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Research Statement

I am a PhD student at the Institute of Mathematics, University of Wroclaw.

My interests focus around forcing and its connections to descriptive set theory. Recently I am working on the various versions of the axiom CPA.

Let $\mathcal{I}$ be the σ-ideal of countable sets on $\mathbb{R}$ and $\mathcal{I}^\alpha$ be its α-th Fubini power on $\mathbb{R}^\alpha$ (see [2]). Consider a game between Adam and Eve which lasts ω_1 rounds. At stage α Adam plays with some countable ordinal ξ_α, a set $B_\alpha \in Bor(\mathbb{R}^{\xi_\alpha}) \setminus \mathcal{I}^{\xi_\alpha}$ and a Borel function $f_\alpha \colon B_\alpha \to 2^\omega$. Then Eve responses with some $C_\alpha \in Bor(\mathbb{R}^{\xi_\alpha}) \setminus \mathcal{I}^{\xi_\alpha}$, $C_\alpha \subseteq B_\alpha$. Adam wins if $\bigcup_{\alpha<\omega_1} f_\alpha[C_\alpha] = 2^\omega$. CPA says that $\mathfrak{c} > \omega_1$ and Eve has no winning strategy in the above game. It holds in the model obtained by the Sacks forcing iterated ω_2 times with countable support. If $\mathfrak{x}$ is some "nice" cardinal invariant then $\mathfrak{x} < cov(\mathcal{I})$ can be forced if and only if it is a consequence of CPA (see Corollary 5.1.8. in [2]). Everything still works if we replace $\mathcal{I}$ with some other $\mathbf{\Pi^1_1}$ on $\mathbf{\Sigma^1_1}$ σ-ideal.

The cardinal $\mathfrak{h}$ denotes the distributivity number of $\mathcal{P}(\omega)/fin$ and $\mathfrak{h}(2)$ the distributivity number of r.o.$(\mathcal{P}(\omega)/fin)^2$. It was proved in [1] that $M \models \mathfrak{h} = \omega_2 \wedge \mathfrak{h}(2) = \omega_1$, where M is the model obtained by the countable support iteration of Mathias forcing of length ω_2.

Recently I am trying to find out some version of CPA, which would hold in the model M, and which would imply $\mathfrak{h}(2) = \omega_1$ and $\mathfrak{h} \geq \omega_2$.

The σ-ideal connected to the Mathias forcing (namely meager sets in the Ellentuck topology) is not "iterable", that means there is no absoluteness enough for this ideal to work in the CPA schema decribed above.

However, a technique used in [1] allows to modify the game providing enough absoluteness. But this still seems to be too weak to imply the results on distributivity numbers. The question is how to strenghten the game in such a way that the axiom would hold in the iterated Mathias model M and would imply the facts about $\mathfrak{h}(2)$ and $\mathfrak{h}$.

References

[1] S. Shelah, O. Spinas *The distributivity numbers of $\mathcal{P}(\omega)/fin$ and its square*, Trans. AMS, 325 (1999), 2023-2047.

[2] J. Zapletal, *Descriptive Set Theory and Definable Forcing.*