Chapter 1
Theoretical Aspects of Graph Models for MANETs*

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Abstract We survey the main theoretical aspects of models for Mobile Ad Hoc Networks (MANETs). We present theoretical characterizations of mobile network structural properties, different dynamic graph models of MANETs, and finally we give detailed summaries of a few selected articles. In particular, we focus on articles dealing with connectivity of mobile networks, and on articles which show that mobility can be used to propagate information between nodes of the network while at the same time maintaining small transmission distances, and thus saving energy.

1.1 Introduction

In 1961 Edward Gilbert [Gil61] defined *random plane networks* as a model to study the communication in networks of shortrange stations spread over a large area. In his model, vertices represent the stations, and edges represent a two-way communication channel between stations. All stations have the same range power, so there is a direct communication between two stations iff the corresponding vertices are connected by an edge. Gilbert distributed the vertices in an infinite plane, by using a Poisson point process in the plane and then connecting two vertices if they

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are separated by at most a distance r. He went to study the asymptotic value of the probability that a vertex belongs to a connected component with all the other vertices.

Nowdays, Gilbert's model is better known as *Random Geometric graphs* (RGG). A random geometric graph can be equivalently defined by distributing n points uniformly on a given surface; thus, a RGG is a graph resulting from placing a set of n vertices independently and uniformly at random on the unit square $[0,1]^2$, and by connecting two vertices if and only if their *distance* is at most the given radius r, the distance depending on the type of metric being used. For convenience, when using a Poisson point process to distribute the vertices, sometimes it is better to scatter the vertices on $[\sqrt{n},\sqrt{n}]^2$, where n is the expected number of points distributed by the Poisson process. It is well known that the results in this model are just rescaled versions of the results on $[0,1]^2$. Some authors consider the torus $[0,1)^2$ to avoid the effects of boundaries, which we will mention in Section 1.3 in more detail. For many properties the boundary effects change the results, see for example [WY06]. We refer to an instance of a RGG with n vertices and radius r as $\mathscr{G}(n,r)$.

The deterministic counterparts of random geometric graphs are called *Unit Disk graphs* (UDG). A graph G is a *Unit Disk graph* for a radius distance r, if its vertices can be put in one-to-one correspondence with the centers of circles of radius r in the plane, in such a way that two vertices in G are connected by an edge if and only if their corresponding circles intersect [Gol80, CCJ90]. The *recognition problem* is to decide whether a given graph G is a UDG. The problem is known to be NP-hard [BK98]. Since the vertices of a UDG are points in the real plane, the problem is not known to be in NP.

Random geometric graphs and unit disk graphs have received quite a bit of attention in the last years both as a particular mathematical structure different from other types of known graphs [Pen03, vL09], and also because of their applications as models for wireless networks, in particular as simplified topological models for *wireless sensor networks* (see for example [SH97], where UDG was also denoted as *point graph model* and [ASSC02, AB02, ZG03, TAGH02, SW06, DPS03]). Further applications of unit discs and random geometric graphs as models for more general *ad hoc networks* are discussed in the references [Hek06, dMCA06, Li04, GY07, YMG08] and in Chapter 1 of [vL09].

Wireless networks consist of a set of *simple* nodes, each one with a wireless transceiver to communicate with their near neighbors, where *near* is understood as the closest in terms of Euclidean distance, and the ability of communication depends on the transmitting power of the transceivers. The goal of a network is to spread information through the network, which is done in a multi-hop fashion. In many ad hoc networks, like sensor networks, due to the simplicity of the nodes, energy consumption is an issue. Therefore, one of the most important questions when modeling a network is to minimize power consumption. That is, the transmission range should be made as small as possible but at the same time large enough to make sure that a packet of information transmitted from a node will arrive to the other nodes in the network. As we mention in the next section, when modeling wireless networks by graph topology, one of the main problems is the trade-off between range of trans-

mission and network connectivity. In Section 1.5.3 we will give examples where mobility boosts message distribution in a network while at the same time maintaining a small range of transmission.

The choice of whether to use a deterministic model, such as UDG, or a randomized model, such as RGG, depends on the application. For example, when using sensors networks, it is usual that the sensors are scattered from some type of vehicle, and hence in this case the random model is the appropriate one. For other kinds of wireless networks, the randomized model also could be interesting to obtain the *average behaviour* of the network. In the next section, we also briefly mention the case where the transmitting power of each node is different, introducing the *range assignment* problem: the problem of assigning different transmission powers to each node in such a way that the power used is minimized, while maintaining the network connected.

The aim of the present work is to survey the recent theoretical results for *Mobile Ad Hoc Networks* (MANETs), with an emphasis of topological models. It is organized as follows: in Section 1.2 we review a few known results about static RGG, mainly those related to connectivity; in Section 1.3 we discuss issues that play a role in dynamic models and present different random mobility models; in Section 1.4 we survey theoretical results concerning a very popular mobility model (the random waypoint model), showing in particular how mathematical tools have been used to identify problems in wireless mobile network simulation, and to solve them; in Section 1.5, we present a few selected recent papers on dynamic MANETs, focusing on papers which present a formal analysis of mobility model properties, and use the analysis to characterize fundamental network properties such as connectivity and information propagation speed. Throughout this paper, "a.a.s." denotes *asymptotically almost surely*, that is, with probability tending to 1, as n goes to ∞. For other concepts in probability, the reader is advised to look into any of the basic references, for ex. [GS01, Pit99].

1.2 Static Properties

In this section, we point out some of the known results about *static* RGG, which will be helpful for the mobility survey. In this line, we skip many of the very interesting recent results on RGG that are of combinatorial nature, such as results about the chromatic number, for example. The main reference on RGG is the book by Mathew Penrose [Pen03]. Moreover, the reader should be aware that since 2002 a lot of work has been done on the topic of static RGG. When considering a RGG as topological model for a wireless network, one of the important issues is to keep the network connected using the minimal amount of energy consumption, i.e., using the smallest transmitting distance. This is called the *critical power* among the networking community [Li04], and the *connectivity threshold* among the mathematical community [Pen03]. In the book of Penrose, the results are exposed in full generality, for any distance norm, and any dimension. To make the basic ideas as clear as

possible, in the present survey we stick to the case of dimension 2 and Euclidean distance norm.

Let $\mathscr{G}(n,r)$ be the graph representing a wireless ad-hoc network with n nodes, where r denotes the transmitting distance. We assume the ideal case where the area covered by a node is exactly a circle. *Topology control* is a technique that uses the tuning of certain parameters, usually the transmitting range r or the maximum degree of the graph, to change/form the topology of the graph representing the network in order to maintain the connectivity while optimizing the energy (or minimizing the interference). There are very good recent surveys on the topic of topology control, see for instance [San05b, Li04]. One of the important problems in topology control is the critical transmitting range for connectivity: *what is the smallest radius, denoted by r_c, that keeps G connected?* If G is a deterministic instance, i.e. a UDG, it is well known that the value of r_c is the length of the longest edge in the *minimum spanning tree* (MST) of G. The case where G is a RGG is more interesting. In this case, Penrose [Pen97] computed the expected length of the longest edge of a MST in a RGG on $[0,1]^2$, yielding the well known connectivity threshold $r_c = \sqrt{\frac{\ln n \pm O(1)}{\pi n}}$ a.a.s., where as usual the abbreviation a.a.s. stands for *asymptotically almost surely*, i.e., with probability $1-o(1)$ as $n \to \infty$. Independently, [GK98] gave the same bounds for r_c, also for the ℓ_2-norm but considering the unit circle as underlying surface. Both proofs are quite different.

Notice that real wireless networks cannot be too dense, because a transmitting node interferes with all the nodes within its interference range. In [SBV01, SB03] the authors have characterized the critical transmission range in the more general model in which the side ℓ of the deployment region is a further parameter, and n and r can be arbitrary functions of ℓ. Note that under this model, the asymptotic behavior of node density (number of nodes per unit area) depends on how n changes with ℓ. In particular, n can be chosen in such a way that the node density asymptotically converges to 0, or to an arbitrary constant greater than 0, or diverges. Under this respect, Santi et al.'s model is more general than the standard RGG model, in which the node density grows to infinity with n. The main finding of [SBV01, SB03] is a proof that, as $\ell \to \infty$, if $r \sim \ell\sqrt{c\frac{\lg \ell}{n}}$ for some constant $c > 0$, then the graph is connected a.a.s.

Going back to the classical model of RGG on $[0,1]^2$, we now try to convey the flavor and intuition behind the value r_c for which a RGG becomes connected a.a.s. Given a set V of n nodes and a positive real $r = r(n)$, each node is placed at some random position in $[0,1]^2$ selected uniformly at random. We define $\mathscr{G}(n,r)$ as the random graph having V as the vertex set, and with an edge connecting each pair of vertices u and v at distance $d(u,v) \leq r$, where $d(\cdot,\cdot)$ denotes the Euclidean distance. We assume that $r = o(1)$, else $\mathscr{G}(n,r)$ is trivially connected a.a.s. Let X be the random variable counting the number of isolated vertices in $\mathscr{G}(n,r)$. Then, by multiplying the probability that one vertex is isolated by the number of vertices we obtain,

$$\mathbf{E}(X) = n(1-\pi r^2)^{n-1} = ne^{-\pi r^2 n - O(r^4 n)}.$$

Define $\mu = ne^{-\pi r^2 n}$. Observe that this parameter μ is closely related to $\mathbf{E}(X)$. In fact, $\mu = o(1)$ iff $\mathbf{E}(X) = o(1)$, and if $\mu = \Omega(1)$ then $\mathbf{E}(X) \sim \mu$.

