About the rotation curves of spiral galaxies, a program

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If we were going to make a little trip in the Milky Way

Abstract: Let us take a rotation curve of a spiral galaxy. This program makes the calculus of the inverse of the matrix of forces in a maximal disk, view as a set of stars (or points) with usual symmetry. After, it realizes the calculus of masses and of the surface density of the disk. No halo of dark matter is need. We added at the end an Annexe: A mathematical background.

```
> restart:
> with(LinearAlgebra):with(CurveFitting):
> k:=160;l:=k/2;Digits:=30:
```

$$k := 160$$

$$l := 80$$

```
> #Tabular: distances and angles
> c:=seq(evalf(cos(Pi*n/l)),n=1..2*l):
d:=seq(evalf((i/(k))^2),i=1..k+1):
> # it is a good choice to simulate a galaxy by point masses
# the number of masses is k^2+1
# we calculate now the forces between the masses
> s:=proc(i,j)
local num,dist,F,n,u;
 u:=0;
 for n to 2*l do
       dist:=evalf(d[i]^2+d[j]^2-2*d[i]*d[j]*c[n]);
       num:=evalf(d[i]-d[j]*c[n]);
       if  (n=2*l)and (i=j) then F:=0
       else F:=evalf(num/(dist^(3/2)))
       fi;
       u:=u+F
  od end:
A:=Matrix([seq([seq(s(ii,jj),jj=1..k),evalf(1/d[ii]^2)],ii=1..k),
[seq(2*l,jj=1..k),1]]):
> #with k=160, around 150 seconds
> #The fundamental matrix A (n-body problem) is established.
#So the result (the inverse matrix) is:
> invA:=MatrixInverse(A);#around 12 seconds
```

$$invA := \begin{bmatrix} \text{161 x 161 Matrix} \\ \text{Data Type: sfloat} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix}$$

```
> #precision:
> X:=evalm(A&*invA):X[1,1];#around 17 seconds
```

0.99999999999999999999999999968

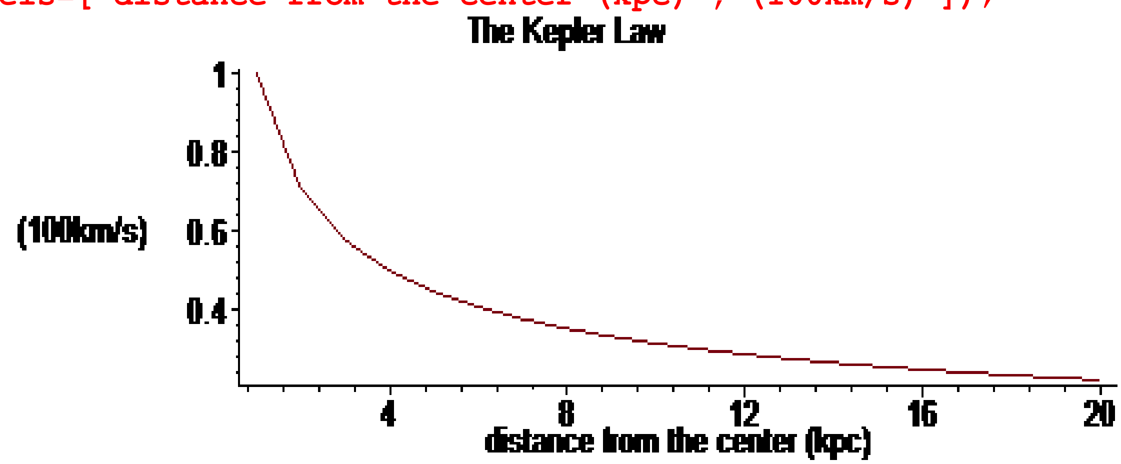
```
> max(seq(abs(X[i,i]-1),i=1..k));
```

$0.141\ 10^{-26}$

```
> max(seq(seq(abs(X[i,j]),j=i+1..k),i=1..k-1));
```

$0.17\ 10^{-20}$

```
> #well, more than 20 exact digits
> Digits:=21:
>
> #We could verify the third Kepler's law.
> fv:=proc(x) 1/sqrt(x) end:R:=20:
> plot([[y,fv(y)]$y=1..R],title=`The Kepler Law`,
labels=[`distance from the center (kpc)`,`(100km/s)`]);
```




```
>  B:=Vector([seq(w*fv(d[i])^2/d[i],i=1..k),1]):
> C:=evalf(MatrixVectorMultiply(invA,B),20):
> wnul:=seq(evalf(solve(C[i]=0,w)),i=1..k+1):
n1:=0:n2:=0:
for j to k+1 do
 N:=seq(subs(w=wnul[j],C[i]),i=1..k+1):truc:=1:
 for i to k+1 do
   if N[i]<-10^(-8) then truc:=0 fi:
 od;
 if truc=1 then if n1=0 then n1:=j else n2:=j fi fi;
od:
if n1=0 then print(`il_y_a_des_masses_negatives`) else
 if wnul[n1]>wnul[n2] then wmax:=wnul[n1];wminim:=wnul[n2]
   else wmax:=wnul[n2];wminim:=wnul[n1] fi;
```

```
 fi:
wmax:=evalf(wmax,25);wminim:=evalf(wminim,25);
wm:=evalf((wmax+wminim)/2,18);
```

$$wmax := 0.9999999999999999999935$$

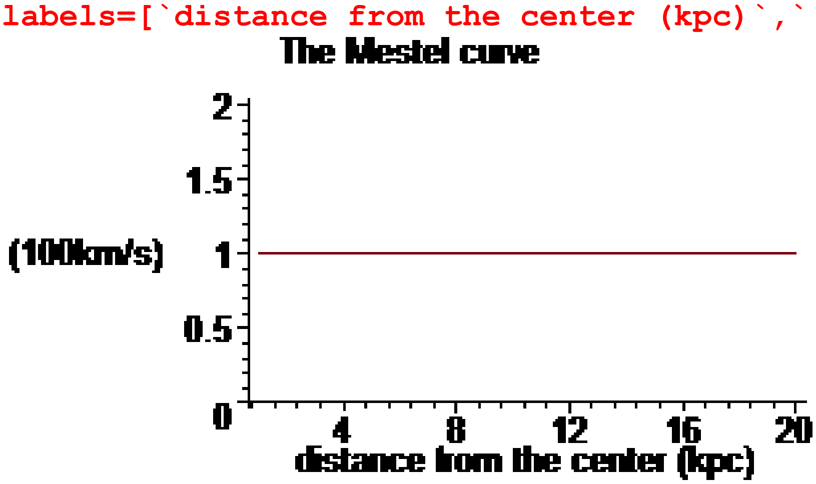
$$wminim := 0.9999999999999999999483$$

$$wm := 1.00000000000000000$$

```
> M:=seq(subs(w=wm,evalf(C[i],18)),i=1..k+1);
```

M := 0., 0., 0., 0.1000 10^{-20}, 0.1000 10^{-20}, 0.1000 10^{-20}, 0., 0.1000 10^{-20}, 0., 0., 0., 0., 0.,
0., 0., 0., 0., -0.1000 10^{-20}, 0., 0., 0.1000 10^{-20}, 0., 0., 0., 0., 0., 0., 0., 0., 0.1000 10^{-20},
0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 1.00000000000000000100

```
> #very well, it's noteworthy!
>
> #For a constant curve (Mestel):
> fv:=proc(x) 1 end:V:=1:R:=20:
> plot([[y,fv(y)]$y=1..R],title=`The Mestel curve`,
labels=[`distance from the center (kpc)`,`(100km/s)`]);
```




```
>  B:=Vector([seq(w*fv(d[i])^2/d[i],i=1..k),1]):
> C:=evalf(MatrixVectorMultiply(invA,B),20):
> wnul:=seq(evalf(solve(C[i]=0,w)),i=1..k+1):
n1:=0:n2:=0:
for j to k+1 do
 N:=seq(subs(w=wnul[j],C[i]),i=1..k+1):truc:=1:
 for i to k+1 do
   if N[i]<-10^(-15) then truc:=0 fi:
 od;
```

```
 if truc=1 then if n1=0 then n1:=j else n2:=j fi fi;
od:
if n1=0 then print(`il_y_a_des_masses_negatives`) else
 if wnul[n1]>wnul[n2] then wmax:=wnul[n1];wminim:=wnul[n2]
   else wmax:=wnul[n2];wminim:=wnul[n1] fi;
 fi:
wmax:=evalf(wmax,25);wminim:=evalf(wminim,25);
wm:=evalf((wmax+wminim)/2,15);
```

$$wmax := 1.557422082131586934942$$

$$wminim := 1.557289724889199123433$$

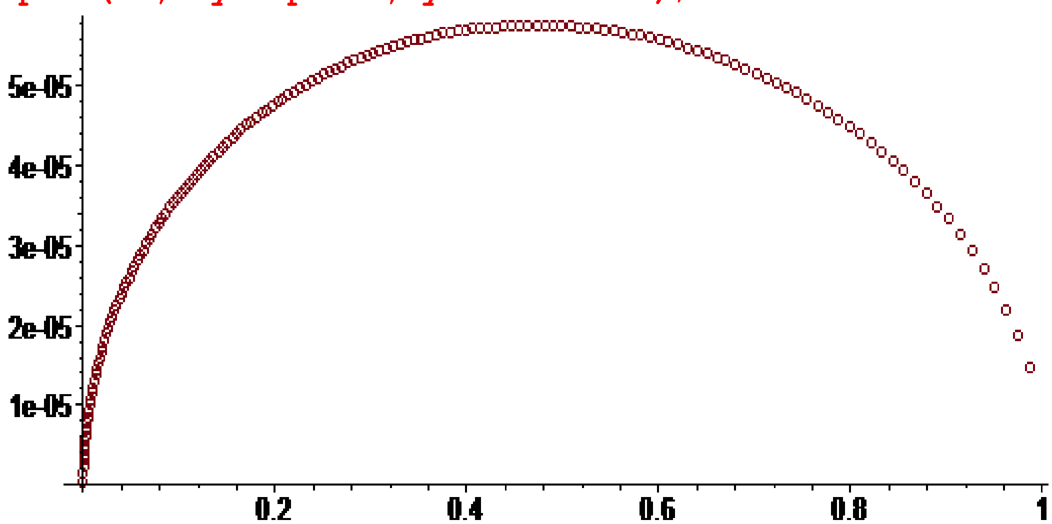
$$wm := 1.55735590351040$$

```
> M:=seq(subs(w=wm,evalf(C[i],15)),i=1..k):M[1..4];MM:=[seq([d[i],s
ubs(w=wm,evalf(C[i],15))],i=1..k-1)]:
```

$0.23824600929753102\ 10^{-6}, 0.149382278651539238\ 10^{-5},$

$0.2192915484633480012\ 10^{-5}, 0.3055410555907115583\ 10^{-5}$

```
> plot(MM,style=point,symbol=circle);
```




```
well, beautiful!
> R:=20;V:=1;MasseMestel:=evalf(0.23*10^10*V^2*R/wm,10):
print(`Masse_de_Mestel`,MasseMestel,`en_Masses_Solaires`);
```

$$R := 20$$

$$V := 1$$

Masse_de_Mestel, $0.2953724315\ 10^{11}$, *en_Masses_Solaires*

```
> Mgal:=MasseMestel;#for R=20 kpc
```

$$Mgal := 0.2953724315\ 10^{11}$$

```
> #mean surface density
> rho:=[seq(Mgal*evalf(8*1*(M[i]+M[i+1])/Pi/((d[i+1]+d[i+2])^2
```

```
-(d[i]+d[i-1])^2)/(R*10^3)^2),i=2..(k-1))]:
> Rho:=[seq(ln(rho[i]),i=1..(k-2))]:nops(%);
```

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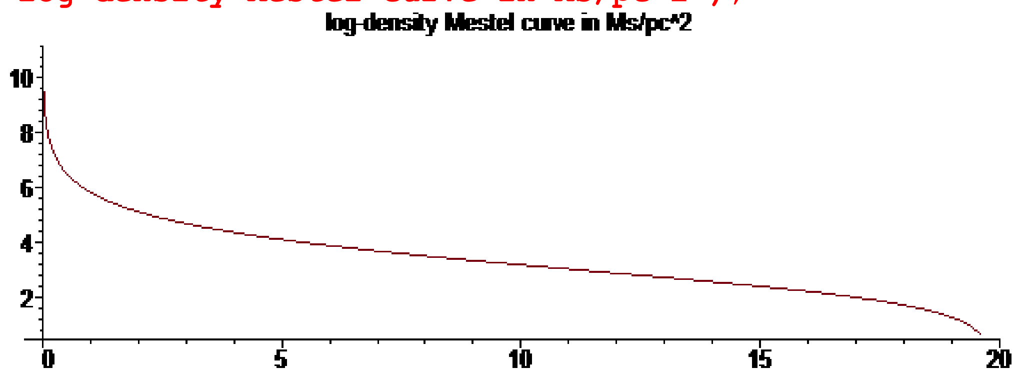
```
> plot([seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-2)],
title=`log-density Mestel curve in Ms/pc^2`);
```




```
>
> #2017, new data for Milky Way curve, coming from Huang (2016)
and Sofué (2015) :
> donneesVL:=[[0,0],[0.20,233.0],[0.38,268.92],[0.66,250.75],
[1.61,217.83],[2.57,219.58],[3.59,223.11],[4.60,231.24],
[5.08,230.46],[5.58,230.01],[6.10,239.61],[6.57,246.27],
[7.07,243.49],[7.58,242.71],[8.04,243.23],[8.34,239.89],
[8.65,237.26],[9.20,235.30],[9.62,230.99],[10.09,228.41],
[10.58,224.26],[11.09,224.94],[11.58,233.57],[12.07,240.02],
[12.73,242.21],[13.72,261.78],[14.95,259.26],[15.52,268.57],
[16.55,261.17],[17.56,240.66],[18.54,215.31],[19.50,214.99],
[21.25,251.68],[23.78,259.65],[26.22,242.02],[28.71,224.11],
[31.29,211.20],[33.73,217.93],[36.19,219.33],[38.73,213.31],
[41.25,200.05],[43.93,190.15]]:

> #For Milky Way, with R=20 kpc and we spline the curve of
velocities.
> donneesVL21:=[[0,0],[0.20,233.0],[0.38,268.92],[0.66,250.75],
[1.61,217.83],[2.57,219.58],[3.59,223.11],[4.60,231.24],
[5.08,230.46],[5.58,230.01],[6.10,239.61],[6.57,246.27],
[7.07,243.49],[7.58,242.71],[8.04,243.23],[8.34,239.89],
[8.65,237.26],[9.20,235.30],[9.62,230.99],[10.09,228.41],
[10.58,224.26],[11.09,224.94],[11.58,233.57],[12.07,240.02],
[12.73,242.21],[13.72,261.78],[14.95,259.26],[15.52,268.57],
[16.55,261.17],[17.56,240.66],[18.54,215.31],[19.50,214.99],
[21.25,251.68]]:nops(%);
```

