SOME MINOR CORRECTIONS TO THE PAPER "CLOPEN TYPE SEMIGROUPS OF ACTIONS ON 0-DIMENSIONAL COMPACT SPACES"

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• **The counterexample in Paragraph** 2.2 **should be amended**.

In paragraph 2.2, below Proposition 2.5, I claim to construct an example of a uniquely ergodic $\mathbf{Z}$-action whose clopen type semigroup is not stably finite. This argument is sloppily written; in the definition of the map there should be a σ acting on the second coordinate. More importantly, the pieces used to define the proposed equidecomposition τ are not clopen, so the construction fails badly. It is actually not hard to check that the clopen type semigroup of this particular action is indeed cancellative.

This construction can however be easily modified by using the two-point compactification instead; and one obtains a topologically transitive $\mathbf{Z}$-action on the Cantor space with exactly two ergodic invariant measures and whose clopen type semigroup is not stably finite. I actually do not know whether one can build a similar example which is uniquely ergodic.

• **A comment on the proof of Proposition** 2.15**.**

At the end of that proof, the key equality $\mu(A \setminus \bigcup_{i=0}^{\infty} A_i) = 0$ is only proved for an ergodic μ, and no further explanation is given. To obtain the desired result one can apply the Choquet–Bishop–de Leeuw theorem, using that $\mu \mapsto \mu(A \setminus \bigcup_{i=0}^{\infty} A_i)$ is a decreasing limit of continuous affine functions.

• **Reformulating Question 6.3**.

I meant to ask Question 6.3 only for actions of *amenable* countable groups. As stated, it actually has a negative answer for any countable nonamenable group Γ!

To see why, let Γ be any nonamenable countable group. By Proposition 4.7, a generic minimal action α of Γ is such that $T(\alpha)$ is unperforated and the algebraic order is a partial ordering; furthermore, a generic minimal α has no invariant Borel probability measure (having no invariant measure is witnessed by an inequality of the form $(n+1)[X] \leq n[X]$, so those actions form an open, conjugacty-invariant subset in $\mathrm{Min}(\Gamma)$, which is nonempty because Γ is not amenable. Since there are dense conjugacy classes in $\mathrm{Min}(\Gamma)$, this open set of actions is dense).

Fix a minimal α with the three generic properties above. Denote by X the ambient Cantor space, and by $[A]$ the type of a clopen subset A of X. Let A be any nonempty clopen subset of X. We know that for some n we have $(n+1)[A] \leq n[A]$, from which we derive $2n[A] \leq n[A]$, which by unperforation gives $2[A] \leq [A]$ whence $n[A] \leq [A]$ for all $n \geq 1$. By minimality, we have that for some n $[X] \leq n[A]$, whence $[X] \leq [A]$; clearly also $[A] \leq [X]$, whence X is equidecomposable with A for *any* nonempty clopen A for a generic minimal action α. But of course, unless $A = X$, $X \setminus X = \emptyset$ is not equidecomposable with $X \setminus A$.

• **A bad typo.** In the glossary, the definition of unperforation is incorrect - the correct one is given in Definition 2.23 : $T(\alpha)$ is unperforated if for all $n \in \mathbf{N}^*$ and all $a, b \in T(\alpha)$ one has $na \leq nb \Rightarrow a \leq b$.

Of course there are other minor typos and inaccurracies, but I felt that those in particular deserved to be pointed out.

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