ (the diagonal) and $\mathbb{G}_0$ is minimal in a certain sense.

So the next question to look at is: could one get similar results by looking at the finite case, and could these results be improved by refining the functions we allow.

References

[1] A. S. Kechris, S. Solecki and S. Todorcevic. Borel chromatic numbers. *Adv. Math.* 141 (1999),1-44

[2] D. Lecomte. On minimal non potentially closed subsets of the plane. *Topology Appl.* 154, 1 (2007) 241-262