Moreover, the asymptotic behavior of μ characterizes the connectivity of $\mathscr{G}(n,r)$. In fact, if $\mu \to 0$, then a.a.s. $\mathscr{G}(n,r)$ is connected, if $\mu = \Theta(1)$, then a.a.s. $\mathscr{G}(n,r)$ consists of one giant component of size $> n/2$ and a number of isolated vertices which follows a Poisson distribution with parameter μ; if $\mu \to \infty$, then a.a.s. $\mathscr{G}(n,r)$ is disconnected. Therefore, from the definition of μ we have that $\mu = \Theta(1)$ iff $r_c = \sqrt{\frac{\ln n \pm O(1)}{\pi n}}$ (see [Pen03]).

Extensions to k-connectivity appear in [Pen99], where the author proves that when the minimum degree of a RGG is k the graph becomes k-connected. Notice that k-connectivity is important in networking as a measure of fault-tolerance of the network. Chapter 13 of [Pen03] presents an extensive treatment of connectivity for RGG, taking into account different norms, higher dimensions and different underlying probability distributions.

Recall that a graph property is *monotone* if it is preserved when edges are added to the graph. A graph property is said to have a *sharp threshold* if the window between having and not having the property can be made arbitrarily small. In [GRK05] the authors prove that every monotone property on a RGG has a *sharp threshold*. As connectivity is a monotone property, we conclude that the property of connectivity in $\mathscr{G}(n,r)$ exhibits a sharp threshold at $r_c = \sqrt{\frac{\ln n}{\pi n}}$.

As mentioned before, for a radius r slightly below the connectivity threshold r_c, $\mathscr{G}(n,r)$ consists a.a.s. of a giant component and some isolated vertices. It is also known that in this situation the probability of having a component of size i at r_c is $O(1/\log^i n)$, and if there exists one, it forms a clique [DMP09b]. A straightforward computation yields that when we consider the *connectivity regime* with $r = r_c$, the expected degree of a vertex is asymptotically $\Theta(\log n)$ (plug r_c in the expected number of neighbors of a vertex, which is $\pi r_c^2(n-1)$). For values of $r > r_c$, $\mathscr{G}(n,r)$ is said to be in the *superconnectivity regime*, and the graph is *dense*[2], while for values of $r < r_c$, $\mathscr{G}(n,r)$ is said to be in the *subconnectivity regime*, and the graph is *sparse*. As we mention in Section 1.5.3, in the subconnectivity regime mobility can help to spread information.

The behavior of RGG for values of r in the subconnectivity regime has been quite thoroughly studied, see Chapter 10 in [Pen03]. It is known that there exists a value $r_t = \frac{c}{\sqrt{n}}$ where a giant component of size $\Theta(n)$ appears in $\mathscr{G}(n,r)$ a.a.s., with c being a constant that experimentally is conjectured to have a value around 2.35 (recall that we focus on the ℓ_2-norm in two dimensions). In the regime where $r < r_t$, each vertex has expected degree $O(1)$. The r_t is denoted as the *thermodynamical threshold*.

The *cover time* C of $\mathscr{G}(n,r)$ is the expected time taken by a simple random walk of $\mathscr{G}(n,r)$ to visit all the nodes in the graph. In [AE07] the authors prove that a.a.s. $C = \Theta(n \log n)$ if $r \geq \sqrt{\frac{c \log n}{n}}$, with $c > 8$. If $r \leq \sqrt{\frac{\log n}{\pi n}}$, the cover time is ∞ with positive probability, bounded away from zero.

[2] Note that a usual graph with n vertices is said to be dense if it has $\Theta(n^2)$ edges.

When dealing specifically with *wireless sensor networks*, an important issue is to assure that sensors properly cover the entire region being monitored, which is known as the *coverage* of the network. Similarly to connectivity, coverage can be modeled using the RGG model, where each vertex represents a sensor, and r is the *sensing range* of the sensors. Given an integer k, a point is said to be *k-covered* if it falls into the sensing range of at least k sensors. If all the points of a region are k-covered, then the region is k-covered. If $\mathscr{C}^k_{n,r}$ denotes the event that every point of $[0,1]^2$ is $(k+1)$-covered by a network with n sensors of range r, the *k-covering problem* consists in giving asymptotic bounds to $\mathbf{Pr}\left[\mathscr{C}^k_{n,r}\right]$, as $n \to \infty$. In [Hal88], the case $k = 1$ is studied, however the author uses a toroidal metric to avoid problems with nodes very near the boundary of the region where the nodes are scattered. Several authors have been working on this problem [MKPS01, WY04, ZH04, KLB08]. In [WY06] the authors give bounds on $\mathbf{Pr}\left[\mathscr{C}^k_{n,r}\right]$ for the unit square, taking into consideration the boundary effect of the unit square, which complicates quite a bit the analytical proof. Sometimes, coverage and connectivity of a wireless sensor network are jointly studied, with the objective of forming a network which not only k-covers the entire monitored region, but it is also connected. It is easy to see that k-coverage implies k-connectivity of the network whenever $r_t \geq 2r_s$, where r_t is the transmission range and r_s is the sensing range of nodes [XXZ+03].

Up to now we have considered that all nodes broadcast at the same transmitting range r, but the efficiency of energy management in a network could be achieved by tuning every node to a different transmitting range. The *range assignment* problem is the following: given a graph with n nodes, each one knowing their position, the goal is to assign a transmitting range r_i to each node i in such a way that the network is connected with minimum energy cost, where the energy e_i used by node i is proportional to r_i^2, i.e., the goal is to minimize $\sum_i e_i$. The problem was first studied in [KKKP00]. Since then, several authors have proposed and studied different variants of the basic model, see Section 5.2 in [San05b].

Another important issue is the design of efficient protocols for disseminating and broadcasting information in wireless ad hoc networks. We refer the reader to one of the multiple surveys treating the topic: [RT99, Raj02, HMKR04, San05b, Li04, YMG08].

1.3 Mobility models for MANETs

After giving a very concise introduction to the results on static random geometric graphs, let us focus our attention on mobility issues. When talking about mobility in MANETs, we mean mobility of the nodes, i.e., the nodes physically move in a region. There is an alternative version of dynamical wireless networks, where the dynamicity is caused by the addition and removal of edges between nodes, due to the temporal evolution of the transmitting range r_i, for each node i. This kind of mobility has been thoroughly studied by the computational geometry community, see for example [AGE+02, GHSZ01]. The main focus of their research is the design and

analysis of sophisticated algorithms and data structures that easily allow deletion or addition of very few edges or nodes at each time. In the case of highly dynamic MANETs, due to the large number of changes in each step, the direct evaluation of the performance of the network is very time-consuming (see for example Section 2 in [BB04]). One way to get an idea of the performance is to use simplified models of the network. Moreover, due to the fact that real MANETs are mostly deployed in environments where it is difficult to control the quality of transmission, simulation could furnish better scenarios to control the experiments. In particular, when designing new protocols for communication, sometimes it is better to start simulating on a simplified topology than a direct implementation on the real network. However, some researchers reason, that low scale simulations are not conclusive and that the final validation of the viability and efficiency of the new proposed protocol must be experimented directly on the network (see for example [KM07]).

In the remainder of this survey, we are going to look at the recent preliminary research done on analytical studies of different mobility models proposed. The goals of the simplified models is to extract the topological properties of mobile networks, which might help both in improving simulation accuracy (see Section 1.4), and in designing new protocols where mobility is used to reduce energy consumption and/or information propagation speed (see Section 1.5). Clearly, this survey does not cover *every* property where mobility helps. For example, for the k-covering problem, in [WSC07] the authors recently proved both analytically and experimentally that, if a fraction of the nodes is mobile with very limited range of mobility, k-coverage can be achieved with less sensors than in the static case.

In the last decade, quite a few models for MANETs have been proposed, see the surveys [Bet01a, CBD02, BB04, Zha06]. Section 2.1.5 of [BB04] gives a detailed taxonomy of the mobility models used in the literature. According to the degree of mobility, there are three types of mobility:

- The *deterministic model*, where nodes move through predetermined paths in a deterministic manner. The model needs to trace the mobility of nodes, which can be cumbersome [THB+02].
- The *hybrid random model* where the model guides the nodes through a predetermined graph, which represents streets, roads, etc. On this graph, however, nodes move randomly. For example, in [JBRAS05] the authors consider a region with obstacles, and force the mobility to take place along the Voronoi tessellation of the obstacles. The *city selection mobility* and the *Graph based mobility* models described in [BB04], are examples of hybrid random models.
- The *pure random model* where the nodes move in a random way in the region. Most of the models, described in the literature, belong to this class. The two most representative models in this class are the *random direction model* and the *random waypoint model*.