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```
> f:=map(u->evalf(op(2,u)/100,5),donneesVL21);
```

$f := [0., 2.3300, 2.6892, 2.5075, 2.1783, 2.1958, 2.2311, 2.3124, 2.3046, 2.3001, 2.3961, 2.4627, 2.4349, 2.4271, 2.4323, 2.3989, 2.3726, 2.3530, 2.3099, 2.2841, 2.2426, 2.2494, 2.3357, 2.4002, 2.4221, 2.6178, 2.5926, 2.6857, 2.6117, 2.4066, 2.1531, 2.1499, 2.5168]$

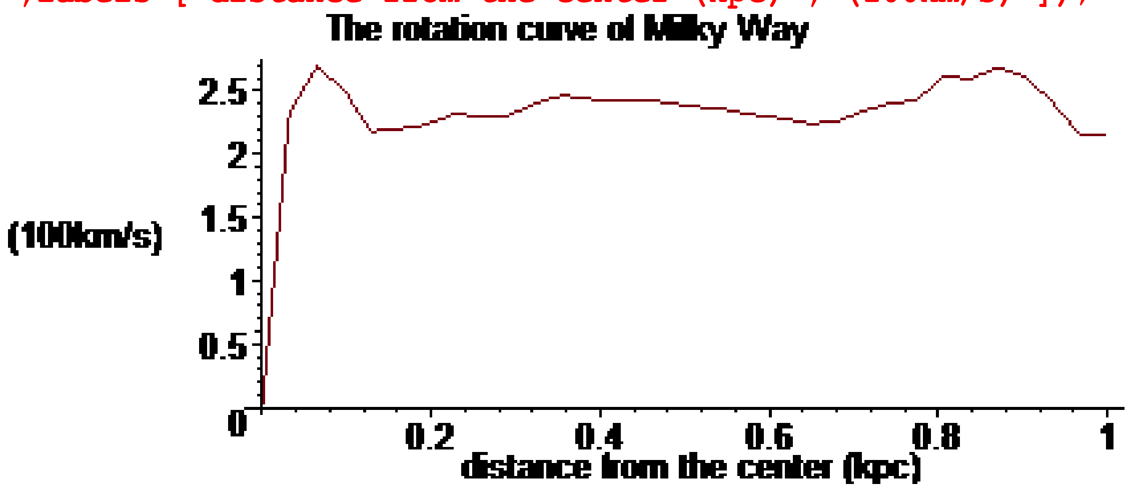
```
> R:=20;a:=nops(f)-2;V:=f[a+1];
```

$$R := 20$$

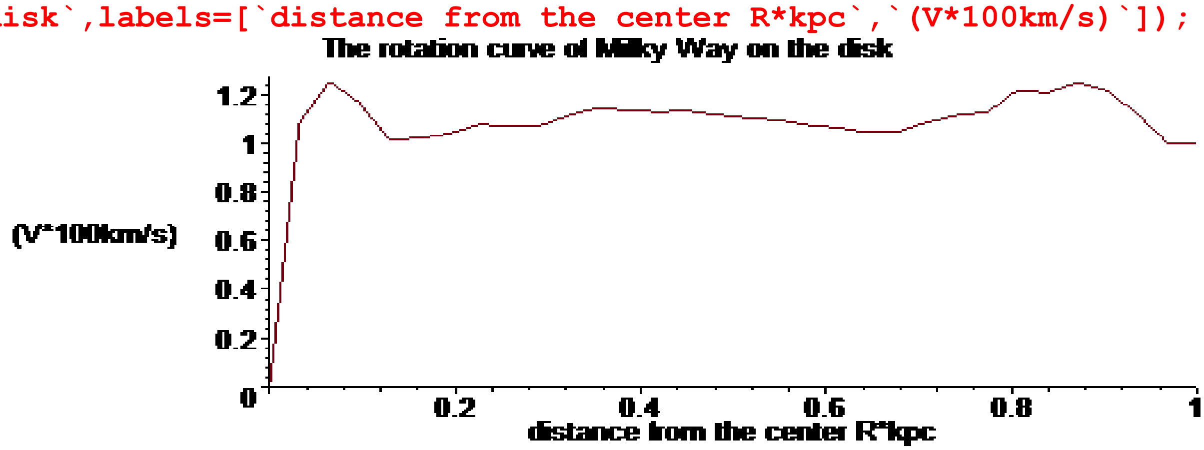
$$a := 31$$

$$V := 2.1499$$

```
> plot([[y/31,f[y+1]]$y=0..31],title=`The rotation curve of Milky
Way`,labels=[`distance from the center (kpc)`,`(100km/s)`]);
```

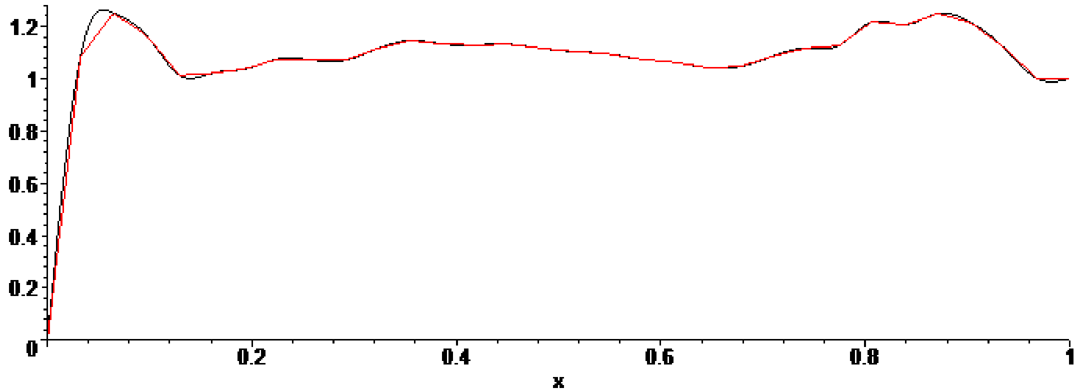



```
> fv:=proc(x)
 (f[trunc(a*x)+1]+(a*x-trunc(a*x))*(f[trunc(a*x)+2]-f[trunc(a*x)
+1]))/V
end:
> plot(fv(x),x=0..1,title=`The rotation curve of Milky Way on the
disk`,labels=[`distance from the center R*kpc`,`(V*100km/s)`]);
```




```
> F:=[seq([(i-1)/31,f[i]/V],i=1..32)]:
>
> g:=x->Spline(F,x):
```

```
> plot({g(x),fv(x)},x=0..1,color=[red,black]);
```




```
>
> #The second member of the linear system, coming from observed
velocities.
   BB:=Vector([seq(w*g(d[i])^2/d[i],i=1..k),1]):
#w as the meaning of the inverse of the mass M of the galaxy
> #calculus of mass as function of w
C:=evalf(MatrixVectorMultiply(invA,BB),30):
> #search of w=wmin and w=wmax such that all the mass are >=0
wnul:=seq(evalf(solve(C[i]=0,w)),i=1..k+1):
n1:=0:n2:=0:
for j to k+1 do
 N:=seq(subs(w=wnul[j],C[i]),i=1..k+1):truc:=1:
 for i to k+1 do
   if N[i]<-10^(-5) then truc:=0 fi:
 od;
 if truc=1 then if n1=0 then n1:=j else n2:=j fi fi;
od:
if n1=0 then print(`il_y_a_des_masses_negatives`) else
 if wnul[n1]>wnul[n2] then wmax:=wnul[n1];wminim:=wnul[n2]
   else wmax:=wnul[n2];wminim:=wnul[n1] fi;
 fi:
wmax:=evalf(wmax,15);wminim:=evalf(wminim,15);
wm:=evalf((wmax+wminim)/2,15);wmax-wminim;
```

$$wmax := 1.29664337753167$$

$$wminim := 1.29664335097775$$

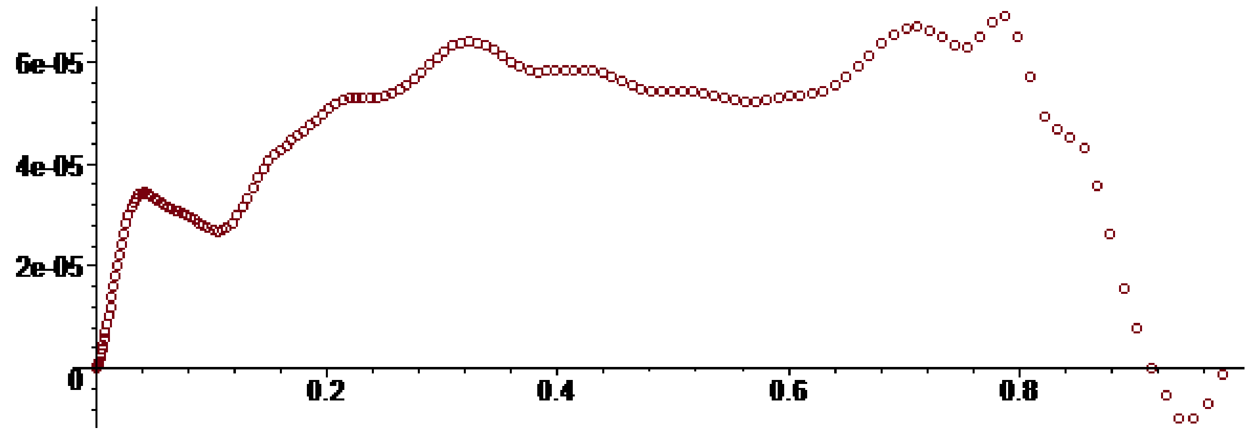
$$wm := 1.29664336425471$$

$$0.2655392\ 10^{-7}$$

```
>
M:=seq(subs(w=wm,evalf(C[i],15)),i=1..k):M[1..3];MM:=[seq([d[i],s
ubs(w=wm,evalf(C[i],15))],i=1..k-2)]:
```

$0.5740575241300\ 10^{-10}, 0.469177057255007\ 10^{-8}, 0.30394016978535919\ 10^{-7}$

```
> plot(MM,style=point,symbol=circle);
```




```
> MasseGalaxie:=evalf(0.23*10^10*V^2*R/wm,10):
print(`Masse_de_la_Galaxie`,MasseGalaxie,`en_Masses_Solaires`);
```

Masse_de_la_Galaxie, 0.1639735538 10^{12}, *en_Masses_Solaires*

```
>
evalf(0.23*10^10*V^2*R/wmax,10),evalf(0.23*10^10*V^2*R/wminim,10)
;
```

0.1639735521 10^{12}, 0.1639735555 10^{12}

```
> #we have a problem at the end of the curve, but
> Mgal:=MasseGalaxie;#for R=20 kpc
```

Mgal := 0.1639735538 10^{12}

```
> #mean surface density
> rho:=[seq(Mgal*evalf(8*l*(M[i]+M[i+1])/Pi/((d[i+1]+d[i+2])^2
-(d[i]+d[i-1])^2)/(R*10^3)^2),i=2..(k-1))]:nops(%);
```

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```
> Rho:=[seq(ln(rho[i]),i=1..(k-9))]:nops(%);
```

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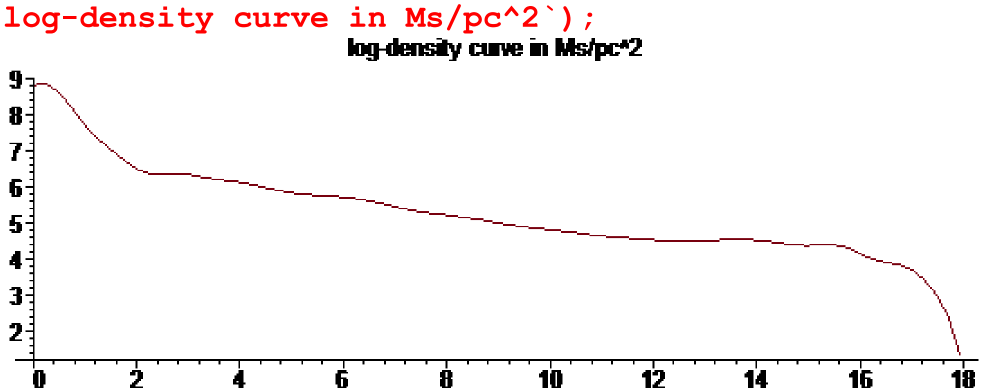
```
> plot([seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-9)],
title=`log-density curve in Ms/pc^2`);
```




```
>
> #Now for an extended Milky Way, (always without spherical halo)
```

```
>
> #For Milky Way, with R=25 kpc and we spline the curve of
velocities.
> donneesVL26:=[[0, 0], [.20, 233.0], [.38, 268.92], [.66,
250.75], [1.61, 217.83], [2.57, 219.58], [3.59, 223.11], [4.60,
231.24], [5.08, 230.46], [5.58, 230.01], [6.10, 239.61], [6.57,
246.27], [7.07, 243.49], [7.58, 242.71], [8.04, 243.23], [8.34,
239.89], [8.65, 237.26], [9.20, 235.30], [9.62, 230.99], [10.09,
228.41], [10.58, 224.26], [11.09, 224.94], [11.58, 233.57],
[12.07, 240.02], [12.73, 242.21], [13.72, 261.78], [14.95,
259.26], [15.52, 268.57], [16.55, 261.17], [17.56, 240.66],
[18.54, 215.31], [19.50, 214.99], [21.25, 251.68], [23.78,
259.65], [26.22, 242.02]]:nops(%);
```

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```
> f:=map(u->evalf(op(2,u)/100,5),donneesVL26);
```

$f := [0., 2.3300, 2.6892, 2.5075, 2.1783, 2.1958, 2.2311, 2.3124, 2.3046, 2.3001, 2.3961, 2.4627, 2.4349, 2.4271, 2.4323, 2.3989, 2.3726, 2.3530, 2.3099, 2.2841, 2.2426, 2.2494, 2.3357, 2.4002, 2.4221, 2.6178, 2.5926, 2.6857, 2.6117, 2.4066, 2.1531, 2.1499, 2.5168, 2.5965, 2.4202]$

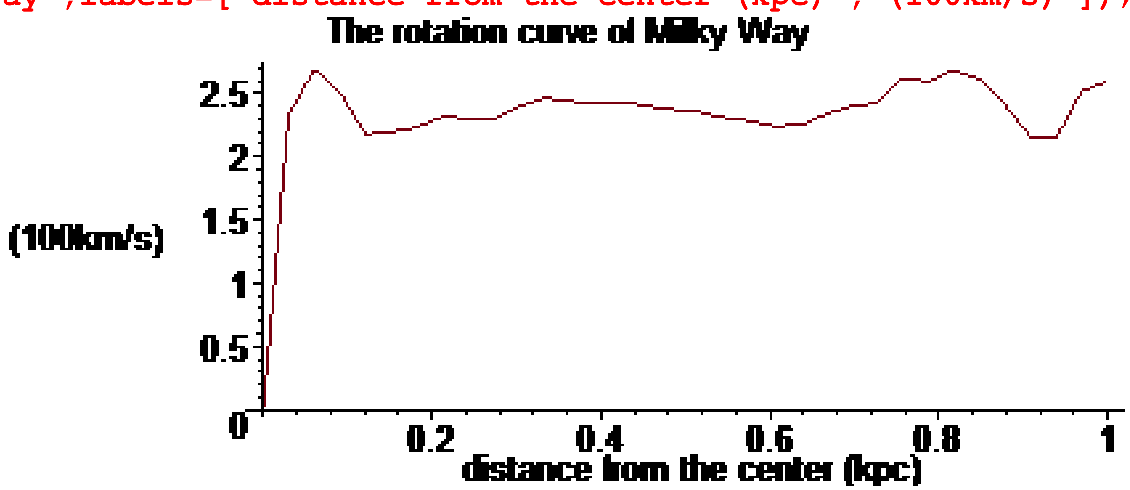
```
> R:=25;a:=nops(f)-2;V:=(f[a+1]+f[a+2])/2;
```

$R := 25$

$a := 33$

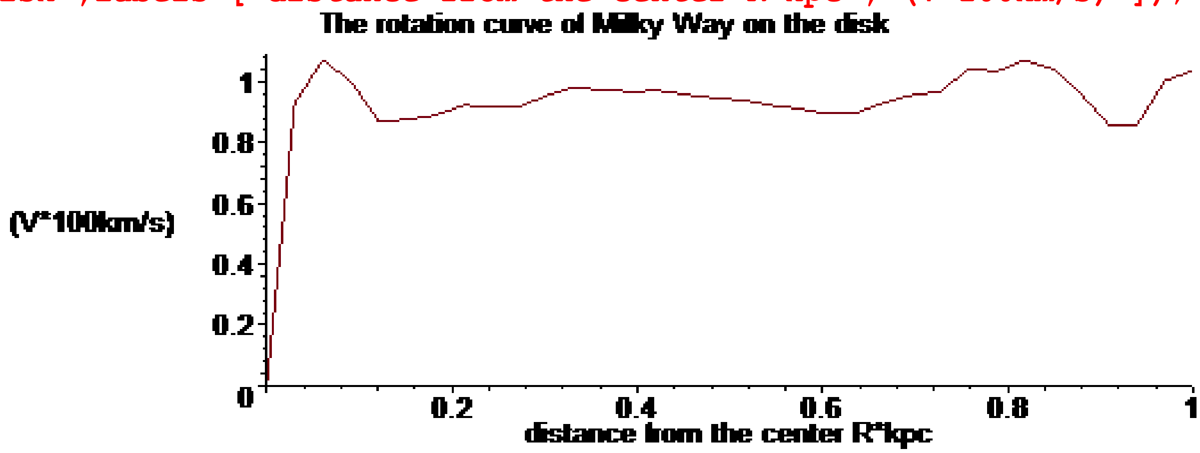
$V := 2.50835000000000000000$

```
> plot([[y/33,f[y+1]]$y=0..33],title=`The rotation curve of Milky
Way`,labels=[`distance from the center (kpc)`,`(100km/s)`]);
```

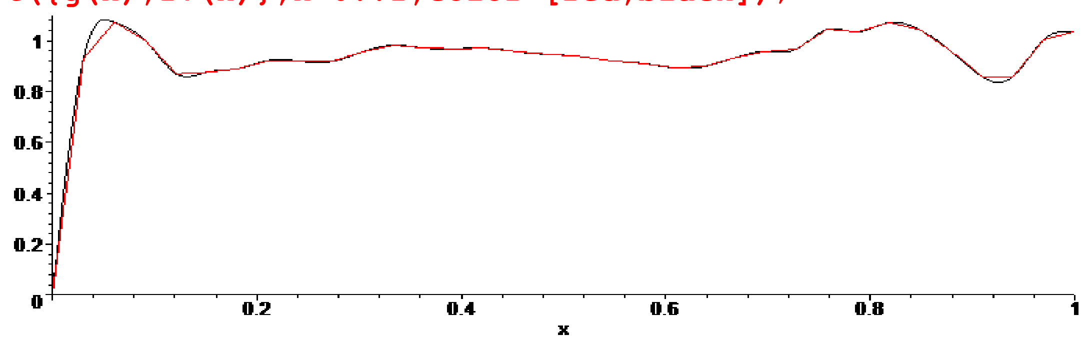



```
> fv:=proc(x)
(f[trunc(a*x)+1]+(a*x-trunc(a*x))*(f[trunc(a*x)+2]-f[trunc(a*x)
+1]))/V
end:
> plot(fv(x),x=0..1,title=`The rotation curve of Milky Way on the
```

```
disk`,labels=[`distance from the center R*kpc`,`(V*100km/s)`]);
```




```
>
> F:=[seq([(i-1)/33,f[i]/V],i=1..34)]:
> g:=x->Spline(F,x):
> plot({g(x),fv(x)},x=0..1,color=[red,black]);
```




```
> #The second member of the linear system, coming from observed
velocities.

