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Corrigendum

Corrigendum to "On the limiting distribution of the metric dimension for random forests" [European J. Combin. 49 (2015) 68–89]

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In the paper "On the limiting distribution of the metric dimension for random forests" the metric dimension $\beta(G)$ of sparse $G(n,p)$ with $p = c/n$ and $c < 1$ was studied (Theorem 1.2). In the proof of this theorem, for the convergence in distribution Stein's Method was applied incorrectly (see comments later). We provide in this corrigendum a right way to prove the theorem. In order to state the corrected version of Theorem 1.2 of the original paper, we need a few definitions. Define $\mathcal{T}$ as the family of all labeled trees. For each $T \in \mathcal{T}$, let $t_n(T)$ be the number of labeled isolated trees isomorphic to T in $G \in G(n,p)$ with $p = c/n$ (the labeling is obtained from the relative ordering of the labels of the vertices and projecting it onto $\{1,\ldots,|T|\}$), and let $\{t_n(T)\}_T$ be the corresponding sequence. Define also $t_n^*(T) := (t_n(T) - \mathbb{E}[t_n(T)])/\sqrt{n}$ and by $\{t_n^*(T)\}$ (or simply t_n^* below) be the corresponding normalized and scaled sequence. Note that for every T with $|T| > n$, $t_n(T) = 0$ and $t_n^*(T) = 0$. Define $X = \{X(T)\}_{T\in\mathcal{T}}$ (the index set here and below is the set of all isolated labeled trees, where the labeling is obtained from the relative ordering and projecting it onto $\{1,\ldots,|T|\}$) to be the Gaussian sequence with 0 means and covariance function $cov(X(T_1), X(T_2)) = h(T_1)\delta(T_1,T_2) + (c-1)k_1k_2h(T_1)h(T_2)$, where $h(T) = c^{-1}(ce^{-c})^k/k!$ with $|T| = k$, and $\delta(\cdot,\cdot)$ is the Kronecker symbol.

Theorem 1.2 in the original paper has then to be replaced by the following theorem:

Theorem 1.1 (*Theorem 1.2 in the Original Paper*)**.** *Let $G \in G(n,p)$.*

(i) *For $p = o\left(n^{-1}\right)$, $\beta(G) = n(1+o(1))$ asymptotically almost surely.*
(ii) *For $p = \frac{c}{n}$ with $0 < c < 1$, we have $\mathbb{E}\left[\beta(G)\right] = Cn(1+o(1))$, where*

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$$C = e^{-c}\left(\frac{3}{2} + c + \frac{c^2}{2} - e^c - \frac{1}{2}e^{ce^{-c}} + \exp\left(c\frac{1-(c+1)e^{-c}}{1-ce^{-c}}\right)\right.$$
$$\left. - c\frac{e^{-c}}{1-ce^{-c}} - \frac{c^2}{2}\left(\frac{1-(c+1)e^{-c}}{1-ce^{-c}}\right)^2\right), \qquad (1)$$

and $\mathbb{V}\mathrm{ar}\,\beta(G) = \Theta(n)$. Moreover, the sequence of random variables $\{(\beta(G) - \mathbb{E}[\beta(G)])/\sqrt{n}\}$ converges in distribution (as $n \to \infty$) to $\sum_T \beta(T)X(T)$, which is a sum of Gaussian variables of computable non-zero covariances as defined above.

Part (i) and the calculation of the asymptotic expectation and the order of the variance of part (ii) in Theorem 1.2 of the original paper was the same and the results still hold, but the convergence to the normal distribution was wrong: indeed, in the original paper, in addition to the convergence to the normal distribution an upper bound on the speed of convergence to the normal distribution was given. This upper bound came from an application of *Stein's Method*:

Theorem 1.2 (*Theorem 1 of [1] and its Following Remarks*)**.** *Let $I \subseteq \mathbb{N}$, $K_i \subseteq I$ and $i \in I$, be finite index sets and suppose that the random variables W, $\{X_i\}_{i\in I}$, $\{W_i\}_{i\in I}$ and $\{Z_i\}_{i\in I}$ have finite second moment. Suppose that $W = \sum_{i\in I} X_i$, with $\mathbb{E}[X_i] = 0$ for $i \in I$, and $\mathbb{E}\left[W^2\right] = 1$. Suppose furthermore that $W = W_i + Z_i$, for any $i \in I$, where W_i is independent of both X_i and Z_i, and let $Z_i = \sum_{k\in K_i} X_k$, for any $i \in I$. Let*

$$\varepsilon = 2\sum_{i\in I}\sum_{k,\ell\in K_i}\left(\mathbb{E}[|X_iX_kX_\ell|] + \mathbb{E}[|X_iX_k|]\,\mathbb{E}[|X_\ell|]\right). \qquad (2)$$

Then, if $\{W^{(n)}\}$ is a sequence of random variables, whose elements can all be decomposed as W, and for which we denote by $\varepsilon^{(n)}$ the corresponding value of ε from (2), we have that $W^{(n)}$ tends in distribution to a standard normal random variable, and

$$d(\mathcal{L}(W^{(n)}), \Phi) \leq K\varepsilon^{(n)},$$

for some universal constant K.