The most frequently used mobility models are the following two and their variations:

- the *Random WayPoint model* (RWP) was first described in [JM96]. In this model, as usual, nodes are initially distributed uniformly at random on the region; then,

each node chooses independently and uniformly at random a destination within the region, as well as a travel speed. The node then starts traveling towards the destination with the selected speed along a linear trajectory. When it reaches the destination (waypoint), it might optionally pause for a certain time, then chooses another waypoint in the region, and continues according to the same pattern. Structural properties of RWP model have been deeply investigated in the literature, and are discussed in detail in Section 1.4.

- the *Random Direction model* (RD): the seed of the RD model is the paper [Gue87], in which each node i in the region under consideration, selects uniformly at random a direction $\theta_i \in [0, 2\pi)$, and chooses a speed that is kept constant during a certain amount of time. After a randomly chosen period of time, each node selects a new direction and speed, and continues moving. As the process evolves over time, some of the nodes might arrive at the boundary of the region, and a *border rule* has to be defined to determine how nodes behave when they hit the border. An easy way to deal with the boundary effect is to consider the toroidal version $[\ell_1, \ell_2)^2$, instead of the unit square $[\ell_1, \ell_2]^2$. In fact, when modelling applications like sensor networks on large terrain, the toroidal model is a fair approximation to reality. For smaller areas, when the boundary effect is significant, an alternative option is to consider the so called *bouncing boundary rule*, where the nodes arriving at a boundary bounce back to the region. When a node hits the boundary, this bouncing could be done either by choosing a random new angle θ', or by following the *mirror reflection* rule, i.e. the node returns to the region at an angle $\theta' = \pi - \theta$, where θ is the incidence angle at which the node hits the boundary. There have been several modifications of the basic RD model, some of them specifically designed to deal with the border effect [HP01] (see below for a definition of border effect). The RD model has been criticized because of the unrealistic behavior caused by uncorrelated changes in direction and speed (see for ex. [HGPC99]). In [Bet01b], the author proposed a variation of the RD model, with two correlated processes, one to define the speed and another one to define the changes of direction (no correlation between different nodes). The authors denoted this variation the *smooth random mobility model*.

Note that the fact that moves in a bounded region gives rise to the so-called *border effect*, which in general can be understood as a modification of the *probability density fucntion (pdf)* describing mobile node positions with respect to the initial *pdf* (typically, uniform), due to the presence of a border. The border effect arises not only in models (such as RD) in which nodes can hit the border and border rules are used to define node behavior in such situation, but also in models (such as RWP) in which nodes can never reach the border of the movement region. Further detailed explanation of the border effect in RWP mobile networks is reported in the next section.

Two further models different to the previous ones are the following:

- The *Brownian motion model*: each of the x- and the y-coordinates describing the current position of each node undergoes a continuous-time stochastic process (these processes are independent for both coordinates, and independent

for all nodes), which is almost surely continuous and the changes in the positions between any two times t_1, t_2 with $0 \leq t_1 \leq t_2$ follow a normal distribution $N(0, t_2 - t_1)$. Moreover, the changes between $t_1 \leq t_2$ are independent from the changes in $t_3 \leq t_4$, if $t_2 \leq t_3$. Brownian motion can be considered as the *limit* case of the Random Direction model, where the period of time after which a new angle is chosen tends to 0 (see for example [CC07]).

- An approach orthogonal to the previous ones was undertaken in [DSW06] in order to accomplish group communication tasks between a set of processors. The model is the following: given n processors executing programs, the communication between the processors is established with the help of an agent who visits the processors. If there are more than one agent and two agents collide at one processor, they merge into one, and if there is no agent, after some time an agent is automatically generated by a processor. The agent performs a random walk on the processors (the next processor could be chosen from some suitably defined neighborhood of the current processor or it could be chosen from the whole set of processors), and whenever it arrives at a processor, the processor stops its current program and replaces it by a new program using the information the agent is carrying. The agent's goal is to broadcast the information in such a way that each processor is visited by the agent at least every M steps, where M depends on n, and that each processor executes a step infinitely often. The authors design agents satisfying these conditions for different group communication tasks and they prove that starting from any arbitrarily chosen node, these agents have an expected cover time of at most $O(n^3)$.

1.4 Structural properties of Random WayPoint mobile networks

In this section, we present theoretical characterizations of structural properties of networks whose nodes move according to a very popular mobility model: the *Random WayPoint* mobility model (RWP). We show how these characterizations have been used to considerably improve accuracy of wireless network simulation. Some of these characterizations (e..g., node spatial distribution) have been used also to study fundamental mobile network properties, such as connectivity (see Section 1.5).

RWP is by far the most commonly used mobility model used in wireless mobile network simulation. Given its popularity, the structural properties of RWP mobile networks have been deeply investigated in the literature, as well as their effects on simulation accuracy.

In the remainder of this section, we focus our attention on two such structural properties, namely *node spatial distribution* and *instantaneous average nodal speed*, and discuss their impact on accuracy of RWP mobile network simulation. We then show how theoretical characterizations of the above properties have been used to define a "perfect" simulation methodology, which completely removes the accuracy issues previously identified.

1.4.1 RWP node spatial distribution

The first structural property of RWP mobile networks that has been formally studied is the asymptotic node spatial distribution, which can be formally defined as follows. Let f_t be the *pdf* describing node position within the movement region at time t of the mobility process. The asymptotic node spatial distribution is a *pdf* formally defined as

$$f_\infty = \lim_{t\to\infty} f_t,$$

whenever the limit on the right hand side exists, i.e., that the mobility model has a *stationary* node spatial distribution. In the literature, it has been proven that most mobility models described in the previous section (e.g., RWP, RD, Brownian, etc.) indeed have a stationary node spatial distribution.

In the following, we present a formal characterization of f_∞ in the presence of RWP node mobility, which we denote by f_{RWP}. In particular, we will survey results that show that $f_{RWP} \neq f_U$ (f_U is the uniform *pdf* on the movement region), unless the expected pause time at the waypoints tends to infinity[3]. Thus, we are in presence of the border effect, which can cause considerable inaccuracies in wireless network simulation. In fact, if simulation results are gathered after a relatively short time after network setup, the node spatial distribution of RWP mobile nodes might not have reached the stationary condition, implying that, from a topological point of view, network conditions are different from those reached at stationary state. To make this point clearer, assume that results of a network simulation are averaged over a time interval starting after 100*secs* since the beginning of simulation, and ending after 900*secs* (these are quite standard simulation intervals in the networking literature). Furthermore, assume that RWP node spatial distribution takes 1000*secs* to stabilize (this is also a reasonable stabilization time, see [BRS03]). Then the outcome of the simulation experiment might be highly inaccurate, since results are gathered *before* the network has reached its stationary state.

Another pitfall of the border effect is on networking protocol performance optimization: typically, networking protocols (e.g., routing protocols) are optimized under the assumption that nodes are uniformly distributed in a certain region. However, if nodes move according to RWP mobility, this assumption is no longer true at stationary state, implying that protocol performance can indeed be highly suboptimal in presence of mobility.

The first analytical study of node spatial distribution under RWP mobility is reported in [BRS03], for the case of nodes moving in the unit square. In that paper, RWP mobility is described as a stochastic process $\{D_i, T_{p,i}, V_i\}$, where D_i is a random variable denoting the two-dimensional coordinates of trip i destination, $T_{p,i}$ is a random variable denoting the pause time at D_i, and V_i is a random variable denoting the node velocity during trip i. The actual value of D_i will be represented by d_i. First, the authors prove a result concerning ergodicity of the sequence of random variables

[3] Note that the fact that the expected pause time at waypoints tends to infinity implies that nodes are asymptotically static, i.e., RWP model under this condition degenerates to a static network.

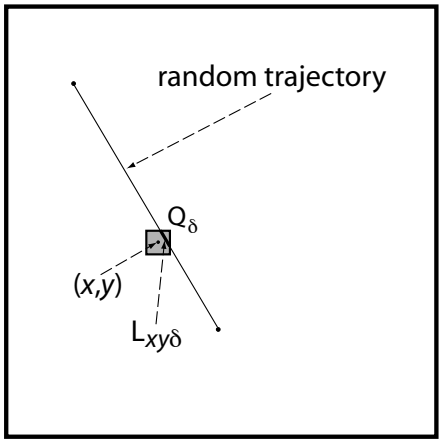

Fig. 1.1 The *pdf* of a RWP mobile node can be characterized by computing the expected length of the segment $L_{xy\delta}$ representing the intersection between a random trajectory and square Q_δ of side δ centered at (x,y) (shaded area).