> BB:=Vector([seq(w*g(d[i])^2/d[i],i=1..k),1]):
#w as the meaning of the inverse of the mass M of the galaxy
> C:=evalf(MatrixVectorMultiply(invA,BB),30):
> #search of w=wmin and w=wmax such that all the mass are >=0
> wnul:=seq(evalf(solve(C[i]=0,w)),i=1..k+1):
n1:=0:n2:=0:
for j to k+1 do
 N:=seq(subs(w=wnul[j],C[i]),i=1..k+1):truc:=1:
 for i to k+1 do
   if N[i]<-10^(-6) then truc:=0 fi:
 od;
 if truc=1 then if n1=0 then n1:=j else n2:=j fi fi;
od:
if n1=0 then print(`il_y_a_des_masses_negatives`) else
 if wnul[n1]>wnul[n2] then wmax:=wnul[n1];wminim:=wnul[n2]
   else wmax:=wnul[n2];wminim:=wnul[n1] fi;
 fi:
```

```
wmax:=evalf(wmax,15);wminim:=evalf(wminim,15);
wm:=evalf((wmax+wminim)/2,15);wmax-wminim;
```

$$wmax := 1.63914534386586$$

$$wminim := 1.63914531059209$$

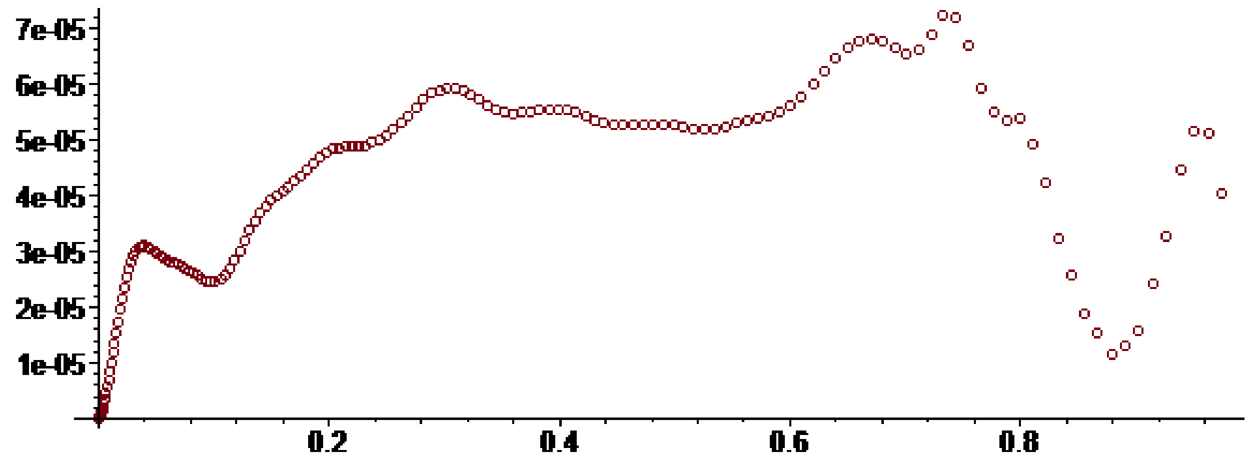
$$wm := 1.63914532722898$$

$$0.3327377\ 10^{-7}$$

```
>
M:=seq(subs(w=wm,evalf(C[i],15)),i=1..k):M[1..3];MM:=[seq([d[i],s
ubs(w=wm,evalf(C[i],15))],i=1..k-2)]:
```

$0.5690259176508\ \ 10^{-10}, 0.465064286684057\ \ 10^{-8}, 0.30127473812319455\ \ 10^{-7}$

```
> plot(MM,style=point,symbol=circle);
```




```
> MasseGalaxie:=evalf(0.23*10^10*V^2*R/wm,5):
print(`Masse_de_la_Galaxie`,MasseGalaxie,`en_Masses_Solaires`);
```

$$Masse\_de\_la\_Galaxie, 0.22072\ 10^{12}, en\_Masses\_Solaires$$

```
>evalf(0.23*10^10*V^2*R/wmax,8),evalf(0.23*10^10*V^2*R/wminim,8);
```

$$0.22071236\ 10^{12}, 0.22071236\ 10^{12}$$

```
> #no problem at the end of the curve
> Mgal:=MasseGalaxie;#for R=25 kpc
```

$$Mgal := 0.22072\ 10^{12}$$

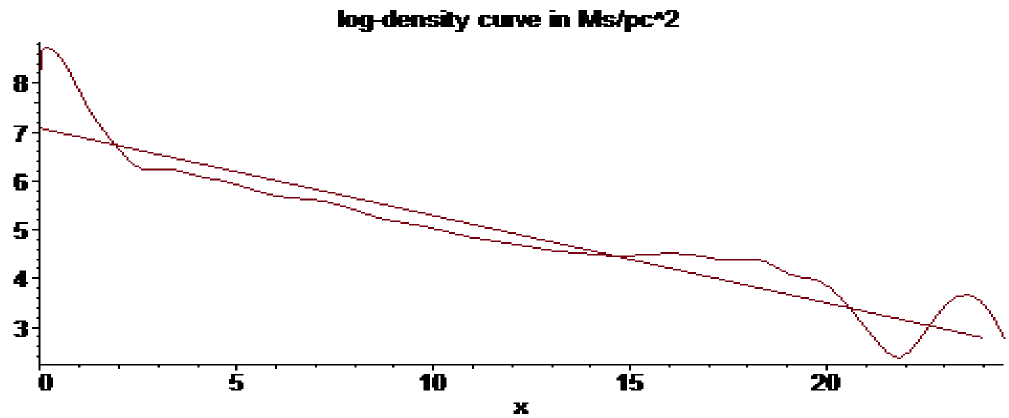
```
> #mean surface density
rho:=[seq(Mgal*evalf(8*1*(M[i]+M[i+1])/Pi/((d[i+1]+d[i+2])^2
-(d[i]+d[i-1])^2)/(R*10^3)^2),i=2..(k-1))]:

> #log-density curve
i:='i':Rho:=[seq(ln(rho[i]),i=1..(k-2))]:
i:='i':courbelog:=plot([[R*(d[i]+d[i+1])/2,Rho[i]]$i=2..k-2],
title=`log-density curve in Ms/pc^2`):
> with(stats):Digits:=5:s:=trunc(k/6);
```

```
liste:=[[seq(R*(d[i]+d[i+1])/2,i=s..(k-5))],[seq(Rho[i+1],i=s-1..
(k-6))]]:
eqfit:=fit[leastsquare[[x,y]]](liste);
eqfonction:=unapply(rhs(eqfit),x):
courberegr:=plot(eqfonction(x),x=0..R-1):
with(plots):
display({courberegr,courbelog});Digits:=20:
```

$$s := 26$$

$$eqfit := y = -0.17884\ x + 7.0770$$




```
>
> #Now for Milky Way, with R=30 kpc
> donneesVL31:=[[0, 0], [.20, 233.0], [.38, 268.92], [.66,
250.75], [1.61, 217.83], [2.57, 219.58], [3.59, 223.11], [4.60,
231.24], [5.08, 230.46], [5.58, 230.01], [6.10, 239.61], [6.57,
246.27], [7.07, 243.49], [7.58, 242.71], [8.04, 243.23], [8.34,
239.89], [8.65, 237.26], [9.20, 235.30], [9.62, 230.99], [10.09,
228.41], [10.58, 224.26], [11.09, 224.94], [11.58, 233.57],
[12.07, 240.02], [12.73, 242.21], [13.72, 261.78], [14.95,
259.26], [15.52, 268.57], [16.55, 261.17], [17.56, 240.66],
[18.54, 215.31], [19.50, 214.99], [21.25, 251.68], [23.78,
259.65], [26.22, 242.02], [28.71, 224.11], [31.29,
211.20]]:nops(%);
```

$$37$$

```
> f:=map(u->evalf(op(2,u)/100,5),donneesVL31);
```

$f := [0., 2.3300, 2.6892, 2.5075, 2.1783, 2.1958, 2.2311, 2.3124, 2.3046, 2.3001, 2.3961, 2.4627, 2.4349, 2.4271, 2.4323, 2.3989, 2.3726, 2.3530, 2.3099, 2.2841, 2.2426, 2.2494, 2.3357, 2.4002, 2.4221, 2.6178, 2.5926, 2.6857, 2.6117, 2.4066, 2.1531, 2.1499, 2.5168, 2.5965, 2.4202, 2.2411, 2.1120]$

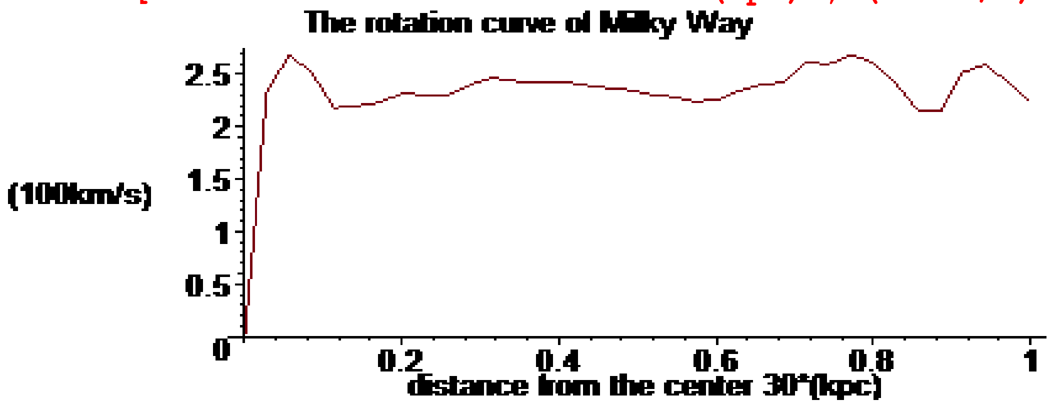
```
>
R:=30;a:=nops(f)-2;V:=evalf((f[a+1]+f[a+2])/2,7);
```

$$R := 30$$

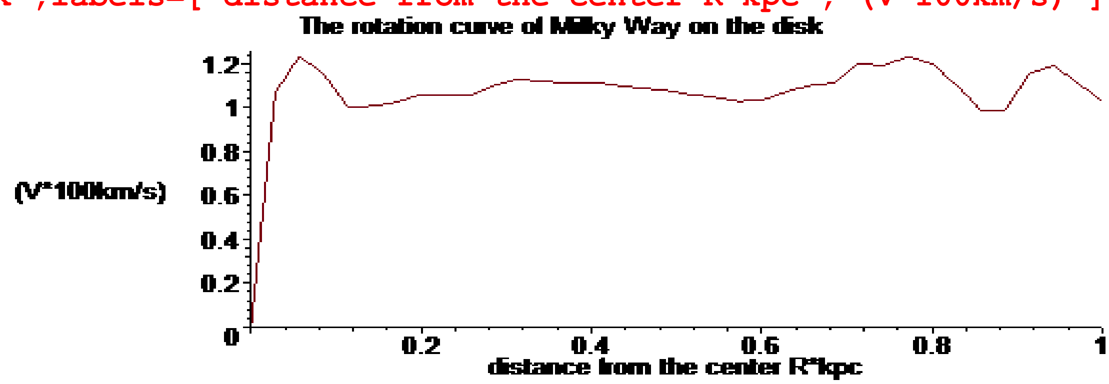
$$a := 35$$

$V := 2.176550$

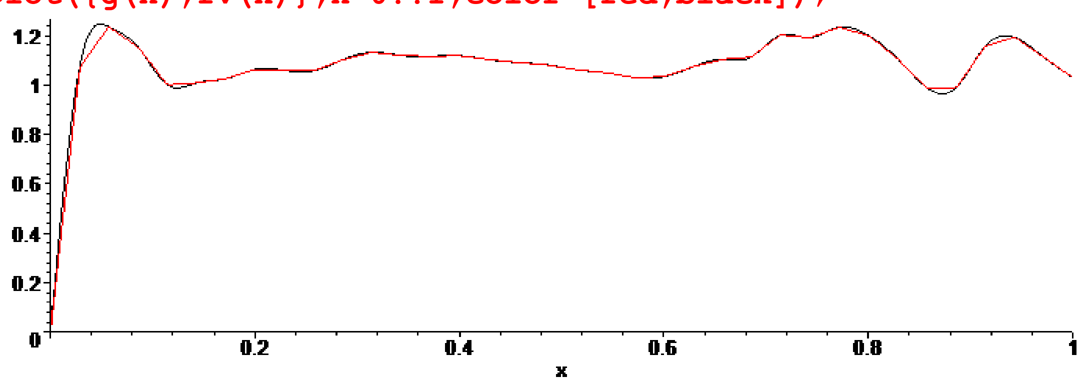
```
> plot([[y/35,f[y+1]]$y=0..35],title=`The rotation curve of Milky Way`,labels=[`distance from the center 30*(kpc)`,`(100km/s)`]);
```




```
> fv:=proc(x)
(f[trunc(a*x)+1]+(a*x-trunc(a*x))*(f[trunc(a*x)+2]-f[trunc(a*x)+1]))/V end:
> plot(fv(x),x=0..1,title=`The rotation curve of Milky Way on the disk`,labels=[`distance from the center R*kpc`,`(V*100km/s)`]);
```




```
> F:=[seq([(i-1)/35,f[i]/V],i=1..36)]:
> g:=x->Spline(F,x):
> plot({g(x),fv(x)},x=0..1,color=[red,black]);
```




```
> #The second member of the linear system, coming from observed velocities.

