Research Statement for Young Set Theory Workshop
CIRM, April 30th - May 4th, 2012

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My research is focused on descriptive set theory with emphasis on effective theory and its applications. A characteristic type of problems of this area is to examine the existence of Borel-measurable choice functions. This is strongly related to the existence of Δ^1_1 (i.e., HYP) points satisfying a given property. To be more specific assume that $\mathcal{X}$ and $\mathcal{Y}$ are recursively presented Polish spaces and that $P \subseteq \mathcal{X} \times \mathcal{Y}$ is a Π^1_1 set such that for all $x \in \mathcal{X}$ there exists a $y \in \Delta^1_1(x)$ such that $(x, y) \in P$. Then there exists a Borel-measurable function $f : \mathcal{X} \to \mathcal{Y}$ such that $(x, f(x)) \in P$ for all $x \in \mathcal{X}$. In a typical example of an application one knows beforehand that for the given set P for each x there exists a y such that $(x, y) \in P$. The proof of the existence of the latter y does not imply though that it can be chosen in a Δ^1_1-way. Hence one has to actually provide a new proof of the statement $(\forall x)(\exists y)[(x, y) \in P]$, where the latter y is actually in $\Delta^1_1(x)$. In other words one has to find an "effective" proof.

A result of this kind that I have obtained recently is the effective version of the theorem about extending the original topology in a Polish space into another Polish topology so that a given Borel-set becomes a clopen set and the Borel structure remains the same. I have also provided conditions under which the extended topology can be encoded in a Δ^1_1-way. As described above this gives rise to a Borel measurable function which carries a code of the original topology to a code of the extended topology, provided that the appropriate conditions are satisfied.