$\{L_i\}$, where $L_i = ||d_i - d_{i-1}||$, that is, L_i denotes the length of the i-th trip. In particular, the authors show that repeatedly sampling from a single random variable in the sequence is statistically equivalent to successively sampling from the sequence $\{L_i\}$. This first result allows reducing the problem of characterizing $f_{RWP,0}$ when the pause time at waypoint is 0 to one of computing the intersection between a random trajectory and an arbitrarily small square of side $\delta > 0$ centered at a certain coordinate (x,y) (see Figure 1.1). This stems from the fact that f_{RWP} can be considered as constant within Q_δ as $\delta \to 0$, implying that

$$f_{RWP,0}(x,y) = \lim_{\delta \to 0} \frac{P(x,y,\delta)}{\delta^2}, \tag{1.1}$$

where $P(x,y,\delta)$ is the probability that an RWP mobile node is located within a square of side δ centered at (x,y). Thus, $f_{RWP,0}$ can be characterized by evaluating $P(x,y,\delta)$. Since ergodicity of $\{L_i\}$ implies for a successively large sample size that

$$P(x,y,\delta) = \frac{E[L_{xy\delta}]}{E[L]},$$

and $E[L]$ (the expected distance between two random uniform points in a square) is well-known from geometric probability, characterizing $f_{RWP,0}$ boils down to computing $E[L_{xy\delta}]$, i.e., the expected length of the intersection between a random trajectory and a square of side δ centered at (x,y). The value of $E[L_{xy\delta}]$ is closely approximated in [BRS03] through computing a set of two-dimensional integrals, yielding

the following expression for f_{RWP} in the region $(0 \le x \le 0.5) \cup (0 \le y \le x)$[4]:

$$f_{RWP,0}(x,y) = 6y + \frac{3}{4}\left(1-2x+2x^2\right)\left(\frac{y}{y-1} + \frac{y^2}{x(x-1)}\right) + \\ + \frac{3y}{2}\left[(2x-1)(y+1)\ln\left(\frac{1-x}{x}\right) + (1-2x+2x^2+y)\ln\left(\frac{1-y}{y}\right)\right] .$$

The density function $f_{RWP,0}$ is drawn in Figure 1.2. As seen from the figure, f_{RWP} is bell-shaped with a higher concentration in the center of the movement region, reflecting the fact that a random trajectory is relatively more likely to cross the center than the boundary of the region.

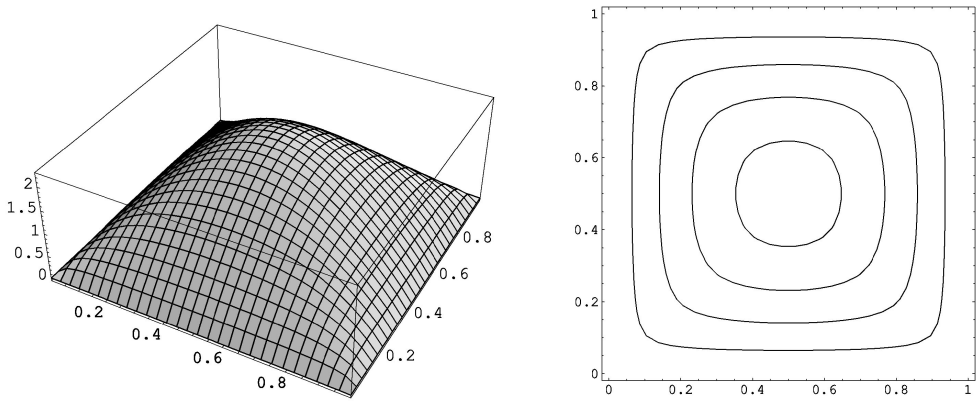

Fig. 1.2 Density function of a RWP mobile network with pause time set to 0: 3D plot (left), and contour lines (right).

After deriving the *pdf* under the assumption of zero pause time, the authors of [BRS03] consider the more general case of pause times chosen according to an arbitrary probability distribution, and show that the resulting node spatial distribution has the following shape:

$$f_{RWP} = p_p f_U + (1-p_p) f_{RWP,0},$$

where $p_p = \lim_{t\to\infty} p_p(t)$, and $p_p(t)$ is the probability that an RWP mobile node is pausing at time t. Thus, f_{RWP} is the sum of two components: a uniform component, accounting for the fact that when nodes are resting at a waypoint they are uniformly distributed, and a non-uniform component, reflecting the fact that when nodes are moving they are more likely located near the center of the movement region. The

[4] Values of $f_{RWP,0}$ in the other regions of the unit square are obtained by symmetry.

derivation of p_p is quite straightforward, and yields:

$$p_p = \frac{E[T_p]}{E[T_p] + \frac{E[L]}{v}},$$

under the hypothesis that the node velocity is fixed to $v > 0$.

In a more recent paper [HLV06], Hyytiä et al. provide the exact characterization of $f_{RWP,0}$, and generalize the previously described results to arbitrary convex shapes of the movement region and arbitrary waypoint distribution.

1.4.2 RWP average nodal speed

A second property of RWP mobile networks that has been extensively studied is the *average nodal speed*, which is formally defined as follows. Assume n nodes move independently within a region according to the RWP mobility model, and denote by $v_i(t)$ the instantaneous speed of the i-th node at time t. The asymptotical average nodal speed v_{RWP} is defined as

$$v_{RWP} = \lim_{t\to\infty} \frac{\sum_{i=1}^{n} v_i(t)}{n} .$$

The first paper that formally investigates the average nodal speed in RWP mobile networks is [YLN03a], where the authors prove that $v_{RWP} \neq v_0$ as long as the trip velocity is randomly chosen in a non-degenerate interval, and v_0 is the average nodal speed at time 0. Before giving some details of the derivation, we observe that the fact that $v_{RWP} \neq v_0$ gives rise to the so-called *speed decay phenomenon*, which displays many similarities with the border effect described in the previous section. In fact, similarly to border effect, speed decay affects both simulation accuracy and optimization of network protocols, for the very same reasons the border effect did, i.e., i) stationary conditions for what concerns node velocity are different from initial ones, and ii) they are reached only after a relatively long stabilization period.

The authors of [YLN03a] derive v_{RWP} under the following three assumptions:

1. nodes move in an unlimited, arbitrarily large area; given the current node location (x,y), the next waypoint is chosen uniformly at random in a circle of radius R_{max} centered at (x,y).
2. the pause time is 0.
3. the node velocity is chosen uniformly at random from $[v_{min}, v_{max}]$.

While the second and third assumption are standard, the first assumption, which is done to simplify analysis, apparently perturbs quite a bit the properties of the mobility model. In the paper it is shown that this assumption has no effect on the value of v_{RWP}, which remain the same as in the case of standard, bounded RWP mobility.

Similarly to [BRS03], the authors of [YLN03a] describe the RWP mobility model as a stochastic process $\{V_i, R_i, S_i\}$, where V_i is the random variable denoting the velocity during trip i, R_i is the random variable denoting travel distance during trip i, and S_i is the random variable denoting travel time during trip i. Setting $\sum_{i=1}^{n} v_i(t)/n = V(t)$, then v_{RWP} can be expressed as follows

$$v_{RWP} = \lim_{t\to\infty} V(t) = \lim_{T\to\infty} \frac{1}{T}\int_{[0,T]} V(t)dt = \lim_{T\to\infty} \frac{\sum_{k=1,\ldots,K(T)} r_k}{\sum_{k=1,\ldots,K(T)} s_k} = \frac{E[R_i]}{E[S_i]} ,$$

where $K(T)$ is the total number of trips undertaken within time T, including the last one (possibly incomplete), and where r_k (resp., s_k) is the travel distance (resp., time) of trip k.

Thus, the computation of v_{RWP} is reduced to the problem of computing the expectation of the random variables. R_i and S_i. In [YLN03a], the authors show that

$$E[R_i] = \frac{2}{3}R_{max} \text{ and } E[S_i] = \frac{2R_{max}}{3(v_{max} - v_{min})} \cdot \ln\left(\frac{v_{max}}{v_{min}}\right) ,$$

yielding

$$v_{RWP} = \frac{v_{max} - v_{min}}{\ln\left(\frac{v_{max}}{v_{min}}\right)} .$$

Furthermore, several interesting implications of the discussed characterization for v_{RWP} are presented in [YLN03a]. First, it is observed that $v_{RWP} \leq v_0$, and that $v_{RWP} = v_0$ if and only if $v_{min} = v_{max}$. This implies that the only way of avoiding speed decay is to avoid randomness in speed selection, imposing the same speed to a node during the entire simulation time. While having constant node velocity may be acceptable in some situations, the range of possible reference application scenarios for simulation is considerably reduced with this assumption. For instance, think about a scenario in which mobile nodes represent vehicles moving in a city: clearly, allowing vehicles to change speed during the travel (e.g., to reflect different speed limits) considerably increase simulation representativeness with respect to a situation in which the vehicle speed is fixed throughout the entire simulation time.

The authors of [YLN03a] observe that v_{RWP} becomes relatively closer to v_0 (thus reducing speed decay intensity, and the time needed to reach stationary node velocity) as the speed range interval becomes smaller. A general recommendation to lessen speed decay is to shrink the allowed node speed interval, which comes at the price, however, of reducing the range of possible application scenarios for simulation.

A final and very interesting implication of the v_{RWP} characterization is that the *pdf* of the random variable S_i becomes heavy tailed when $v_{min} \to 0$, implying that $E[S_i]$ becomes infinite, and $v_{RWP} \to 0$. Thus, if v_{min} is set to 0, the stationary regime of an *RWP* mobile network actually coincides with a static network ($v_{RWP} = 0$), and is reached only after infinite time. It is clear then that setting $v_{min} = 0$, as it is actually very common in wireless network simulation, severely impacts simulation accuracy,

since simulation results cannot be gathered before the node velocities have reached the stationary state.