> BB:=Vector([seq(w*g(d[i])^2/d[i],i=1..k),1]):
```

```
#w as the meaning of the inverse of the mass M of the galaxy
> C:=evalf(MatrixVectorMultiply(invA,BB),30):
> #search of w=wmin and w=wmax such that all the mass are >=0
> wnul:=seq(evalf(solve(C[i]=0,w)),i=1..k+1):
n1:=0:n2:=0:
for j to k+1 do
 N:=seq(subs(w=wnul[j],C[i]),i=1..k+1):truc:=1:
 for i to k+1 do
   if N[i]<-10^(-5) then truc:=0 fi:
 od;
 if truc=1 then if n1=0 then n1:=j else n2:=j fi fi;
od:
if n1=0 then print(`il_y_a_des_masses_negatives`) else
 if wnul[n1]>wnul[n2] then wmax:=wnul[n1];wminim:=wnul[n2]
   else wmax:=wnul[n2];wminim:=wnul[n1] fi;
 fi:
wmax:=evalf(wmax,15);wminim:=evalf(wminim,15);
wm:=evalf((wmax+wminim)/2,15);wmax-wminim;
```

$$wmax := 1.29626222698922$$

$$wminim := 1.29626219763040$$

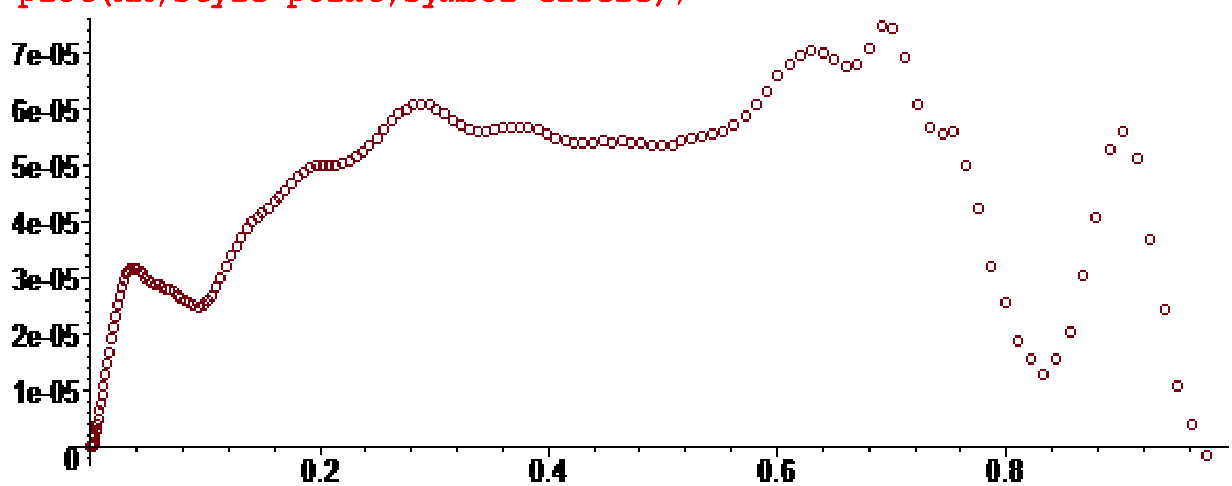
$$wm := 1.29626221230981$$

$$0.2935882\ 10^{-7}$$

```
>
M:=seq(subs(w=wm,evalf(C[i],15)),i=1..k):M[1..3];MM:=[seq([d[i],s
ubs(w=wm,evalf(C[i],15))],i=1..k-2)]:
```

$0.6348821720099\ 10^{-10}, 0.518888344801622\ 10^{-8}, 0.33614133380241097\ 10^{-7}$

```
> plot(MM,style=point,symbol=circle);
```




```
> MasseGalaxie:=evalf(0.23*10^10*V^2*R/wm,8):
print(`Masse_de_la_Galaxie`,MasseGalaxie,`en_Masses_Solaires`);
```

Masse_de_la_Galaxie, $0.25217007\ 10^{12}$, *en_Masses_Solaires*

```
>evalf(0.23*10^10*V^2*R/wmax,8),evalf(0.23*10^10*V^2*R/wminim,8);
```

$$0.25217007\ 10^{12}, 0.25217007\ 10^{12}$$

```
> #a very little problem at the end.
> Mgal:=MasseGalaxie;#for R=30 kpc
```

$$Mgal := 0.25217007\ 10^{12}$$

```
> #mean surface density
> rho:=[seq(Mgal*evalf(8*l*(M[i]+M[i+1])/Pi/((d[i+1]+d[i+2])^2
-(d[i]+d[i-1])^2)/(R*10^3)^2),i=2..(k-1))]:nops(%);
```

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```
> Rho:=[seq(ln(rho[i]),i=1..(k-3))]:nops(%);
```

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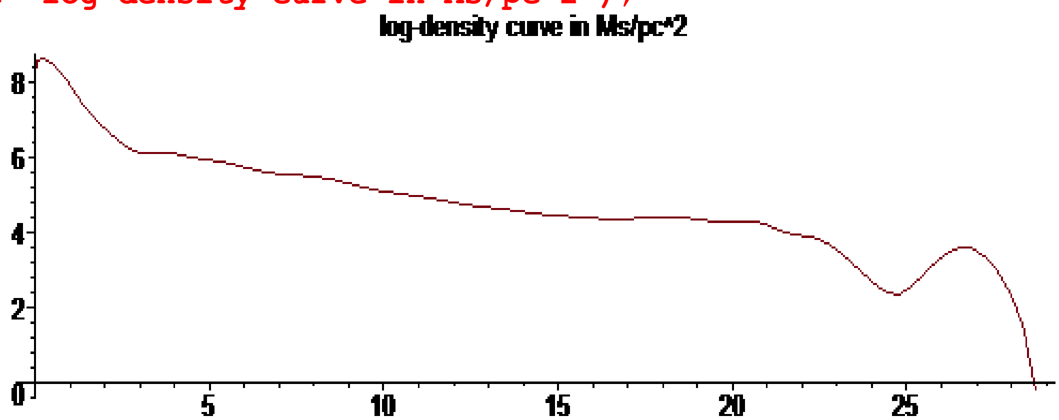
```
> plot([seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-4)],
title=`log-density curve in Ms/pc^2`);
```




```
> densitéVL_30:=[seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-4)]:
> #And for Milky Way, with R=40 kpc ? see Fosué and also Huang
> donneesVL41:=[[0,0],[0.20,233.0],[0.38,268.92],[0.66,250.75],
[1.61,217.83],[2.57,219.58],[3.59,223.11],[4.60,231.24],
[5.08,230.46],[5.58,230.01],[6.10,239.61],[6.57,246.27],
[7.07,243.49],[7.58,242.71],[8.04,243.23],[8.34,239.89],
[8.65,237.26],[9.20,235.30],[9.62,230.99],[10.09,228.41],
[10.58,224.26],[11.09,224.94],[11.58,233.57],[12.07,240.02],
[12.73,242.21],[13.72,261.78],[14.95,259.26],[15.52,268.57],
[16.55,261.17],[17.56,240.66],[18.54,215.31],[19.50,214.99],
[21.25,251.68],[23.78,259.65],[26.22,242.02],[28.71,224.11],
[31.29,211.20],[33.73,217.93],[36.19,219.33],[38.73,213.31],
[41.25,200.05]]:nops(%);
```

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```
> f:=map(u->evalf(op(2,u)/100,5),donneesVL41);
```

$f := [0., 2.3300, 2.6892, 2.5075, 2.1783, 2.1958, 2.2311, 2.3124, 2.3046, 2.3001, 2.3961, 2.4627, 2.4349, 2.4271, 2.4323, 2.3989, 2.3726, 2.3530, 2.3099, 2.2841, 2.2426, 2.2494, 2.3357, 2.4002, 2.4221, 2.6178, 2.5926, 2.6857, 2.6117, 2.4066, 2.1531, 2.1499, 2.5168, 2.5965, 2.4202, 2.2411, 2.1120, 2.1793, 2.1933, 2.1331, 2.0005]$

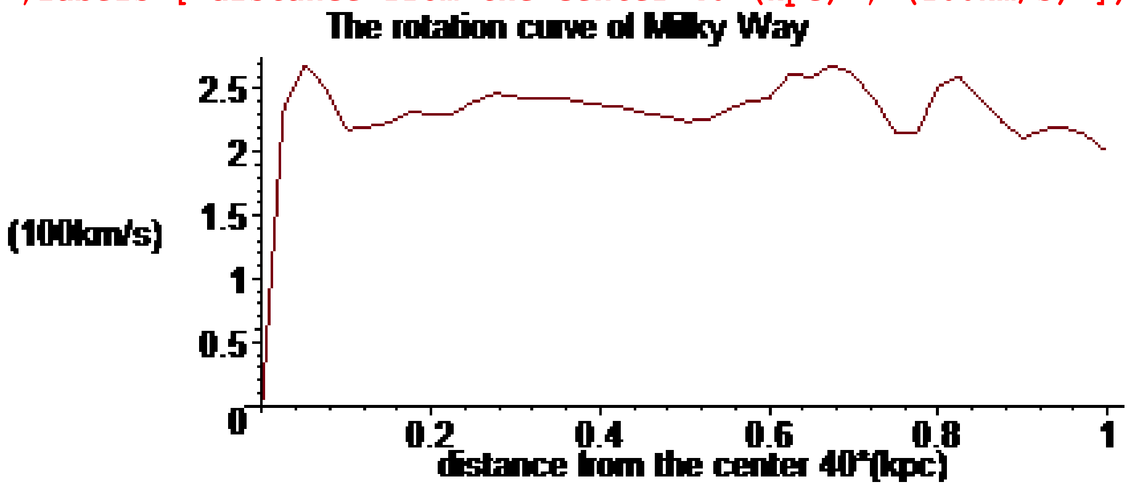
```
R:=40;a:=nops(f)-2;V:=evalf((f[a+1]+f[a+2])/2,7);
```

$$R := 40$$

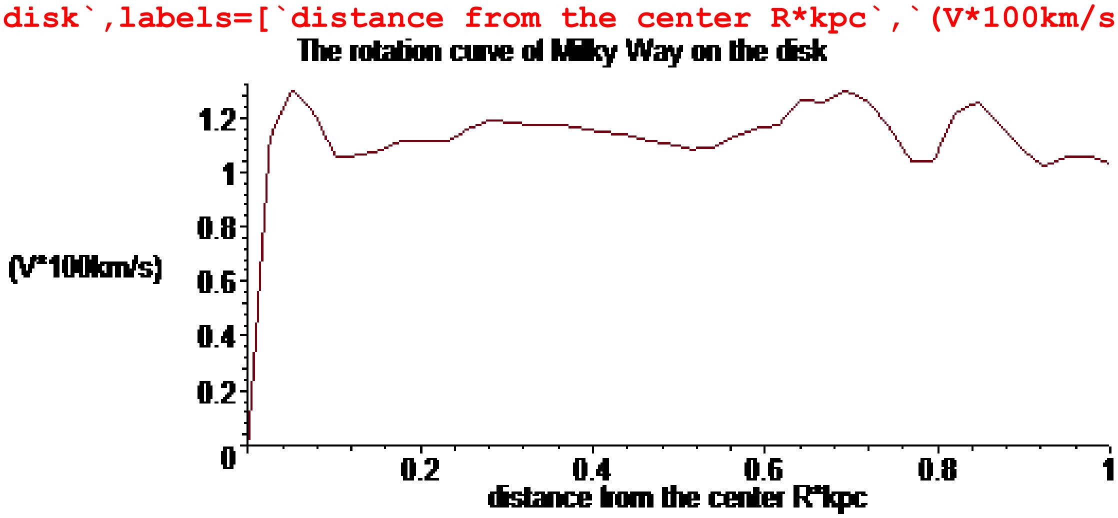
$$a := 39$$

$$V := 2.066800$$

```
> plot([[y/40,f[y+1]]$y=0..40],title=`The rotation curve of Milky Way`,labels=[`distance from the center 40*(kpc)`,`(100km/s)`]);
```

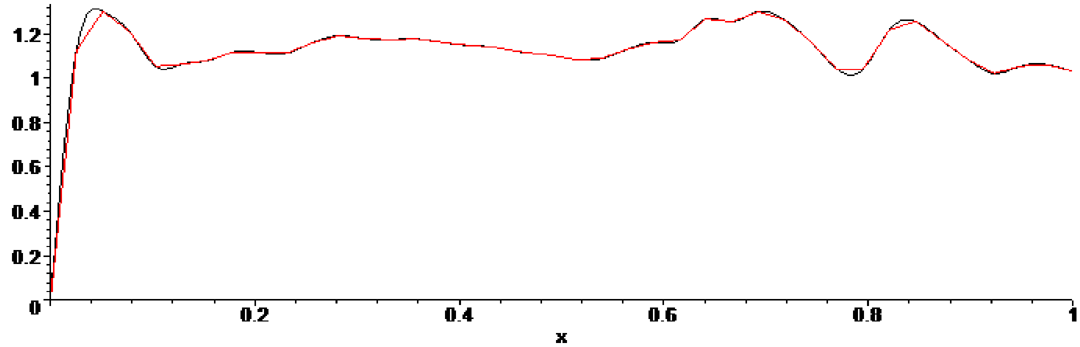



```
> fv:=proc(x)
 (f[trunc(a*x)+1]+(a*x-trunc(a*x))*(f[trunc(a*x)+2]-f[trunc(a*x)+1]))/V  end:
> plot(fv(x),x=0..1,title=`The rotation curve of Milky Way on the disk`,labels=[`distance from the center R*kpc`,`(V*100km/s)`]);
```




```
> F:=[seq([(i-1)/39,f[i]/V],i=1..40)]:
> g:=x->Spline(F,x):
```

```
> plot({g(x),fv(x)},x=0..1,color=[red,black]);
```




```
> #The second member of the linear system, coming from observed velocities.

> BB:=Vector([seq(w*g(d[i])^2/d[i],i=1..k),1]):
#w as the meaning of the inverse of the mass M of the galaxy
> C:=evalf(MatrixVectorMultiply(invA,BB),30):
> #search of w=wmin and w=wmax such that all the mass are >=0
> wnul:=seq(evalf(solve(C[i]=0,w)),i=1..k+1):
n1:=0:n2:=0:
for j to k+1 do
 N:=seq(subs(w=wnul[j],C[i]),i=1..k+1):truc:=1:
 for i to k+1 do
   if N[i]<-10^(-6) then truc:=0 fi:
 od;
 if truc=1 then if n1=0 then n1:=j else n2:=j fi fi;
od:
if n1=0 then print(`il_y_a_des_masses_negatives`) else
 if wnul[n1]>wnul[n2] then wmax:=wnul[n1];wminim:=wnul[n2]
   else wmax:=wnul[n2];wminim:=wnul[n1] fi;
 fi:
wmax:=evalf(wmax,15);wminim:=evalf(wminim,15);
wm:=evalf((wmax+wminim)/2,15);wmax-wminim;
```

$$wmax := 1.25770737549743$$

$$wminim := 1.25770734123387$$

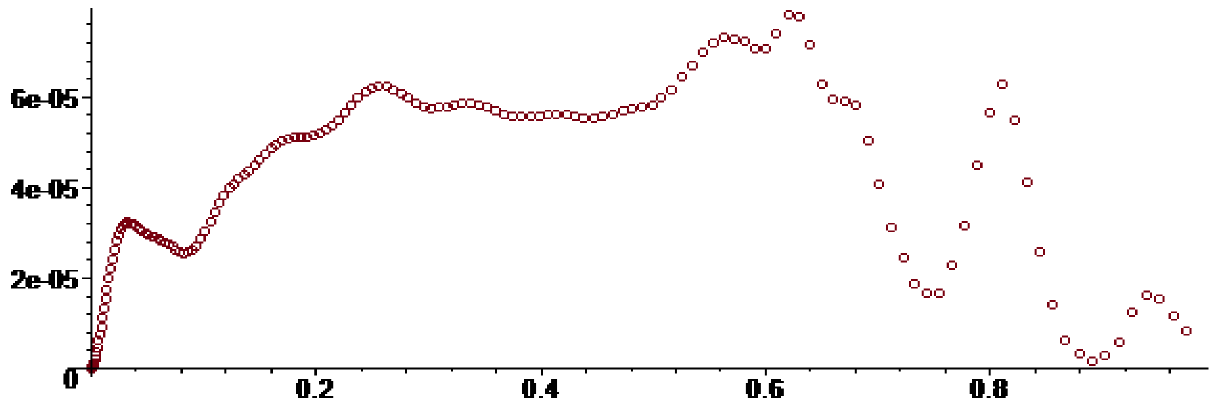
$$wm := 1.25770735836565$$

$$0.3426356\ 10^{-7}$$