As mentioned, Stein's Method was applied with $K_i \subseteq [n]$ being the set of indices of those vertices belonging to the same connected component as the vertex with index i. Unfortunately, in order to apply this method, the index sets have to be initially fixed and cannot be random sets, and the proof given there is not correct.

However, the convergence to the normal distribution still holds as an application of the following result of Pittel [3] (without an upper bound on the speed of convergence, however): Define for each $\delta > 0$ the space $\ell_{1,\delta}$ as the Banach space of all sequences $x = \{x(T)\}_T$ with the norm $\|x\|_\delta := \sum_T |T|^\delta |x(T)| < \infty$. Note that in particular $t_n^* \in \ell_{1,\delta}$ for each $\delta > 0$, since for $|T| = k$ with $k > n$, clearly $t_n^*(T) = 0$. Adapted to our setup of $G(n,p)$ with $p = c/n$ and $0 < c < 1$, Pittel's theorem reads as follows:

Theorem 1.3 (*Theorem 2 and the Following Proposition of [3]*)**.** *Let $G \in G(n,p)$ with $p = c/n$ and $0 < c < 1$. For each $\delta > 0$, $X \in \ell_{1,\delta}$ almost surely, and t_n^* converges to X in distribution. By Portmanteau's theorem, the latter means that for every bounded continuous function $f : \ell_{1,\delta} \to \mathbb{R}$, $\mathbb{E}\left[f(t_n^*)\right]$ converges to $\mathbb{E}[f(X)]$.*

For our concrete purpose of the metric dimension, for any $x \in \ell_{1,\delta}$ define $g(x) = \sum_T \beta(T)x(T)$. Since $\beta(T) \leq |T|$, we have $|g(x)| \leq \sum_T |T||x(T)| = \|x\|_1$. Since g is linear, g is a continuous functional in $\ell_{1,\delta}$ with $\delta = 1$. Let G' be the graph consisting of the connected components of $G \in G(n,p)$ that are not trees. By Theorem 5.7 of [2], $\mathbb{E}\left[|G'|\right] = O(1)$, and hence, since $\beta(G') \leq |G'|$, we have $\mathbb{E}\left[\beta(G')\right] = O(1)$. Therefore, for any function ω_n tending to infinity with n, $\beta(G')/\omega_n$ converges to 0 in distribution. We thus have

$$(\beta(G) - \mathbb{E}[\beta(G)])/\sqrt{n} = \Big(\sum_T \beta(T)t_n(T) - \sum_T \beta(T)\mathbb{E}[t_n(T)]\Big)/\sqrt{n} + \frac{\beta(G') - \mathbb{E}\left[\beta(G')\right]}{\sqrt{n}}$$
$$= \sum_T \beta(T)t_n^*(T) + o(1) = g(t_n^*) + o(1).$$

Now, we aim to show that $(\beta(G) - \mathbb{E}[\beta(G)])/\sqrt{n}$ converges in distribution to $g(X)$, or equivalently $g(t_n^*)$ to $g(X)$. Seeing g as a function $g : \ell_{1,\delta} \to \mathbb{R}$, g is continuous but not bounded. Let $Y_n = g(t_n^*)$ and $Y = g(X)$. To show that Y_n converges to Y, by Portmanteau's theorem, it suffices to show that for every bounded continuous function $f : \mathbb{R} \to \mathbb{R}$, $\mathbb{E}[f(Y_n)]$ converges to $\mathbb{E}[f(Y)]$. Note that $f(Y_n) = f \circ g(t_n^*)$ and $f(Y) = f \circ g(X)$. Observe that $f \circ g$ is continuous as a composition of two continuous functions, and it is also bounded, since f is bounded. Thus, $\mathbb{E}\left[f \circ g(t_n^*)\right]$ converges to $\mathbb{E}[f \circ g(X)]$, since by Theorem 1.3, t_n^* converges to X in distribution. Thus, Y_n converges to Y, and the remaining part of item (ii) of Theorem 1.1 is proven.

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