1.4.3 The "perfect" simulation

In the previous sections, we have shown how theoretical characterization of RWP mobile network properties can disclose sources of inaccuracy in wireless network simulation. Possible countermeasures have also been discussed, which essentially amounts to:

(a)*simulation "warm-up"*: run the simulation for a relatively long time interval before starting collecting simulation results;
(b)*reducing speed range*: choose velocity from a smaller speed interval.

Unfortunately, both approaches for improving wireless network simulation approaches have considerable drawbacks, which discourage their usage in simulation practice. In particular, (a) causes considerable wastage of computational resources. Furthermore, estimating the time needed for the network to reach stationary conditions is difficult, and in some situations the time needed to reach stationarity can actually be infinite, for instance when v_{min}=0. Moreover, the approach (b) also has considerable drawbacks, as it considerably reduces the range of possible reference application scenarios for simulation. Furthermore, (b) has effect only on the speed decay phenomenon, but cannot be used to mitigate the border effect.

Motivated by the above observations, researchers have made efforts to design a "perfect" simulation methodology, in which issues with simulation accuracy can be solved without incurring the drawbacks of approaches (a) and (b). A first noteworthy contribution in this direction is [YLN03b], where the authors present a methodology to remove the speed decay effect without reducing the speed range interval, with the only constraint that $v_{min} > 0$. The authors' goal is to initialize the system directly in the stationary state, without the need of a "warm-up" period. The authors start deriving the *pdf* of the stationary average node velocity $\mathscr{V}_{RWP}$, and show that $\mathscr{V}_{RWP}$ cannot be directly used to initialize the system: if $\mathscr{V}_{RWP}$ is used instead of a uniform distribution in $[v_{min} > 0, v_{max}]$ to select initial node velocities, the *pdf* of the resulting stationary average node velocity *changes*, and it is no longer $\mathscr{V}_{RWP}$. Then, the authors show that a possible way of avoiding this problem is using a *composite* mobility model, where the *pdf* used to select initial node speed is different from that used to select the speed of next trips. In particular, the authors of [YLN03b] formally prove that the following methodology completely removes speed decay:

1. use $\mathscr{V}_{RWP}$ to select speed of the first trip;
2. use default speed distribution (uniform in $[v_{min} > 0, v_{max}]$) to select speed of next trips.

In [LV06], the authors generalize the results of [YLN03b] to a wide class of mobility models (including RWP model, RD model, etc.), and show that the "perfect"

simulation methodology defined in [YLN03b] can be used not only for average node speed, but also for any structural network property admitting a stationary distribution. With respect to this, the authors of [LV06] show that a necessary and sufficient condition for a mobility model to admit stationary structural distributions is that the expected trip duration is finite. Thus, for models such as RWP, the "perfect" simulation methodology of [YLN03b] can be used not only to remove speed decay, but also the border effect.

1.5 Formal studies of connectivity on MANETs' models

1.5.1 Connectivity threshold for mobility models

As described in the previous section, the border effect may considerably impact simulation accuracy. In this section, we analyze the consequence of the border effect on the formal analysis of properties for MANETs, in particular referring to the critical transmission range for connectivity.

• *Connectivity threshold for mobile models*. Using the previous result, Santi [San05a] studies the connectivity threshold for mobile networks. His model is the following: There are n vertices deployed uniformly at random in the unit square $[0,1]^2$. The nodes move randomly, but the mobility model is not fixed, it only must meet two conditions: it must be *bounded* and *obstacle-free*. A mobility model $\mathscr{M}$ is said to be *bounded* if the support of the probability density function *pdf* of the long-term distribution of the nodes is contained in $[0,1]^2$. Similarly, $\mathscr{M}$ is said to be *obstacle-free* if the support of the *pdf* contains $[0,1]^2 \setminus \partial[0,1]^2$, where $\partial[0,1]^2$ denotes the boundary. In other words, every subregion with non-zero measure has to have positive probability to contain at least one node at a given time. Notice that the random direction model, the random waypoint model and Brownian motion are all bounded and obstacle-free. Moreover, not necessarily all nodes have to move at the same speed, each one can choose its speed from an interval $[v_{\min}, v_{\max}]$. Also, the nodes can pause for a predefined amount of time t_p after having reached their destination.

In particular, due to border effects and due to different node velocities, the long-term spatial distribution of the nodes might be different from the starting distribution, even if they start with the uniform distribution. Define the *mobile threshold for connectivity* $r_{\mathscr{M}}$ as the minimum value of the radius r, such that when taking a snapshot of the graph chosen from the long-term spatial distribution of the nodes, the graph is connected. Notice $r_{\mathscr{M}}$ might be different from the threshold of the static case $r_c = \sqrt{\frac{\log n}{\pi n}}$. In fact, the first result of the paper states that if the *pdf* of the mobility model $f_{\mathscr{M}}$ is continuous on $\partial[0,1]^2$ and $\min f_{\mathscr{M}} > 0$, then a.a.s. $r_{\mathscr{M}} = c\sqrt{\frac{\log n}{\pi n}}$ with $c \geq 1$. The proof uses the fact that in the static case, a.a.s. the threshold of connectivity equals the longest edge of the Euclidean minimum spanning tree built on the n points (see [Pen97]).

The second result the paper states that in the random waypoint model with pause time t_p and $v = v_{\min} = v_{\max}$, a.a.s. the connectivity threshold of the long-term spatial distribution $r^w_{t_p} = \frac{t_p + \frac{0.521405}{v}}{t_p}\sqrt{\frac{\log n}{\pi n}}$, for $t_p > 0$, and $r_0 \gg \frac{\log n}{n}$ for $t_p = 0$. Intuitively the results says that when nodes stop at the waypoint for a positive amount of time before choosing the next waypoint, the connectivity threshold of the long-term distribution differs from the static case by only a constant factor. In the case when $t_p \to \infty$, $r^w_{t_p} \to r_c$, and the long-term spatial distribution becomes the uniform distribution. On the other hand, if the nodes start travelling towards the next waypoint immediately after touching the current waypoint, the connectivity threshold is asymptotically larger than in the static case. The intuition behind this result is as follows: the formula for the *pdf* contains two components; one for the time a node is resting at a waypoint, which is uniform since the waypoint is chosen uniformly at random, and a mobility component responsible for border effects. If the uniform component of the *pdf* is not 0, it asymptotically dominates over the mobility component, and the connectivity threshold is asymptotically the same as in the static case. On the contrary, if the uniform component is 0, the *pdf* coincides with the mobility component, which has a different asymptotic behavior than uniform, implying a larger connectivity threshold.

1.5.2 Connectivity periods on mobile models

• *The walkers' model on the grid*. The authors in [DPSW08] present a model of establishment and maintenance of communication between mobile nodes, denoted *walkers* in the paper, where the nodes move in a fixed environment modeled by a toroidal grid T. Therefore, the authors present a hybrid random model. The model is defined as follows: given a toroidal square grid in the plane $T = (V,E)$ with $|V| = N = n^2$, a set W of *walkers* with $|W| = w$, and a "transmitting distance" d (the same for all the walkers), the w walkers are sprinkled randomly and independently on the N vertices of T (a vertex may contain more than one walker). Two walkers w_1 and w_2 can communicate in one hop if the Euclidean distance between the position of the walkers is at most d. Two walkers can communicate if they can reach each other by a sequence of such hops.

Then, in a synchronized way, each walker performs an independent standard random walk on the nodes of T. That is, each walker moves at each time step to one of the four neighboring vertices, all chosen with equal probability $1/4$. Hence, for any time $t \in \mathbb{N}$, one can define the *random graph of walkers* $\mathscr{W}_t(T,w,d)$: the vertices of this graph are the w walkers together with their position they are occupying on T at time t, and there is an edge between two walkers if their Euclidean distance is at most d (if more than one walker occupies a vertex of the grid, the authors do not consider the corresponding multigraph and consider that position of the grid as if there was only one walker). The authors then study the behavior (as $N \to \infty$) of the connectivity and disconnectivity of $\mathscr{W}_t(T,w,d)$ for any $t \in \mathbb{N}$, where $\mathscr{W}_0(T,w,d)$

is formed by the initial distribution of the walkers on T (see Figure 1.3 for a toy example of one step).

The paper first examines the initial *static* case $\mathscr{W}_0(T,w,d)$, which is a snapshot of the process at one point in time: in particular, the paper studies the distribution of the number of isolated vertices of $\mathscr{W}_0(T,w,d)$, as well as some other information which helps to answer the dynamic questions. Define h to be the number of grid points within distance d of any fixed point in T. Clearly, $h=\Theta(d^2)$. If $d=\Omega(n)$, then $\mathscr{W}_0(T,w,d)$ is connected a.a.s., so the interesting case is $d=o(n)$, i.e., $h=o(N)$. Furthermore denote by $\rho=w/N$ be the expected number of walkers at a vertex and define the parameter $\mu=N(1-e^{-\rho})e^{-h\rho}$. The authors first prove that in the static initial case at time $t=0$, $\mathbf{Pr}[\mathscr{W}_0(T,w,d)$ is connected$]=e^{-\mu}+o(1)$.