```
>M:=seq(subs(w=wm,evalf(C[i],15)),i=1..k):M[1..3];MM:=[seq([d[i],
subs(w=wm,evalf(C[i],15))],i=1..k-2)]:
```

$0.763660545193\ 10^{-10}, 0.62413761483094\ 10^{-8}, 0.4043194233463329\ 10^{-7}$

```
> plot(MM,style=point,symbol=circle);
```




```
> MasseGalaxie:=evalf(0.23*10^10*V^2*R/wm,8):
print(`Masse_de_la_Galaxie`,MasseGalaxie,`en_Masses_Solaires`);
```

Masse_de_la_Galaxie, $0.31246770\ 10^{12}$, *en_Masses_Solaires*

```
>evalf(0.23*10^10*V^2*R/wmax,8),evalf(0.23*10^10*V^2*R/wminim,8);
```

$0.31246770\ 10^{12}, 0.31246772\ 10^{12}$

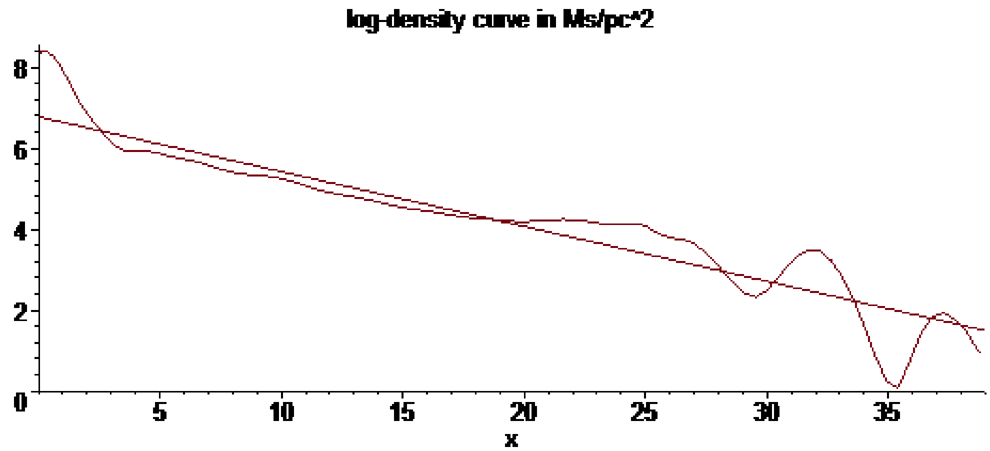
```
> #no problem at the end.
> Mgal:=MasseGalaxie;#for R=40 kpc
```

$Mgal := 0.31246770\ 10^{12}$

```
> #mean surface density
rho:=[seq(Mgal*evalf(8*1*(M[i]+M[i+1])/Pi/((d[i+1]+d[i+2])^2
-(d[i]+d[i-1])^2)/(R*10^3)^2),i=2..(k-2))]:

> #log-density curve
i:='i':Rho:=[seq(ln(rho[i]),i=1..(k-3))]:
i:='i':courbelog:=plot([[R*(d[i]+d[i+1])/2,Rho[i]]$i=2..k-3],
title=`log-density curve in Ms/pc^2`):
> with(stats):Digits:=5:s:=trunc(k/6);
liste:=[[seq(R*(d[i]+d[i+1])/2,i=s..(k-5))],[seq(Rho[i+1],i=s-1..
(k-6))]]:
eqfit:=fit[leastsquare[[x,y]]](liste);
eqfonction:=unapply(rhs(eqfit),x):
courberegr:=plot(eqfonction(x),x=0..R-1):
with(plots):
display({courberegr,courbelog});Digits:=25:
```

$s := 26$



> #For Fosué and also Huang, the Keplerian mass of the Milky Way is around 0.5 10^12 solar mass. So the radius of the disk of the Milky Way seems to be around 40 kpc, (no need of a hypothetical halo).

>

> #Thus : no need of a spherical halo to explain the flatness problem. Moreover the plane of dwarf galaxy satellites could be understood.

> #This program is robust and fast! If the galaxy is now in a universe, it is not difficult to modify this program (a correction from 1% to 3% for the mass which is less than the uncertainties coming from velocities).

> #with k=160, total time is less than 200 seconds.

Is this the end of the trip?

> #Now let us go to Andromeda.

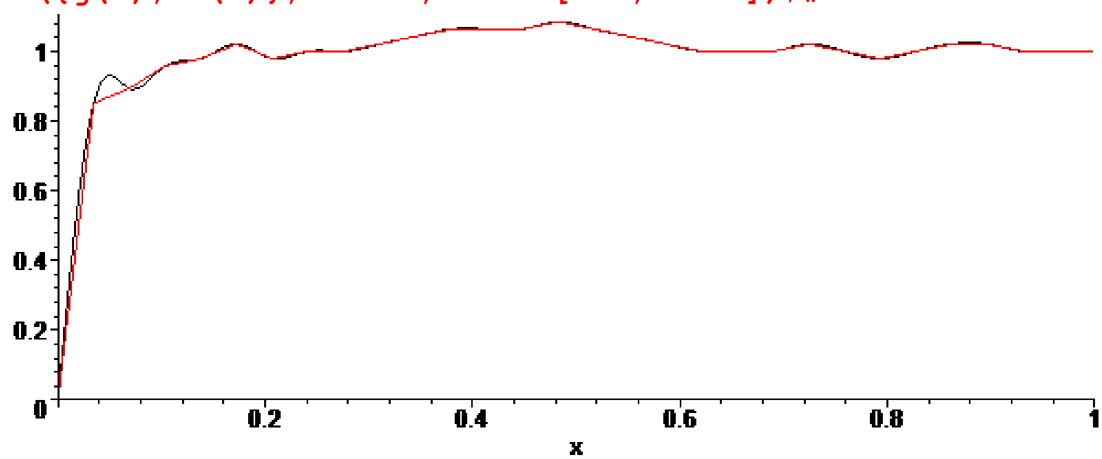
```
> #For Andromeda NE
R:=30;V:=2.35;
f:=[0,2.,2.1,2.25,2.3,2.4,2.3,2.35,2.35,2.4,2.45,2.5,2.5,2.5,2.55
,2.5,2.45,2.4,2.35,2.35,2.35,2.4,2.35,2.3,2.35,2.4,2.4,2.35,2.35,
2.35,2.35]:
a:=nops(f)-2;
fv:=proc(x)
(f[trunc(a*x)+1]+(a*x-trunc(a*x))*(f[trunc(a*x)+2]-f[trunc(a*x)
+1]))/V
end:
```

$$R := 30$$

$$V := 2.35$$

$$a := 29$$

```
> F:=[seq([(i-1)/29,f[i]/V],i=1..30)]:
> g:=x->Spline(F,x):
> plot({g(x),fv(x)},x=0..1,color=[red,black]);#well
```



```
> #The second member of the linear system, coming from observed
velocities.
   BB:=Vector([seq(w*g(d[i])^2/d[i],i=1..k),1]):
> #calculus of mass as function of w
C:=evalf(MatrixVectorMultiply(invA,BB),30):
> #search of w=wmin and w=wmax such that all the mass are >=0
wnul:=seq(evalf(solve(C[i]=0,w)),i=1..k+1):
n1:=0:n2:=0:
for j to k+1 do
 N:=seq(subs(w=wnul[j],C[i]),i=1..k+1):truc:=1:
 for i to k+1 do
   if N[i]<-10^(-5) then truc:=0 fi:
 od;
 if truc=1 then if n1=0 then n1:=j else n2:=j fi fi;
od:
if n1=0 then print(`il_y_a_des_masses_negatives`) else
```

```
 if wnul[n1]>wnul[n2] then wmax:=wnul[n1];wminim:=wnul[n2]
   else wmax:=wnul[n2];wminim:=wnul[n1] fi;
 fi:
wmax:=evalf(wmax,15);wminim:=evalf(wminim,15);
wm:=evalf((wmax+wminim)/2,15);wmax-wminim;
```

$$wmax := 1.52637658960144$$

$$wminim := 1.52637656731640$$

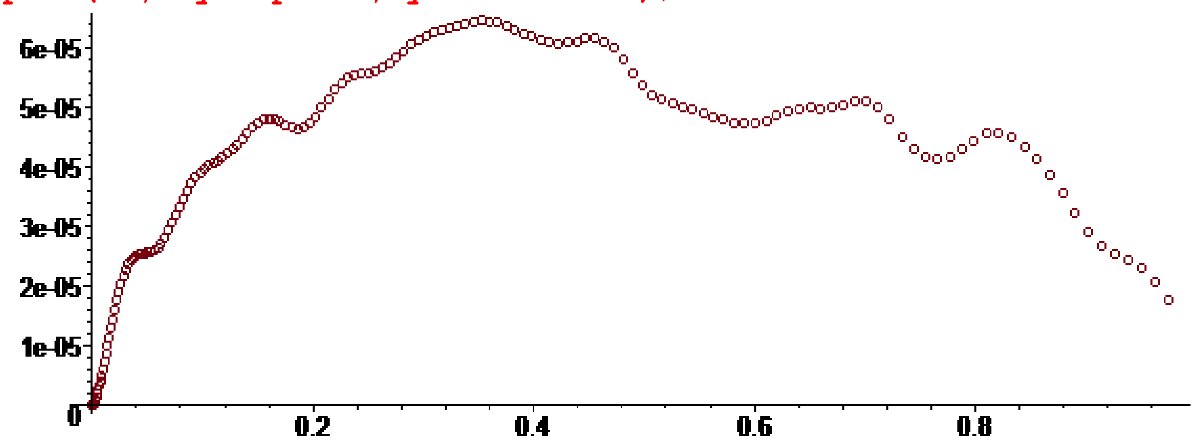
$$wm := 1.52637657845892$$

$$0.2228504\ 10^{-7}$$

```
>
M:=seq(subs(w=wm,evalf(C[i],15)),i=1..k):M[1..3];MM:=[seq([d[i],s
ubs(w=wm,evalf(C[i],15))],i=1..k-2)]:
```

$0.4092599432665\ 10^{-10}, 0.334487997012523\ 10^{-8}, 0.216686670762919 49\ 10^{-7}$

```
> plot(MM,style=point,symbol=circle);
```




```
> MasseGalaxie:=evalf(0.23*10^10*V^2*R/wm,5):
print(`Masse_d'Andromède NE`,MasseGalaxie,`en_Masses_Solaires`);
```

Masse_d'Andromède NE, $0.24964\ 10^{12}$, *en_Masses_Solaires*

```
>
evalf(0.23*10^10*V^2*R/wmax,8),evalf(0.23*10^10*V^2*R/wminim,8);
```

$$0.24964515\ 10^{12}, 0.24964515\ 10^{12}$$

```
> Mgal:=MasseGalaxie:
> #mean surfacic density
> rho:=[seq(Mgal*evalf(8*l*(M[i]+M[i+1])/Pi/((d[i+1]+d[i+2])^2
-(d[i]+d[i-1])^2)/(R*10^3)^2),i=2..(k-1))]:
> Rho:=[seq(ln(rho[i]),i=1..(k-2))]:nops(%);
```

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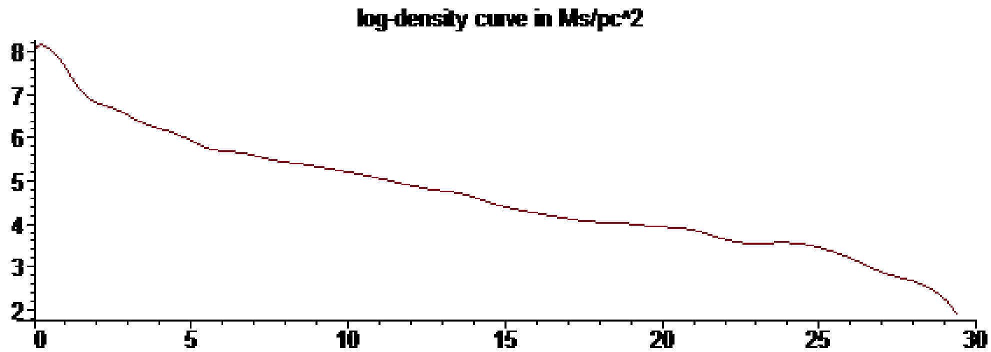
```
> plot([seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-2)],
title=`log-density curve in Ms/pc^2`);
```



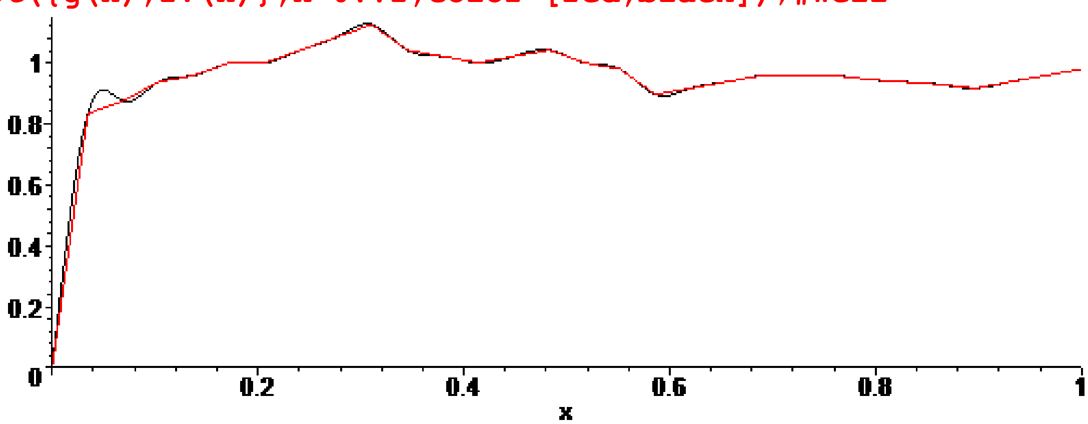

```
>densitéAndromède_NE:=[seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-2)]:
> #For Andromede SW :
#The observed rotation curve for Andromeda SW
R:=30;V:=2.4;
f:=[0,2.,2.1,2.25,2.3,2.4,2.4,2.5,2.6,2.7,2.5,2.45,2.4,2.45,2.5,2
.4,2.35,
2.15,2.2,2.25,2.3,2.3,2.3,2.27,2.25,2.23,2.2,2.25,2.3,2.35,2.4]:
a:=nops(f)-2;
fv:=proc(x)
(f[trunc(a*x)+1]+(a*x-trunc(a*x))*(f[trunc(a*x)+2]-f[trunc(a*x)
+1]))/V
end:
```

$$R := 30$$

$$V := 2.4$$

$$a := 29$$

```
> F:=[seq([(i-1)/29,f[i]/V],i=1..30)]:
> g:=x->Spline(F,x):
> plot({g(x),fv(x)},x=0..1,color=[red,black]);#well
```




```
> #The second member of the linear system, coming from observed
velocities.
BB:=Vector([seq(w*g(d[i])^2/d[i],i=1..k),1]):
#w as the meaning of the inverse of the mass M of the galaxy
> #calculus of mass as fonction of w
C:=evalf(MatrixVectorMultiply(invA,BB),30):
```

