Université Claude Bernard Lyon 1 & ENS de Lyon

2^{nd} year Master program Advanced Mathematics

Calculus of variations and elliptic partial differential equations

Petru Mironescu
Institut Camille Jordan
Université Claude Bernard Lyon 1
http://math.univ-lyon1.fr/~mironescu/
mironescu@math.univ-lyon1.fr

Description. This is an intermediate + course presenting some basic tools in the qualitative analysis, existence, and regularity theory for solutions of elliptic partial differential equations (PDEs). A first part, related to the direct method in the calculus of variations, goes beyond elliptic PDEs.
http://math.univ-lyon1.fr/~mironescu/enseignement/edp_MA2.html

Prerequisites

1. Good knowledge of general measure theory and integration.
2. Reasonable knowledge of geometric aspects of the integration theory (Gauss-Ostrogradskii...) and of the local theory of submanifolds of $\mathbb{R}^n$.
3. Good knowledge of the basic results concerning the Laplace equation.

Syllabus

1. The direct method in the calculus of variations
   (a) Basic examples.
   (b) Notions of convexity.
   (c) Passing to the weak limits in nonlinear quantities. Compensation phenomena.
   (d) Gap phenomena.
2. Maximum principles and applications
   (a) Maximum principles for elliptic partial differential equations (PDEs) in non divergence and divergence form.
   (b) Iterative methods based on monotonicity (sub- and supersolutions).
   (c) Symmetry properties of solutions of semilinear elliptic PDEs.

(d) Krasnoselskii's uniqueness result.

3. Regularity theory
   (a) Serrin's example.
   (b) Singular integrals.
   (c) L^p-theory for elliptic equations in non-divergence form.
   (d) A glimpse of the C^α-theory for elliptic equations in non-divergence form.
   (e) De Giorgi-Nash regularity theory for elliptic equations in divergence form.
   (f) Bootstrap. Regularity in the critical case.
   (g) A limiting case : Wente estimates. A glimpse of other compensation phenomena.
4. Other (non-)existence methods
   (a) Concentration-compactness.
   (b) Mountain pass solutions.
   (c) Other topological methods.
   (d) Pohozaev's identity.
5. A glimpse of phase-transition problems
   (a) A glimpse of the BV space.
   (b) Abstract Γ-convergence.
   (c) The Modica-Mortola functional in the limit $\varepsilon \to 0$.
   (d) Vector-valued variants.

Evaluations

1. Article presentation : 20 $\%$.
2. Partial examination (2 hours) : 30 $\%$.
3. Final examination (3 hours) : 50 $\%$.

Schedule. All the courses are on the LyonTech-la Doua campus, in the Braconnier building.

1. 2 hours courses on Fridays 2–4 PM : 15/09, 22/09, 29/09, 10/11, 17/11, 24/11.

2. 2 hours courses on Wednesdays 9–11 AM : 04/10, 11/10, 18/10, 25/10, 08/11, 22/11.
3. Office hours (1 h) : Fri 29/09 4–5 PM, Wed 11/10 11–12 AM, Wed 25/10 11–12 AM, Wed 08/11 11–12 AM, Fri 10/11 4–5 PM, Fri 24/11 4–5 PM.
4. Partial examination : Fri 27/10 2–4 PM.
5. Articles presentation : Fri 08/12 2–5 PM.
6. Final examination : Wed 13/12, 9–12 AM.