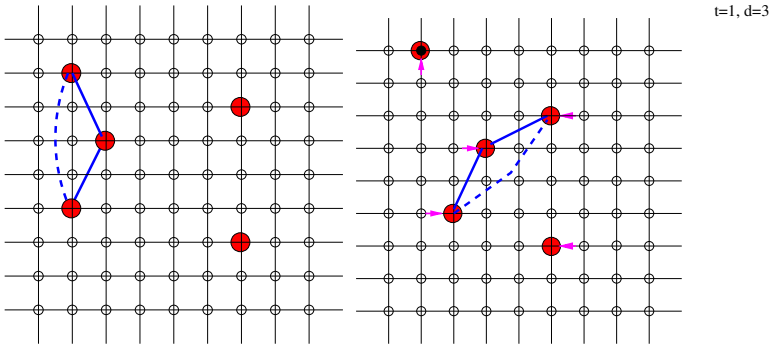

Fig. 1.3 A step of the walkers' problem on the grid. The solid line represents direct communications of the ad-hoc network, the dashed line represents communication between nodes that are at distance more than d.

Using the information from the static case, in the dynamic setting, the crux of the paper is the study, as t evolves, of the birth and death of isolated vertices, and the sudden connection and disconnection of $\mathscr{W}_t(T,w,d)$. Let LD_t be the random variable counting the length of the disconnected period (similarly, a random variable LC_t counting the length of the connected period is considered) starting at time step t provided that it really starts to be disconnected at t. Define the *average length* of a disconnected period starting at time t to be $LD_{\text{av}} := \mathbf{E}(LD_t \mid LD_t > 0)$, which is independent of t, and is a function of N, d and w. The authors show that the following hold about LD_{av}:

$$LD_{\text{av}} \sim \begin{cases} \frac{e^{\mu}-1}{\mu b \rho} & \text{if } d\rho \to 0, \\ \frac{e^{\mu}-1}{1-e^{-\lambda}} & \text{if } d\rho \to c, \\ e^{\mu} & \text{if } d\rho \to \infty, \end{cases}$$

where $b=\Theta(d)$ is a function related to the boundary of the ball of radius r in T, and $\lambda=\left(1-e^{-b\rho}\right)\mu$ with $0<\lambda<\mu$ for $d\rho\to c$. Furthermore, LD_t converges in probability for $t\to\infty$ (N fixed) to a random variable LD, where $LD\sim LD_{\text{av}}$

a.a.s. Similar results can be given for the average length of connected periods. For the proof, the authors calculate joint factorial moments of variables accounting for births, deaths and survivals of isolated vertices, and they show that the connectivity (disconnectivity, respectively) of the graph is asymptotically equivalent to the non-existence (existence, respectively) of isolated vertices.

The results in the paper are proved in full generality, under any norm and for $T = [0,1)^m$ for $m = \Theta(1)$. Also, the paper proves results on the connectivity and disconnectivity periods for the case when the the underlying graph of fixed paths is a cycle.

• *The DRGG model with radii r_c.* The paper [DMP09a] studies the connectivity of a Random Direction type model for MANETs. The model is a RGG at the connectivity threshold r_c, where all vertices move at the same speed. This dynamic model is denoted by the authors as the Dynamic Random Geometric Graph. More formally, the model is the following: at the starting of the process ($t = 0$), n nodes are scattered independently and uniformly at random in the unit torus $[0,1)^2$. At any time $t \in \{0,1,2,3,\ldots\}$, two nodes are connected if their Euclidean distance is at most r. The authors fix the value of r to be the value at the connectivity threshold for static RGG, i.e., $r = r_c = \sqrt{\frac{\log n \pm O(1)}{\pi n}}$. The dynamic model is the following: given two positive reals $s = s(n)$ and $m = m(n)$, at any time step t, each node i jumps a distance s in some direction $\alpha_{i,t} \in [0,2\pi)$. The initial angle $\alpha_{i,0}$ is chosen independently and uniformly at random for each node i, and then at each time step each node changes its angle independently with probability $1/m$. Thus, the number of steps a node has to wait before changing its direction follows a geometric distribution with expectation m. New angles are also selected independently and uniformly at random in $[0,2\pi)$ (see Figure 1.4 for a toy example of the changes of the graph in a single step).

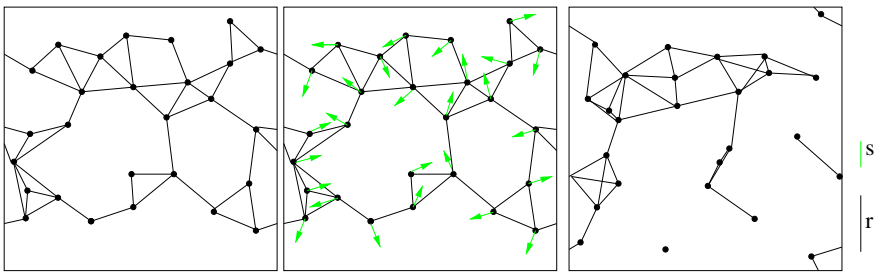

Fig. 1.4 A step in the DRGG. Starting at a given ad-hoc graph (left picture), every node chooses a new direction chosen at random (center picture), creating a new ad-hoc graph (right picture).

The goal of the paper is to analyze the expected length of (dis)connectivity periods of the underlying graph. To state the main result more formally, denote by $\mathscr{C}_t$ the event that the random graph is connected at time t, and similarly denote by $\mathscr{D}_t$ the event that the graph is disconnected at time t. Furthermore, define by $L_t(\mathscr{C})$ to be the random variable counting the number of consecutive steps that $\mathscr{C}$ holds

starting from time t (possibly ∞ and also 0 if $\mathscr{C}_t$ does not hold). $L_t(\mathscr{D})$ is defined analogously by interchanging $\mathscr{C}$ with $\mathscr{D}$. It can be shown that the distribution of $L_t(\mathscr{C})$ and $L_t(\mathscr{D})$ is independent of t. Define also

$$\lambda_{\mathscr{C}} = \mathbf{E}\left(L_t(\mathscr{C}) \mid \mathscr{D}_{t-1} \wedge \mathscr{C}_t\right) \quad \text{and} \quad \lambda_{\mathscr{D}} = \mathbf{E}\left(L_t(\mathscr{D}) \mid \mathscr{C}_{t-1} \wedge \mathscr{D}_t\right),$$

that is, $\lambda_{\mathscr{C}}$ ($\lambda_{\mathscr{D}}$, respectively) count the expected number of steps that the graph stays connected (disconnected, respectively) starting at time t conditional upon the fact that it becomes connected (disconnected) precisely at time t. The main result of the paper is the following: if $srn = \Theta(1)$, then

$$\lambda_{\mathscr{C}} \sim \frac{1}{1-e^{-\mu(1-e^{-4srn/\pi})}} \quad \text{and} \quad \lambda_{\mathscr{D}} \sim \frac{e^{\mu}-1}{1-e^{-\mu(1-e^{-4srn/\pi})}}.$$

Otherwise, it is

$$\lambda_{\mathscr{C}} \sim \begin{cases} \frac{\pi}{4\mu srn} & \text{if } srn = o(1), \\ \frac{1}{1-e^{-\mu}} & \text{if } srn = \omega(1), \end{cases} \quad \text{and} \quad \lambda_{\mathscr{D}} \sim \begin{cases} \frac{\pi(e^{\mu}-1)}{4\mu srn} & \text{if } srn = o(1), \\ e^{\mu} & \text{if } srn = \omega(1). \end{cases}$$

One can observe that for $srn = o(1)$ and $srn = \omega(1)$ the results of $\lambda_{\mathscr{C}}$ and $\lambda_{\mathscr{D}}$ correspond to the respective limits in the case when $srn = \Theta(1)$. These results have various consequences; on the one hand the expected number of steps in a period of connectivity (disconnectivity) does not depend on m, that is, it does not depend on how often the nodes change their direction. On the other hand, $\lambda_{\mathscr{C}}$ and $\lambda_{\mathscr{D}}$ are non-decreasing in s. The intuition behind this is as follows: if the distances between two time steps are big, the correlations between two consecutive steps are smaller, and connectivity/disconnectivity changes more frequently. For a very large s (case $srn = \omega(1)$), $\lambda_{\mathscr{C}}$ and $\lambda_{\mathscr{D}}$ do not depend on s anymore, since for such a value of s two consecutive steps are roughly independent. Finally, one can observe that in the case $srn = o(1)$ models the underlying continuous-time model very well: denote by $\tau_{\mathscr{C}} = s\lambda_{\mathscr{C}}$ ($\tau_{\mathscr{D}} = s\lambda_{\mathscr{D}}$, respectively) the distance covered by each vertex during a connectivity (disconnectivity) period. Then,

$$\tau_{\mathscr{C}} \sim \frac{\pi}{4\mu rn} \sim \frac{\pi\sqrt{\pi}}{4\mu\sqrt{n\ln n}}, \qquad \tau_{\mathscr{D}} \sim \frac{\pi(e^{\mu}-1)}{4\mu rn} \sim \frac{\pi\sqrt{\pi}(e^{\mu}-1)}{4\mu\sqrt{n\ln n}},$$

which asymptotically do not depend on s. Since these results also hold if s tends to 0 arbitrarily fast, the related continuous-time model has a similar behavior: in that model the traveled distance during periods of connectivity (disconnectivity) also does not depend on the average distance sm between changes of angle.