```
> #search of w=wmin and w=wmax such that all the mass are >=0
wnul:=seq(evalf(solve(C[i]=0,w)),i=1..k+1):
n1:=0:n2:=0:
for j to k+1 do
 N:=seq(subs(w=wnul[j],C[i]),i=1..k+1):truc:=1:
 for i to k+1 do
   if N[i]<-10^(-5) then truc:=0 fi:
 od;
 if truc=1 then if n1=0 then n1:=j else n2:=j fi fi;
od:
if n1=0 then print(`il_y_a_des_masses_negatives`) else
 if wnul[n1]>wnul[n2] then wmax:=wnul[n1];wminim:=wnul[n2]
   else wmax:=wnul[n2];wminim:=wnul[n1] fi;
 fi:
wmax:=evalf(wmax,15);wminim:=evalf(wminim,15);
wm:=evalf((wmax+wminim)/2,15);wmax-wminim;
```

$$wmax := 1.67975395286903$$

$$wminim := 1.67975392689909$$

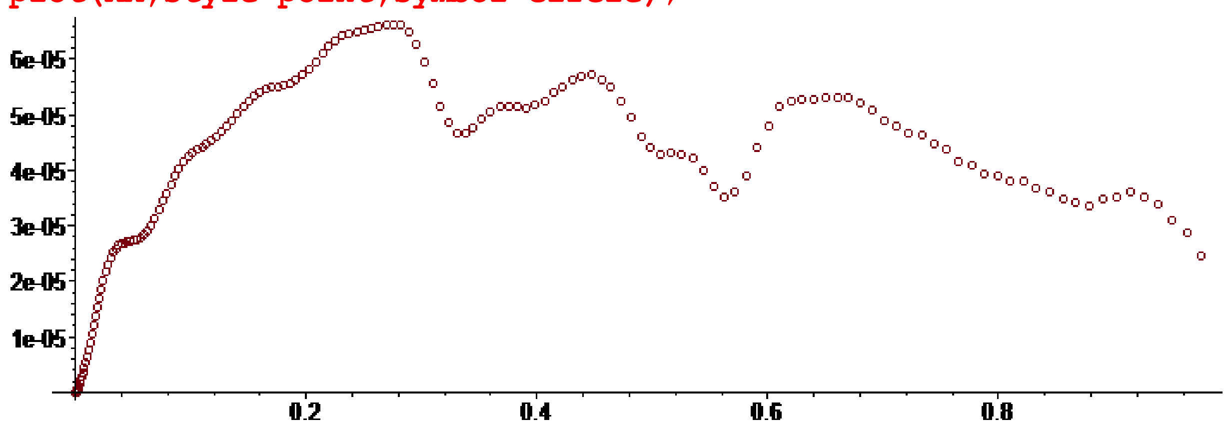
$$wm := 1.67975393988406$$

$$0.2596994\ 10^{-7}$$

```
>M:=seq(subs(w=wm,evalf(C[i],15)),i=1..k):M[1..3];MM:=[seq([d[i],
subs(w=wm,evalf(C[i],15))],i=1..k-2)]:
```

$0.4333838421490\ 10^{-10}, 0.354204531074112\ 10^{-8}, 0.22945938137795726\ 10^{-7}$

```
> plot(MM,style=point,symbol=circle);
```




```
> MasseGalaxie:=evalf(0.23*10^10*V^2*R/wm,5):
print(`Masse_d'Andromède SW`,MasseGalaxie,`en_Masses_Solaires`);
```

Masse_d'Andromède SW, $0.23660\ 10^{12}$, *en_Masses_Solaires*

```
>evalf(0.23*10^10*V^2*R/wmax,8),evalf(0.23*10^10*V^2*R/wminim,8);
```

$$0.23660608\ 10^{12}, 0.23660608\ 10^{12}$$

```
> Mgal:=MasseGalaxie:
```

```
> #mean surface density
> rho:=[seq(Mgal*evalf(8*l*(M[i]+M[i+1])/Pi/((d[i+1]+d[i+2])^2
-(d[i]+d[i-1])^2)/(R*10^3)^2),i=2..(k-1))]:nops(%);
```

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```
> Rho:=[seq(ln(rho[i]),i=1..(k-2))]:nops(%);
```

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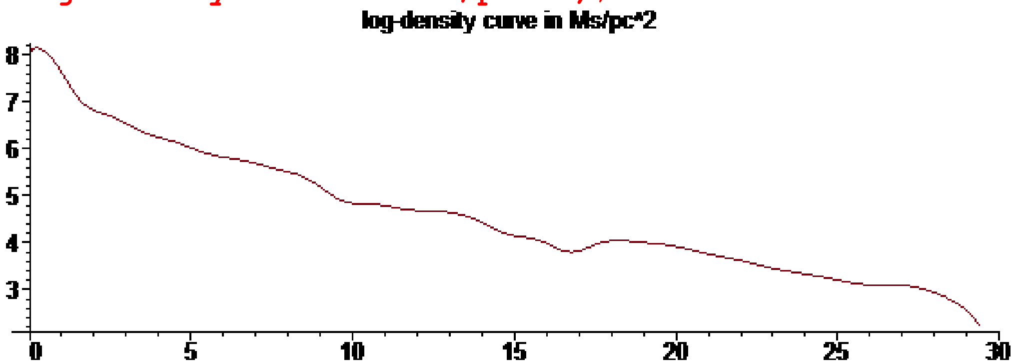
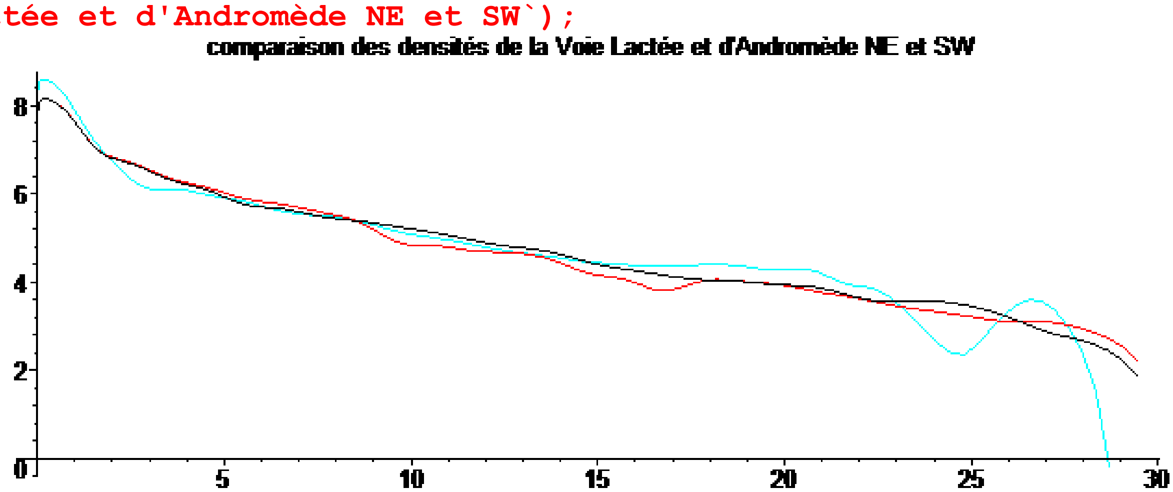
```
> plot([seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-2)],
title=`log-density curve in Ms/pc^2`);
```



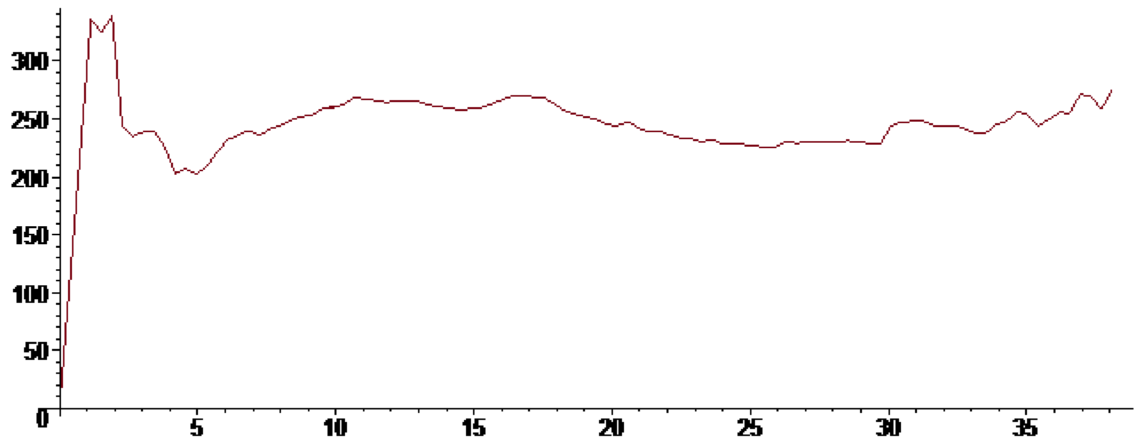

```
>densitéAndromède_SW:=[seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-2)]:
>
>plot([densitéAndromède_NE,densitéAndromède_SW,densitéVL_30],colo
r=[black,red,cyan], title=`comparaison des densités de la Voie
Lactée et d'Andromède NE et SW`);
```




```
> #the surface densities are similar up to 24 kpc from the
center; the color and mass are respectively for Andro_NE,
Andro_SW, Milky Way, [black,red,cyan],
[.24964e12,.23660e12,.25217007e12]; so Milky Way and Andromeda
are twin galaxies. This fact is new since 2014.
```

Is this the end of the journey?

```
> #But the data for Andromède are old (1996) so let us now take
the recent data (Chemin, L., Carignan, C., & Foster, T. 2009,
ApJ, 705, 1395) valid up to 38 kpc for the entire galaxy.
>Andro:=[[0,0],[1.14,336.],[1.52,324.5],[1.90,339.],[2.28,243.5],
[2.66,235.],[3.04,239.],[3.43,239.],[3.81,226.],[4.19,203.],
[4.57,207.],[4.95,202.5],[5.33,209.],[5.71,221.5],[6.09,232.],
[6.85,240.],[7.23,235.5],[7.61,241.5],[7.99,244.5],[8.37,249],
[8.75,252],[9.13,253.],[9.51,259.],[9.90,259.],[9.51,259.],
[10.28,262.],[10.66,269.],[11.04,267.],[11.42,266.],
[11.80,264.5],[12.18,264.7],[12.56,265.3],[12.94,265.2],
[13.32,262.],[13.32,262.],[13.70,261.],[14.08,259.],[14.46,258.],
[14.84,258.5],[15.23,259.2],[15.61,262.7],[15.99,266.],
[16.37,270.],[16.75,270.],[17.13,269.],[17.51,268.5],
[17.89,263.],[18.27,257.],[18.65,254.],[19.03,252.],
[19.41,249.5],[19.79,245.7],[20.18,243.7],[20.56,247.9],
[20.94,242.3],[21.32,239.2],[21.70,239.5],[22.08,236.1],
[22.46,233.8],[22.84,233.1],[23.22,230.1],[23.60,232.1],
[23.98,228.7],[24.36,229.1],[24.75,227.9],[25.13,226.9],
[25.51,225.1],[25.89,225.4],[26.27,230.3],[26.65,229.],
[27.03,229.9],[27.41,230.1],[27.79,229.8],[28.17,230.4],
[28.56,230.9],[28.94,229.8],[29.32,228.8],[29.70,228.3],
[30.08,243.6],[30.46,247.3],[30.84,247.8],[31.22,248.4],
[31.61,244.5],[31.99,244.5],[32.37,244.4],[32.75,241.7],
[33.13,237.7],[33.51,237.6],[33.89,244.9],[34.27,247.9],
[34.66,256.3],[35.04,253.5],[35.42,244.3],[35.80,249.3],
[36.18,255.7],[36.56,255.0],[36.94,271.1],[37.32,269.8],
[37.71,258.2],[38.09,275.1]]:
> plot(Andro);
```




```
> Comparison between Andromeda, with these new data, and the
Milky Way for 3 radius R=30, 35 and 38 kpc. we take now k:=240.
>For R=30 kpc
>donneesVL30:=[[0,0],[0.20,233.0],[0.38,268.92],[0.66,250.75],
[1.61,217.83],[2.57,219.58],[3.59,223.11],[4.60,231.24],
[5.08,230.46],[5.58,230.01],[6.10,239.61],[6.57,246.27],
[7.07,243.49],[7.58,242.71],[8.04,243.23],[8.34,239.89],
[8.65,237.26],[9.20,235.30],[9.62,230.99],[10.09,228.41],
[10.58,224.26],[11.09,224.94],[11.58,233.57],[12.07,240.02],
[12.73,242.21],[13.72,261.78],[14.95,259.26],[15.52,268.57],
[16.55,261.17],[17.56,240.66],[18.54,215.31],[19.50,214.99],
[21.25,251.68],[23.78,259.65],[26.22,242.02],[28.71,224.11],
[31.29,211.20]]:nops(%);
```

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```
> f:=map(u->evalf(op(2,u)/100,5),donneesVL30);
```

$f := [0., 2.3300, 2.6892, 2.5075, 2.1783, 2.1958, 2.2311, 2.3124, 2.3046, 2.3001, 2.3961, 2.4627, 2.4349, 2.4271, 2.4323, 2.3989, 2.3726, 2.3530, 2.3099, 2.2841, 2.2426, 2.2494, 2.3357, 2.4002, 2.4221, 2.6178, 2.5926, 2.6857, 2.6117, 2.4066, 2.1531, 2.1499, 2.5168, 2.5965, 2.4202, 2.2411, 2.1120]$

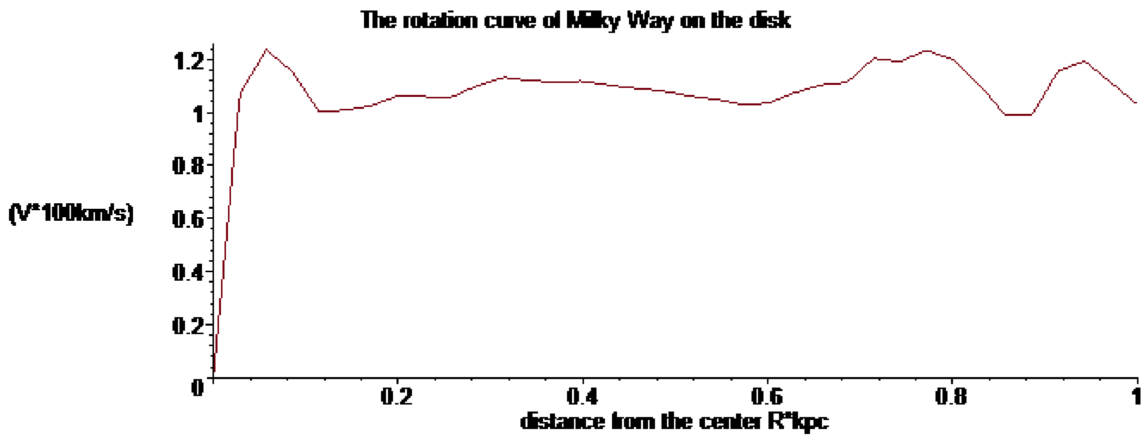
```
>
R:=30;a:=nops(f)-2;V:=evalf((f[a+1]+f[a+2])/2,7);
```

$R := 30$

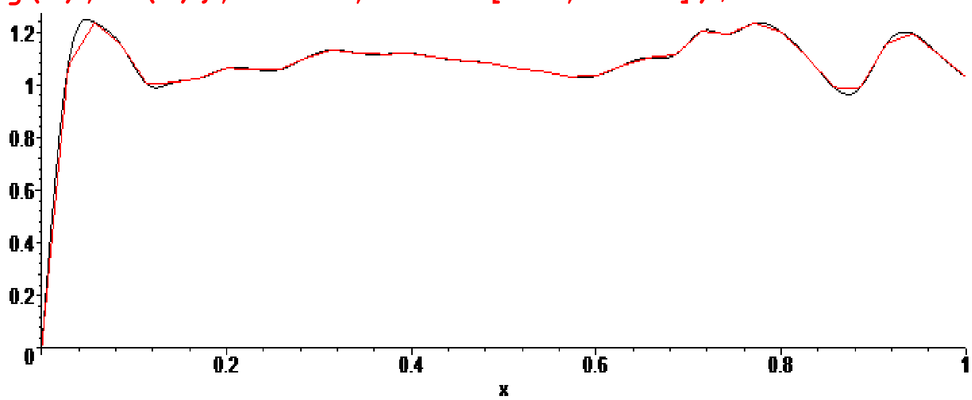
$a := 35$

$V := 2.176550$

```
> fv:=proc(x)
(f[trunc(a*x)+1]+(a*x-trunc(a*x))*(f[trunc(a*x)+2]-f[trunc(a*x)
+1]))/V  end:
> plot(fv(x),x=0..1,title=`The rotation curve of Milky Way on the
disk`,labels=[`distance from the center R*kpc`,`(V*100km/s)`]);
```




```
> F:=[seq([(i-1)/35,f[i]/V],i=1..37)]:
> g:=x->Spline(F,x):
> plot({g(x),fv(x)},x=0..1,color=[red,black]);
```



