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I am interested in descriptive set theory, and more specifically the study of measurable functions. I have been searching developments and precisions of partitions theorems such as the Jayne-Rogers theorem. Those come from a simple and natural question ; given a measurable function, is it possible to partition its domain in a countable amount of pieces, each piece of bounded topological complexity, such that the function is simpler on each piece of the partition? An instance of this question is : is every Borel function piecewise continuous? This specific question of Lusin has been answered negatively several times, but the general problem is still open.

I am also interested in ordering functions. I recently began to study the following quasi-order on functions: given two functions f and g from a topological space X into itself, set $f \leq g$ iff there are two continuous functions σ and τ such that $f = \tau \circ g \circ \sigma$. The nice property of this order is that it preserves the Borel complexity of functions at a really precise level (the Wadge one namely). The downside is the the complexity of the qo itself. I have restricted my attention to continuous functions from the Baire space ω^ω into itself. I have tried to prove that the qo is in fact a well-quasi-order (wqo) on continuous functions. A wqo is a qo with only finite antichains and finite strictly descending chains. This apparently simple question is quite complex. I have found a sufficient condition for the continuous fragment to be a better-quasi-order (bqo). Remind that the bqo condition is a strengthening of the wqo condition that satisfies much more regularity properties. I conjecture that the continuous fragment is a bqo.