The main ingredient of the proof is the fact that the probabilities needed to compute $\lambda_{\mathscr{C}}$ and $\lambda_{\mathscr{D}}$ can be expressed in terms of the probabilities of events involving only two consecutive steps. This is surprising, since in this case (in contrast to the article [DPSW08]) the sequence of connected/disconnected states is not Markovian - staying connected for a long period of time makes it more likely to remain connected for one more step. As in the article [DPSW08], it turns out

that the existence/non-existence of isolated vertices is asymptotically equivalent to the disconnectivity/connectivity of the graph, both in the static case and for two consecutive steps. Although the proof is technically very different from the one in [DPSW08], it is similar in spirit: the characterization of the changes of the number of isolated vertices between two consecutive steps is based on the computation of the joint factorial moments of the variables accounting for these changes (births/deaths/survivals of isolated vertices). As in [DPSW08], it is not obvious that the probability of existence of components of larger sizes in the dynamic model is negligible compared to the probability of sudden appearance of isolated vertices, but in the paper it is shown to be the case.

1.5.3 The effect of mobility to speed up message dissemination in sparse networks

In this section we survey in chronological order three results which show that high mobility of nodes helps in disseminating information.

• *The source-destination pairs-model.* The work [GT02] can be considered as the first attempt to formally analyze a model of mobility. The model is the following: there are n nodes ($n \to \infty$) all lying in the disk of unit area. The location of the i-th node at time t is given by the random variable $X_i(t)$. Each of the n nodes is a source node for one session and a destination node for another session, and each node i has an infinite stream of packets to send to its destination $d(i)$. The source-destination (S-D) association is established initially and does not change over time. The nodes are mobile, but the mobility model described by the authors is non-constructive: the process $\{X_i(\cdot)\}$ is stationary and ergodic with stationary distribution uniform on the disk, and trajectories of different nodes are independent and identically distributed. It is a drawback of the paper, that the exact movement of the nodes is not explained: in particular, it is not clear what happens when a node touches the boundary of the disk. Recall that as mentioned before, boundary effects can change the distribution. The information exchange is not restricted to nodes within a certain distance, but it is the following: at slotted time t, node i has transmission power $P_i(t)$. Denote by $\gamma_{ij}(t)$ the channel gain from node i to node j, such that the received power at node j is $P_i(t)\gamma_{ij}(t)$. Formally, $\gamma_{ij}(t)$ is defined as $\frac{1}{|X_i(t)-X_j(t)|^\alpha}$, where α is a parameter greater than 2. Node i can transmit to node j if

$$\frac{P_i(t)\gamma_{ij}(t)}{N_0 + \frac{1}{L}\sum_{k\neq i} P_k(t)\gamma_{kj}(t)} > \beta, \tag{1.2}$$

where β is the signal-to-interference ratio requirement for successful communication, N_0 is the background noise power, and L is the processing gain of the system, it can be taken to be 1. Intuitively speaking, on the one hand, the closer j to i at time t, the bigger $\gamma_{ij}(t)$, and the more likely it is that node i can transmit a packet to node j. On the other hand, relative distances between nodes also play a role: if

a node i is close to neighbour j, but j has many other neighbors very close, and at the same time i is further away from another node j', whose neighbors are all further away than i, it might happen that i is able to transmit to j' and not to j. In the following it is assumed that all nodes transmit at the same power P. Whether or not a node transmits to another one is decided by an external scheduler. Every node is assumed to have an infinite buffer to store packets, and when packets are transmitted from source to destination, they can go through one or more other nodes serving as relays. The goal is to find a scheduling policy with high long-term throughput. To make this concept more precise, define by $M_i^\pi(t)$ the number of source node i packets that $d(i)$ receives at time t under the scheduling policy π. A throughput $\lambda(n)$ is feasible, if there exists a policy π such that for *every* S-D pair i we have

$$\lim_{T\to\infty} \inf \sum_{t=1}^{T} M_i^\pi(t) \geq \lambda(n),$$

and the goal is to maximize $\lambda(n)$.

The authors first prove a lower bound in a dynamic model where relay nodes are forbidden. More precisely, they show that there exists a constant $c > 0$ such that the probability of having a throughput of at least $cn^{-(1/(1+\alpha/2))}$ tends to 0 for n sufficiently large. The theorem is stronger if α is closer to 2: if $\alpha \to 2$, the probability of a throughput of $c/\sqrt{n}$ tends to 0. This is the same lower bound as in the static model [GK00]. The intuition behind this result is the following: if long distances are allowed, then interference limits the number of concurrent transmissions. If a scheduling policy allows only short transmissions, then only a small fraction of S-D pairs is sufficiently close to transmit a packet.

Next, as a main result of their paper, the authors show that mobility helps if intermediate relay nodes are permitted. If for every S-D pair every other node can serve as intermediate relay (that is, at different time slots different nodes may contain part of the packet stream between i and $d(i)$), an asymptotically optimal throughput of $\lambda(n) = c$ for some $c > 0$ can be attained. To prove this the authors consider the following scheduling policy: every packet is relayed at most once. For every time slot t, the set of nodes is randomly partitioned into a set of potential senders (of size sn for some constant $s > 0$) and potential receivers. Each sender node may transmit packets to its nearest neighbor among all receiver nodes, and the sender indeed transmits if the interference generated by other senders is sufficiently small (according to the formula given in (1.2)). The algorithm runs in two interleaved phases: in phase 1 (in odd time slots, say) packets are sent only from source nodes to relays (or directly to the destination node), in phase 2 (in even time slots, say) packets are sent only from relays to destination nodes. The proof of the result uses the fact that at any particular moment in time the distribution of the points is uniform on the disk, together with some results on the asymptotic distribution of extrema of i.i.d. random variables. We recall once again, that is not clear how the nodes move and what happens when touching the boundary.

• *The DRGG model below r_t.* In the work [JMR09] the authors study a very general Random Direction type model with a radius below the threshold of the existence of

a giant component. More precisely, the authors consider the following model: at the beginning n nodes ($n \to \infty$) are distributed uniformly at random in a square $\mathscr{A} = \mathscr{L} \times \mathscr{L}$, where $L = c\sqrt{n}$ for some large constant $c > 0$. Two nodes can exchange information if they are within Euclidean distance 1. It is assumed that information exchange takes zero time, once two nodes are at distance ≤ 1. By the choice of L, $n/\mathscr{A}$ tends to a small constant ($n/\mathscr{A} < 1/\pi$), which in the static case corresponds to a random geometric graph below the thermodynamical limit $r_t = c/\sqrt{n}$. Recall in Section 2 we already pointed that for a radius r below the thermodynamical limit r_t, the RGG is disconnected and it does not have yet a giant component. The mobility model is the following: the nodes follow random trajectories with Poisson rate τ, keeping uniform speed between direction changes. When a node hits the boundary at an incidence angle θ, it follows the mirror reflection policy, i.e., the node bounces back at angle $\pi - \theta$. Therefore, the probability density for a node to travel a time t in a certain direction before changing the direction is

$$\frac{1}{2\pi}\tau \exp(-\tau t),$$

where τ is a parameter controlling the speed of change. Notice that if $\tau \to \infty$ then the mobility represents Brownian motion, while if $\tau \to 0$ the mobility represents a random waypoint model with the mirror reflection policy, where the nodes only change direction when touching the boundary of the square. The factor $\frac{1}{2\pi}$ comes from the fact that every angle has the same probability to be chosen.

The authors give an upper bound on the speed at which information can be propagated between any pair of nodes. Recall that in the static case information between most pairs of nodes cannot be propagated since the largest connected component for the value of $\nu := n/\mathscr{A}$ to be considered has size $O(\log n)$. The authors show that mobility helps to propagate information. In order to state the result more precisely, consider a node that starts at coordinate $z_0 = (x_0, y_0)$ at time $t = 0$ that wants to propagate information to a destination node starting at coordinate $z_1 = (x_1, y_1)$. The authors show that the destination node can be assumed to be fixed without changing the asymptotic results of the analysis. Denote by $q_\nu(z_0, z_1, t)$ the probability that the destination receives the information before time t (n is assumed to be large, but the density ν is a constant). A scalar $s_0 > 0$ is called an upper bound for the propagation speed, if for all $s > s_0$, $\lim q_\nu(z_0, z_1, \frac{|z_0 - z_1|}{s}) = 0$ whenever $|z_1 - z_0| \to \infty$. Using this definition, the authors show that an upper bound on the information propagation speed is

$$\min_{\rho, \Theta > 0} \left\{ \frac{\Theta}{\rho} \text{ with } \Theta = \sqrt{\rho^2 \nu^2 + \left(\tau + \frac{\frac{n}{\mathscr{A}} 4\pi \nu I_0(\rho)}{1 - \frac{n}{\mathscr{A}} \pi \frac{2}{\rho} I_1(\rho)} \right)^2} - \tau \right\}, \qquad (1.3)$$

where v is the maximum node speed, $I_0()$ and $I_1()$ are *modified Bessel functions* defined by $I_0(x) = \sum_{k \geq 0} (\frac{x}{2})^{2k} \frac{1}{(k!)^2}$, and $I_1(x) = \sum_{k \geq 0} (\frac{x}{2})^{2k+1} \frac{1}{(k+1)!k!}$. To get some intuition about this bound and its involved parameters, note that the quantities $I_0(x)$ and

$\frac{2}{x}I_1(x)$ are both larger than 1, and therefore the expression has meaning if $\frac{n}{\mathscr{A}} < \frac{1}{\pi}$, as above the thermodynamical limit there is a giant component, and therefore the information propagation speed is infinity. Observe also that the obtained value is larger if τ is larger. Such a behavior is expected, since changing directions more frequently may result in faster information propagation, and therefore the propagation speed might be higher. Finally, ρ and Θ are parameters that correspond to the Laplace transform of the sequence of nodes such that a piece of information is visiting on its way from source to destination (see below for a rough explanation).