```
> BB:=Vector([seq(w*g(d[i])^2/d[i],i=1..k),1]):
#w as the meaning of the inverse of the mass M of the galaxy
> C:=evalf(MatrixVectorMultiply(invA,BB),30):
> #search of w=wmin and w=wmax such that all the mass are >=0
> wnul:=seq(evalf(solve(C[i]=0,w)),i=1..k+1):
n1:=0:n2:=0:
for j to k+1 do
 N:=seq(subs(w=wnul[j],C[i]),i=1..k+1):truc:=1:
 for i to k+1 do
   if N[i]<-10^(-5) then truc:=0 fi:
 od;
 if truc=1 then if n1=0 then n1:=j else n2:=j fi fi;
od:
if n1=0 then print(`il_y_a_des_masses_negatives`) else
 if wnul[n1]>wnul[n2] then wmax:=wnul[n1];wminim:=wnul[n2]
   else wmax:=wnul[n2];wminim:=wnul[n1] fi;
 fi:
wmax:=evalf(wmax,15);wminim:=evalf(wminim,15);
wm:=evalf((wmax+wminim)/2,15);wmax-wminim;
```

$$wmax := 1.29609974554824$$

$$wminim := 1.29609971619675$$

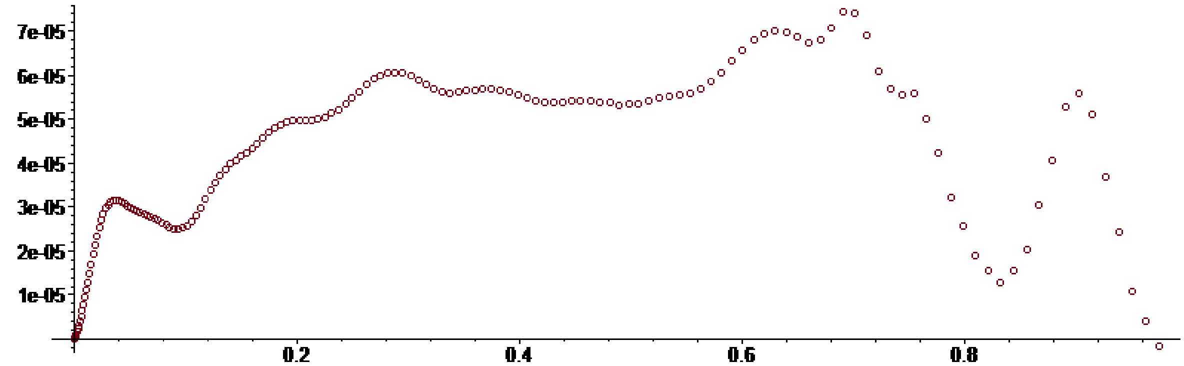
$$wm := 1.29609973087250$$

$$0.2935149\ 10^{-7}$$

```
>M:=seq(subs(w=wm,evalf(C[i],15)),i=1..k):M[1..3];MM:=[seq([d[i],
subs(w=wm,evalf(C[i],15))],i=1..k-2)]:
```

$0.6348035233179\ \ 10^{-10}, 0.518823756785924\ \ 10^{-8}, 0.33609949360446503\ \ 10^{-7}$

```
> plot(MM,style=point,symbol=circle);
```




```
> MasseGalaxie:=evalf(0.23*10^10*V^2*R/wm,8):
print(`Masse_de_la_Galaxie`,MasseGalaxie,`en_Masses_Solaires`);
```

Masse_de_la_Galaxie, $0.25220169\ 10^{12}$, *en_Masses_Solaires*

```
>evalf(0.23*10^10*V^2*R/wmax,8),evalf(0.23*10^10*V^2*R/wminim,8);
```

$$0.25220169\ 10^{12}, 0.25220169\ 10^{12}$$

```
> Mgal:=MasseGalaxie;
```

$$Mgal := 0.25220169\ 10^{12}$$

```
> MgalVL30:=.25220e12;
```

$$MgalVL30 := 0.25220\ 10^{12}$$

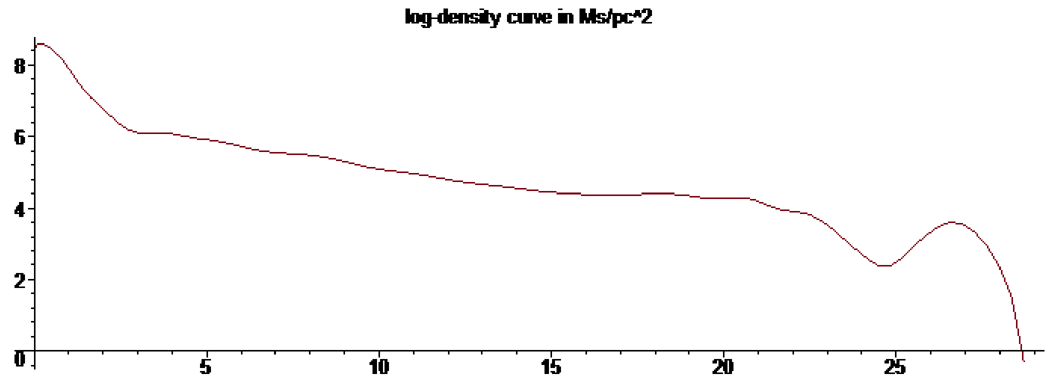
```
> #mean surface density
rho:=[seq(Mgal*evalf(8*1*(M[i]+M[i+1])/Pi/((d[i+1]+d[i+2])^2
-(d[i]+d[i-1])^2)/(R*10^3)^2),i=2..(k-2))]:
> rho[150..157];
```

[36.550157768358683832 3 , 35.300125271617676307 7 , 28.386415878465065032 0 ,
19.403335234199181972 5 , 10.996795736244568683 0 ,
4.6111850673300306404 1 , 0.7327481842839546855 02 ,
-1.0400694788358788095 7]

>

```
#log-density curve

Rho:=[seq(ln(rho[i]),i=1..(k-4))]:
> plot([seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-4)],
title=`log-density curve in Ms/pc^2`);
```




```
> densitéVL30:=[seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-4)]:
>
> Andro30:=[[0,0],[1.14,336.],[1.52,324.5],[1.90,339.],
[2.28,243.5],[2.66,235.],[3.04,239.],[3.43,239.],[3.81,226.],
[4.19,203.],[4.57,207.],[4.95,202.5],[5.33,209.],[5.71,221.5],
[6.09,232.],[6.85,240.],[7.23,235.5],[7.61,241.5],[7.99,244.5],
[8.37,249],[8.75,252],[9.13,253.],[9.51,259.],[9.90,259.],
[9.51,259.],[10.28,262.],[10.66,269.],[11.04,267.],[11.42,266.],
[11.80,264.5],[12.18,264.7],[12.56,265.3],[12.94,265.2],
[13.32,262.],[13.32,262.],[13.70,261.],[14.08,259.],[14.46,258.],
[14.84,258.5],[15.23,259.2],[15.61,262.7],[15.99,266.],
[16.37,270.],[16.75,270.],[17.13,269.],[17.51,268.5],
[17.89,263.],[18.27,257.],[18.65,254.],[19.03,252.],
[19.41,249.5],[19.79,245.7],[20.18,243.7],[20.56,247.9],
[20.94,242.3],[21.32,239.2],[21.70,239.5],[22.08,236.1],
[22.46,233.8],[22.84,233.1],[23.22,230.1],[23.60,232.1],
[23.98,228.7],[24.36,229.1],[24.75,227.9],[25.13,226.9],
[25.51,225.1],[25.89,225.4],[26.27,230.3],[26.65,229.],
[27.03,229.9],[27.41,230.1],[27.79,229.8],[28.17,230.4],
[28.56,230.9],[28.94,229.8],[29.32,228.8],[29.70,228.3],
[30.08,243.6]]:
> #For Andromeda30
R:=30;V:=2.40;
f:=map(u->op(2,u)/100,Andro30):
a:=nops(f)-2;
fv:=proc(x)
 (f[trunc(a*x)+1]+(a*x-trunc(a*x))*(f[trunc(a*x)+2]-f[trunc(a*x)
+1]))/V
end:
```

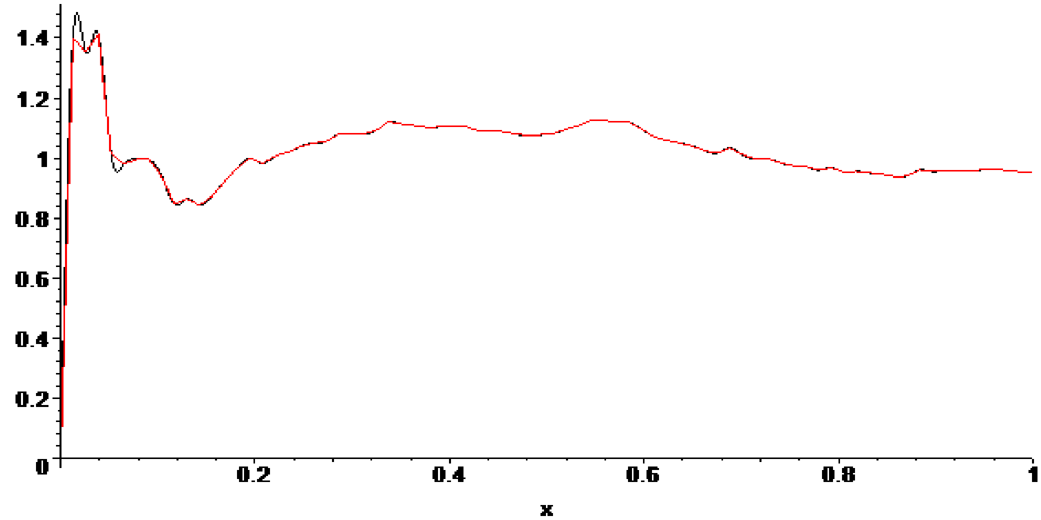
$$R := 30$$

$$V := 2.40$$

$$a := 77$$

```
> F:=[seq([(i-1)/77,f[i]/V],i=1..78)]:
> g:=x->Spline(F,x):
```

```
> plot({g(x),fv(x)},x=0..1,color=[red,black]);
```




```
> BB:=Vector([seq(w*g(d[i])^2/d[i],i=1..k),1]):
> C:=evalf(MatrixVectorMultiply(invA,BB),30):
> wnul:=seq(evalf(solve(C[i]=0,w)),i=1..k+1):
n1:=0:n2:=0:
for j to k+1 do
 N:=seq(subs(w=wnul[j],C[i]),i=1..k+1):truc:=1:
 for i to k+1 do
   if N[i]<-10^(-5) then truc:=0 fi:
 od;
 if truc=1 then if n1=0 then n1:=j else n2:=j fi fi;
od:
if n1=0 then print(`il_y_a_des_masses_negatives`) else
 if wnul[n1]>wnul[n2] then wmax:=wnul[n1];wminim:=wnul[n2]
   else wmax:=wnul[n2];wminim:=wnul[n1] fi;
 fi:
wmax:=evalf(wmax,15);wminim:=evalf(wminim,15);
wm:=evalf((wmax+wminim)/2,15);wmax-wminim;
```

$$wmax := 1.58684496047333$$

$$wminim := 1.58684479838473$$

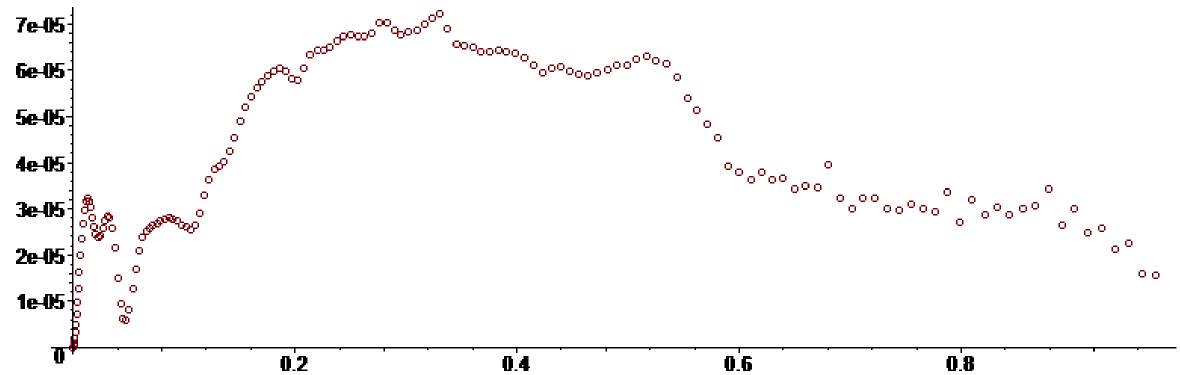
$$wm := 1.58684487942903$$

$$0.16208860\ 10^{-6}$$

```
>M:=seq(subs(w=wm,evalf(C[i],15)),i=1..k):M[1..3];MM:=[seq([d[i],
subs(w=wm,evalf(C[i],15))],i=1..k-2)]:
```

$0.28632900277784\ 10^{-9}, 0.2340073285137875\ 10^{-7}, 0.1515614373437346 99\ 10^{-6}$

```
> plot(MM,style=point,symbol=circle);
```




```
> MasseGalaxie:=evalf(0.23*10^10*V^2*R/wm,5):
print(`Masse_d'Andromède`,MasseGalaxie,`en_Masses_Solaires`);
```

$Masse\_d'Andromède,\ 0.25047\ 10^{12},\ en\_Masses\_Solaires$

```
> MgalAndro30:=.25047e12;
```

$MgalAndro30 := 0.25047\ 10^{12}$

```
> rho:=[seq(MgalAndro30*evalf(8*l*(M[i]+M[i+1])/Pi/
((d[i+1]+d[i+2])^2
-(d[i]+d[i-1])^2)/(R*10^3)^2),i=2..(k-1))]:nops(%);
```

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```
> rho[150..158];
```

[18.8202629941841191441 , 17.9042367161536068983 , 16.2306684089766064085 ,
14.8298265462072842882 , 13.5229207371455547674 ,
11.6634467504719020297 , 9.42836959439959923588 ,
7.52745871867826191984 , 5.53210191320532546773]

```
>Rho:=[seq(ln(rho[i]),i=1..(k-2))]:nops(%);
```

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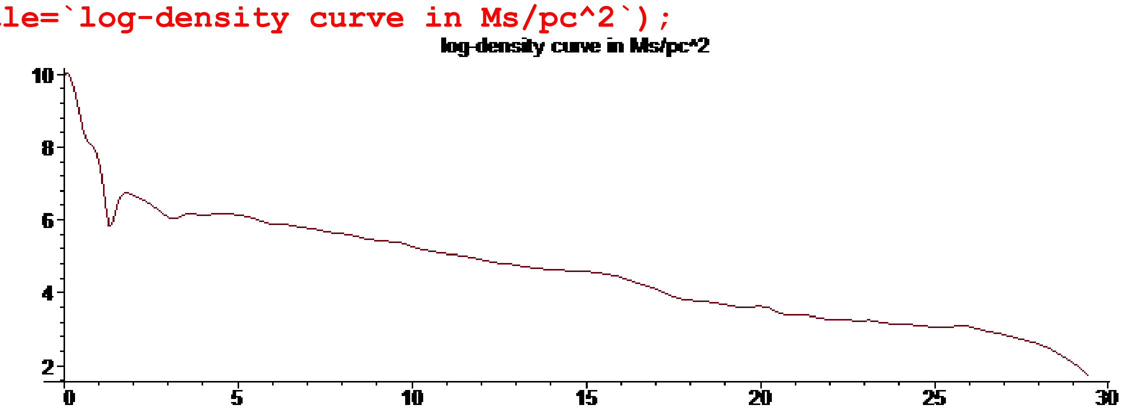
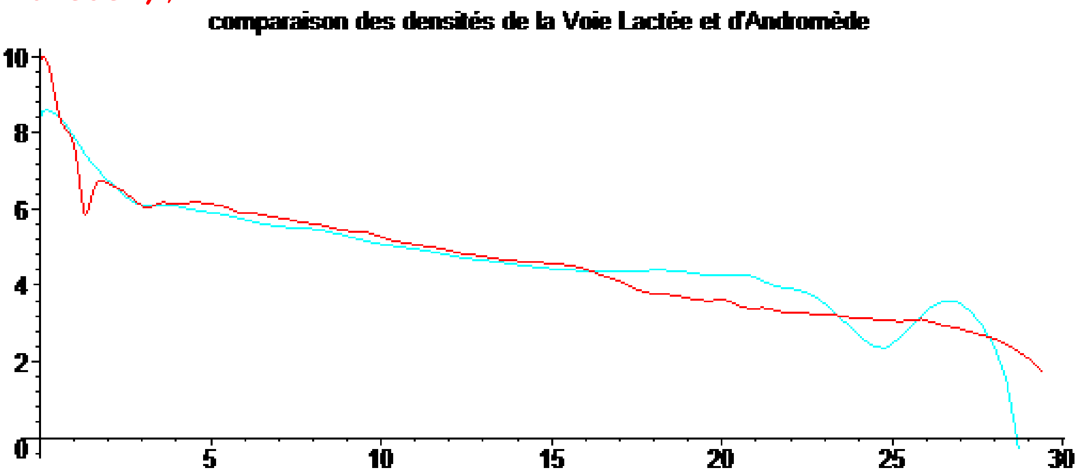
```
> plot([seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-2)],
title=`log-density curve in Ms/pc^2`);
```




```
> densitéAndromède30:=[seq([R*(d[i]+d[i+1])/2,Rho[i]],i=1..k-2)]:
> plot([densitéAndromède30,densitéVL30],color=[red,cyan],
title=`comparaison des densités de la Voie Lactée et
```

d'Andromède`);