To prove the result (1.3), the authors decompose the journey (which is the sequence of nodes a piece of information undergoes from the source to the destination) into different segments. These segments either correspond to node movements through which the information is propagated or to direct propagations between two nodes, when a node immediately, without movement, propagates the information to another one due to the fact that the two nodes are at distance ≤ 1. The authors consider the segments as independent, which is not true, since for example two consecutive nodes in the sequence are more likely to move in opposite directions or node speeds are different, and a faster moving node meets more nodes, but they show that in this way they prove an upper bound on the propagation speed for the real model, and hence the assumption is justified.

On the technical side, the authors compute the Laplace transform of the probability density of a fixed journey of length k, defined as a journey where $k+1$ nodes participate in the process of information propagation from the source node to the destination node. Since the segments are considered to be independent, the Laplace transform of the journey is the product of the Laplace transform of the segments. In particular, the Laplace transform of such a journey does not depend on the particular nodes participating, but only on the length of the journey. As the journey, however, is not known in advance, the authors consider the Poisson generating function $G(Z,(\rho,\Theta))$ whose n-th coefficient is the Laplace transform of all journeys in a network with n nodes in a square of size $\mathscr{A}$. They show that this generating function is equivalent to an ordinary generating function whose k-th coefficient is the Laplace transform of the probability density of a fixed journey of length k. Hence, for $n \to \infty$ an upper bound for the asymptotic behavior of $q_n(z_0,z_1,t)$ can be calculated from simpler expressions for journeys composed of independent segments. The asymptotic growth of the Laplace transform of $q_\nu(z_0,z_1,t)$ is then obtained by those values of (ρ,Θ) for which the denominator corresponding to the n-th coefficient of the Poisson generating function $G(\nu,(\rho,\Theta))$ vanishes. The final expression for $q_\nu(z_0,z_1,t)$ is then obtained using the inverse Laplace transform.

One has to point out that the conference version of the article, although sounding very plausible, is not easy to read. In particular, the probability spaces are not clearly defined.

• *The hybrid grid model approximating DRGG, for $r > r_t$.* In the Chapter: *Information Spreading in Dynamic Networks: An Analytical Approach*, Andrea Clementi and Francesco Pasquale give an extensive presentation of this model and other previous related models in the specific framework of information spreading in dynamic networks. However, for completeness of our survey, we also briefly sketch

the model. We refer the reader to the mentioned chapter in the present book. In the model used by [CPS09] a RGG is approximated by a very fine grid on which the nodes are restricted to move. Hence, it is a discretized version (with respect to both time and space) of the models used in [JMR09]: there are n nodes ($n \to \infty$) moving on the corner points of a grid inside a square of size $\sqrt{n}$. In more detail, for some given $\varepsilon > 0$, at any time t the nodes occupy one position of $L(n,\varepsilon)$, where

$$L(n,\varepsilon) = \left\{ (i\varepsilon, j\varepsilon) \mid i,j \in \mathbb{N} \wedge i,j \leq \frac{\sqrt{n}}{\varepsilon} \right\}.$$

The position at time $t = 0$ is chosen uniformly at random, independently for all nodes, and at any fixed time slot t two nodes are connected by an edge if their Euclidean distance is less than r. Here $r \geq r_0$, where r_0 is a sufficiently large constant. Therefore, the graph contains a giant component, but is not necessarily connected a.a.s., which would happen only for $r \geq c \log n$. The mobility model is the following: for a given *move radius* ρ, define the *move graph* $M_{n,\rho,\varepsilon} = (L_{n,\varepsilon}, E_{n,\rho,\varepsilon})$, where

$$E_{n,\rho,\varepsilon} = \{(p,q) \mid p,q \in L_{n,\varepsilon},\ (p,q) \leq \rho\},$$

and $d(\cdot,\cdot)$ is the Euclidean distance. Furthermore, for any position p in the square, define by $\Gamma(p) = \{q \mid (p,q) \in E_{n,\rho,\varepsilon}\}$. A node at position p at time t chooses uniformly at random its position at time $t+1$ among all elements of $\Gamma(p)$. In other words, it chooses a random node in a ρ-vicinity of the original position (see Figure 1.5 for toy example of one step in the present model). Initially, at time $t = 0$,

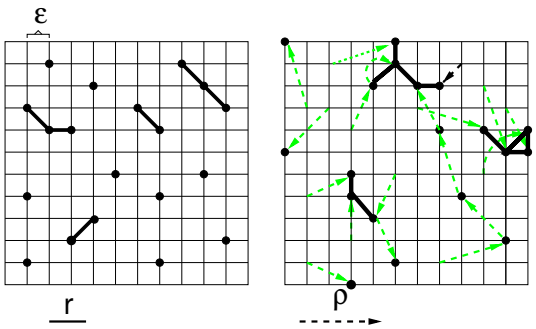

Fig. 1.5 Two consecutive time steps in the model of [CPS09]. On the left the graphs at some fixed time t, where a node connected with all the other nodes at distance $\leq r$. Right picture: the resulting graph after a movement of each vertex of a distance $\leq \rho$. The trajectory of movement is indicated by the light dotted arrows.

one node, the source node, contains a message that should be broadcast to every other node of the network. Whenever at a certain time slot t one node u contains the message and there is another node v within distance r that does not yet contain it, the message is broadcast from u to v. It is assumed that transmission takes zero

time. Recall that the flooding time is the number of time steps required to broadcast the message to all nodes in the network.

The authors prove the following: if $\rho \geq c \log n$ for some constant $c > 0$, then the flooding is a.a.s. completed after

$$O\left(\frac{\sqrt{n}}{\rho} + \log n\right)$$

time steps, which is asymptotically almost tight since the expected flooding time is $\Omega(\sqrt{n}/\rho)$. That is, if the move radius is sufficiently large (i.e., the node velocity is sufficiently high), the flooding time is independent of r (as long as $r \geq r_0$). This is especially interesting for r below the connectivity threshold: flooding can be completed although at every time step the graph is disconnected.

The proof of the result uses a tessellation argument; the square is subdivided into supercells of side length $\Theta(\rho)$. The proof proceeds in the following three steps: first, it is shown that after $O(\log n)$ time steps there is a.a.s. at least one supercell which contains $\Theta(\rho^2)$ informed nodes (the supercell is called quasi-informed). Next, in a second phase, it is shown that, with high-probability, any quasi-informed supercell at time t makes all its adjacent supercells quasi-informed at time $t+1$. Since any supercell set D has a boundary of size at least $\Theta(\sqrt{|D|})$, after $O(\sqrt{n}/\rho)$ time steps all supercells are quasi-informed a.a.s. Finally, in a last phase, it is shown that in $O(\log n)$ time steps a.a.s., any quasi-informed cell becomes completely informed. That is, all nodes of that cell contain the message that should be broadcast.

1.6 Conclusions

We surveyed the main theoretical issues when studying models for MANETs. We described some of the models, where properties have been investigated with a certain degree of formal rigor.

In particular, in Section 4 we have presented theoretical characterizations of fundamental properties such as node spatial distribution and average velocity, under the assumption that nodes move according to the RWP mobility model. In the same section, we have shown how such characterizations have been used to disclose accuracy issues with wireless network simulation practice, and to design a "perfect" simulation methodology solving these issues.

In section 1.5 we presented recent papers dealing with connectivity issues of dynamical models, where nodes move synchronously on $[0,1)^2$. The goal in [San05a] is to study how mobility affects the threshold of connectivity. The author gives the threshold under certain conditions affecting mobility parameters. The papers [DPSW08] and [DMP09a] compute the expected lengths of connectivity and disconnectivity periods of vertices that are moving on a predetermined grid (in the case of [DPSW08]), and of vertices of a dynamic geometric graph whose radius is at the threshold of connectivity (in the case of [DMP09a]). The remaining three papers deal with

the issue of how mobility can be used to maintain the transmission range small while at the same time allowing for connectivity properties. The papers of [JMR09] and [CPS09] are complementary: whereas the authors in [JMR09] study random geometric graphs with a radius below the thermodynamical threshold, the paper [CPS09] considers the case of radii between the thermodynamical threshold and the threshold of connectivity. The third paper studied here, the work of [GT02] is orthogonal to these two since there is no absolute bound on the radius of transmission, but it also supports the hypothesis that mobility can help in propagating information.

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