>

> MgalAndro30;MgalVL30;

$$0.25047\ 10^{12}$$

$$0.25220\ 10^{12}$$

> #recall the mass of Andromeda with the old data, for the same radius:

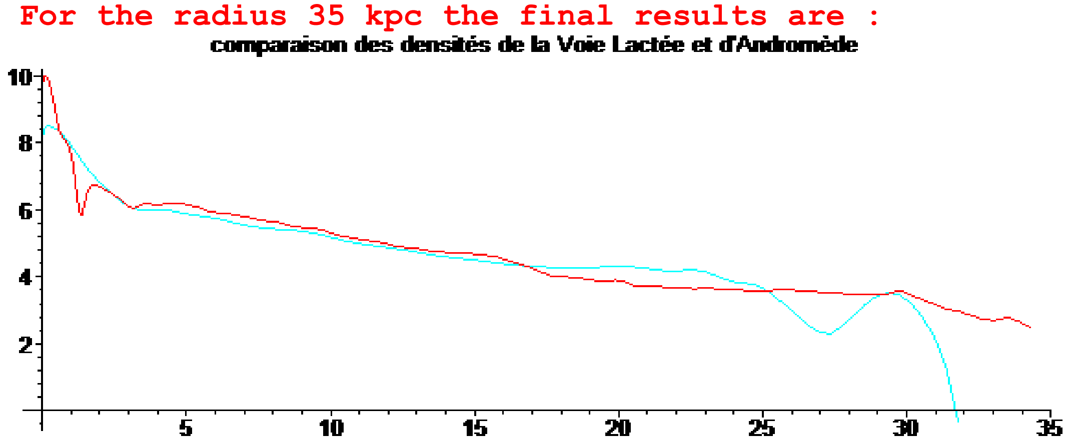
> `Masse_d'Andromède NE`= .24964e12;`Masse_d'Andromède SW`= .23660e12;

$$Masse\_d'Andromède\ NE = 0.24964\ 10^{12}$$

$$Masse\_d'Andromède\ SW = 0.23660\ 10^{12}$$

> #Now if the two galaxies have a bigger radius, R=35 or R=38 kpc

> # For the radius 35 kpc the final results are :

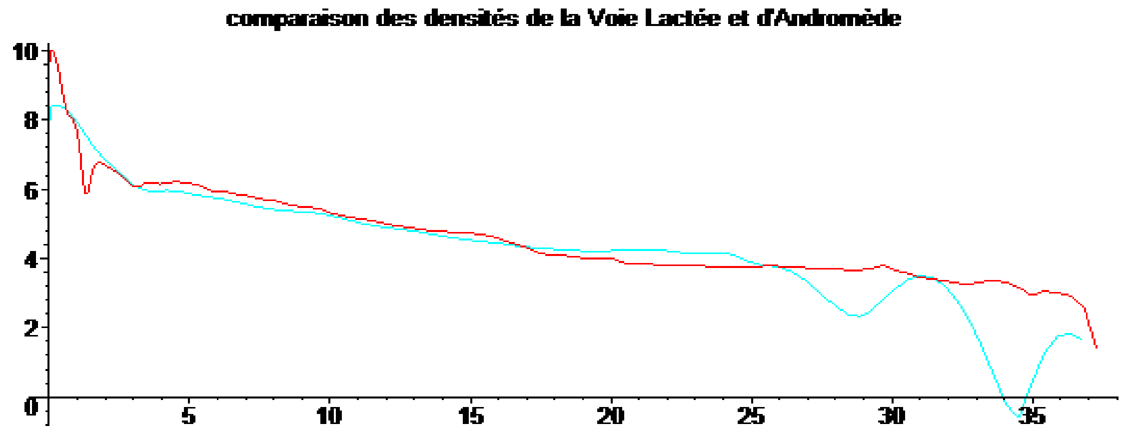


> MgalAndro35=.30873e12;MgalVL35=.28104e12;

$$MgalAndro35 = 0.30873\ 10^{12}$$

$$MgalVL35 = 0.28104\ 10^{12}$$

> #And for the radius 38 kpc the final results are :



> **MgalAndro38=.34705e12;MgalVL38=.302285e12;**

$$MgalAndro38 = 0.34705\ 10^{12}$$

$$MgalVL38 = 0.302285\ 10^{12}$$

Some recent studies show that the Kepler radius of these two galaxies, with a halo of dark matter, would be of the order of 40 to 50 kpc with a Kepler mass of the order of 0.5 10 ^ 12 solar masses. Our results confirm these facts **but without halo of dark matter.**

Why?

It is a long story of mathematical errors in the use of the theory of integration. Even if for about 6 years, one of the errors is corrected by the use of the functions of Bessel, there are still others. A disk of matter is not a sphere!

Until now we worked with the Newton theory of the gravitation. But it is probably that relativistic theories could provide similar results. Indeed the Einstein theory and also the conformal theory have been tested by many scientists around the world. A first problem, the speed of light is finish. Another problem, we worked with a spiral galaxy in an empty universe. The first problem has an aftermath, not for the relativist gravitation but for the trip of Gaston with his gaffophone because the radius of the Milky Way is greater than 50 000 light years. Probably this journey is a dream.

The second problem, with a spiral galaxy in an empty universe, is difficult to overcome. Indeed the equality between the inertial mass mI and the gravitational mass mG is a consequence of the Newton's Laws but in Einstein gravitational theory it is a basic

principle. How to explain that mI is not null in an empty universe? In a non-empty universe and endowed with the Mach's principle, it is possible. But we have a small aftermath: the Kepler's law, so the Kepler's radius, don't exist. Nevertheless we have another radius, the radius of attraction of the galaxy which was defined by J. M. Souriau (cf. Mizony 2005). I made a calculus for the Milky Way in a de Sitter universe 20 years ago, the relativist corrections are very small, for example the mass is around 1% heavier. Since, many scientists made also alternative proofs (cf. Mannheim, Marmet, Cooperstock , … , and these last years a Chinese group and an Australian). All, they find, using different forms for the disk and often Einstein equations, similar results: the Bessel's functions, similar mass, …, with no need of a dark matter halo. So no "core-cusp" problem for the "vast rotating disk of dwarf galaxies surrounding the Andromeda galaxy".
As a **conclusion** we have to note that the mass of the twin galaxies for the same radius from 30 to 38 kpc are similar and in the interval **[2.3 10^11, 3.5 10^11] times the solar mass.**

The creation of the "dark matter" fiction is similar to the "gaffophone" story.

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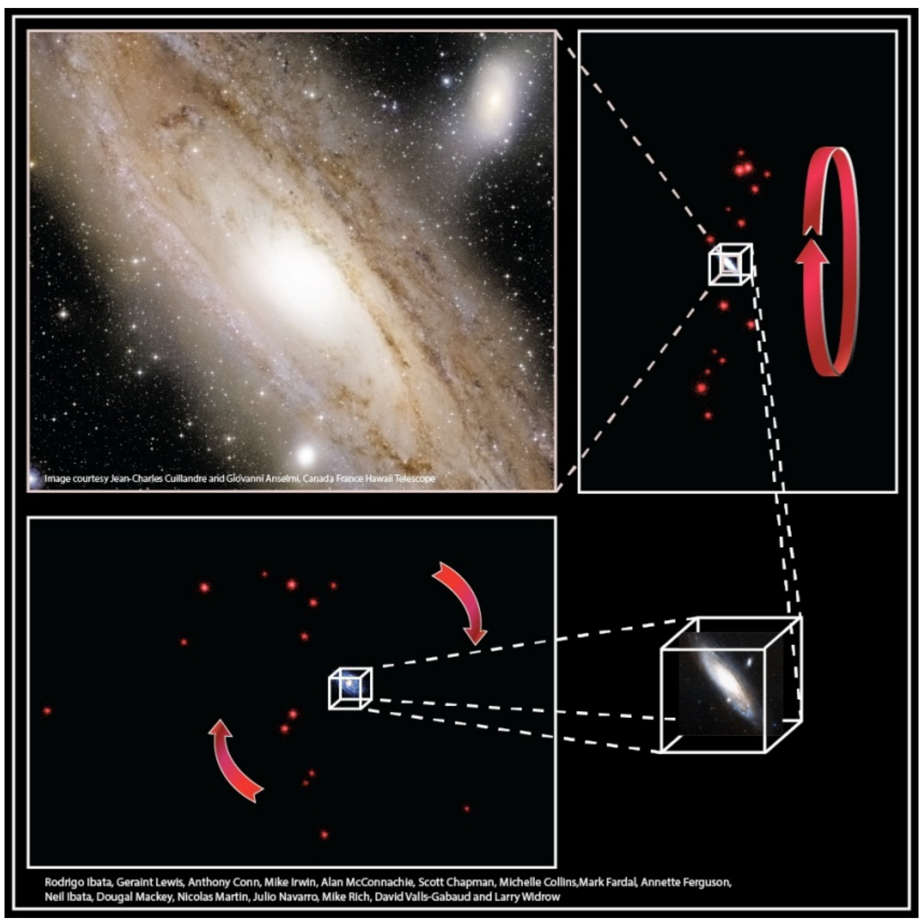

Rotation curves for spiral galaxies and maximal disk

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Résumé

We use a new original method to calculate the surface density of a galaxy from its rotation curve assuming that all the mass is in the galactic plane. We show that this method gives accurate masses for the disk. Moreover, given a rotation curve for $r < R_g$ only (R_g is the galaxy radius), we find that the surface density is nearly uniquely determined except near the edge of the disk. We also demonstrate that the common assumption that a flat rotation curve leads to a density profile $\Sigma(R) \propto 1/R$ is *false* for a finite disk. We derive for the Milky Way a mass of $1.4 \times 10^{11} M_{\odot}$. The exponential profile is well reproduced with a scale length between 3.6 and 5 kpc. The local surface density is found to be $125 \pm 10 M_{\odot}.pc^{-2}$, compatible with other independant determinations.

From this paper, written in 1996, we just shall take the Annexe, "A mathematical background". For this we have two references and a short explanation of our method.

[BT] Binney, J., and Tremaine, S. 1987, *Galactic dynamics* (Princeton : Princeton University Press) (BT)

[Erdelyi 1953] Erdelyi A., Higher transcendental functions, Vol 1, McGraw Hill, New-York, 1953

Our method is based on a numerical resolution of the Poisson equation. For a maximal disk, given the surface density $\Sigma(R)$, the force acting on a given point of the disk reads :

$$\vec{F} = \int \frac{\Sigma(R)}{R^3} \vec{R} d\vec{R}. \qquad (1)$$

Such an integral is mathematically divergent, but there are in fact symmetric compensations near the singular point, so that the integral is well defined. Recall that all numerical methods to calculate integrals introduce various discretisations of the integration volume. We will evaluate this integral, and use the discretized equation to derive $\Sigma(R)$ from $\vec{F}$.

A mathematical background

Let suppose that the rotation velocity $v(r)$ is known up to infinity. Using Bessel functions J_0 and J_1 we have the following formula [BT] :

$$\Sigma(r) = \frac{1}{2\pi G}\int_0^\infty J_0(kr)S(k)k\,dk$$

where

$$S(k) = \int_0^\infty v^2(x)J_1(kx)\,dx.$$

And conversely, using Bessel transformations :

$$S(k) = 2\pi G\int_0^\infty \Sigma(x)J_0(kx)x\,dx$$

$$v^2(r) = r\ \int_0^\infty S(k)J_1(kr)k\,dk.$$

For the Mestel's disk or for the exponential disk these integrals can be solved analytically.

This method has a theoretical interest : indeed the correspondence $v \leftrightarrow \Sigma(r)$ is a bijection, but since the rotation curve is know from observations up to $r \leq R_g$ only, several surface densities can in principle lead to the same rotation curve $v(r)$ for $r \leq R_g$, but with different $v(r)$ for $r \geq R_g$. Since most of the mass is concentrated in the center of the galaxy, the rotation curves for $r \geq R_g$ must be nearly Keplerian. It is the reason why our method provides a very narrow range $[\omega_{min}, \omega_{max}]$ of solutions.

If we want to compute $\Sigma(r)$ from $v(r)$, we have to consider the following double singular integral :

$$\Sigma(r) = \frac{1}{2\pi G}\int_0^\infty J_0(kr)\left[\int_0^\infty v^2(x)J_1(kx)\,dx\ \right]\ k\,dk.$$

In order to carry out the integrations, it is necessary to introduce a principal value integral.

Using properties of hypergeometric series ${}_2F_1$ [Erdelyi 1953], we write :

$$\begin{aligned} 4\pi G r\Sigma(r) \quad &= \quad \lim_{a\to 1}\int_0^a [v^2(r/\sqrt{z})\,\frac{1}{\sqrt{z}}\ {}_2F_1([3/2,1/2],[1],z) \\ &\quad -\frac{1}{2}v^2(r\sqrt{z})\ {}_2F_1([3/2,3/2],[2],z)\ ]dz. \end{aligned} \tag{8}$$

Conversely, $v(r)$ can be written as a function of $\Sigma(r)$ in the following form :

$$\begin{aligned} v^2(r)/r \quad &= \quad \pi G\lim_{a\to 1}\int_0^a [\Sigma(r\sqrt{z})\ \ {}_2F_1([3/2,1/2],[1],z) \\ &\quad -\frac{1}{2\sqrt{z}}\Sigma(r/\sqrt{z})\ {}_2F_1([3/2,3/2],[2],z)\ ]dz. \end{aligned} \tag{9}$$

It is also possible to rewrite this latter formula using elliptic integrals. Actually elliptic integrals are hypergeometric functions, so the formula (2-146) of [BT] leads to the above principal value integral in the plane of the disk.

But the most important consequence is the theoretical formula giving the mass of the galaxy from the rotation curve : If R_g is the radius of the disk then :

$$M = \frac{R_g}{4G}\int_0^1 [v^2(R_g\sqrt{z}) + v^2(\frac{R_g}{\sqrt{z}})/\sqrt{z}]\ {}_2F_1([3/2,1/2],[2],z)\,dz.$$

This formula can not be used to compute the mass of a given galaxy because of the unknown term $v^2(R_g/\sqrt{z})$, but we shall use it to prove that our method gives accurate results.

Notice first that if $r \geq R_g$ then the relation (9) reduces to :

$$v^2(r)/r = \pi G \int_0^1 \Sigma(r\sqrt{z})\ \ {}_2F_1([3/2,1/2],[1],z)dz,$$

or

$$v^2(r) = \frac{2\pi G}{r}\int_0^{R_g}\ {}_2F_1([3/2,1/2],[1],u^2/r^2)\,u\,\Sigma(u)\,du. \qquad (10)$$

Let $v_k^2(r) = \frac{GM}{r} = \frac{2\pi G}{r}\int_0^{R_g} u\,\Sigma(u)\,du$ be the Keplerian rotation curve. Then (10) can be written :

$$v^2(r) - v_k^2(r) = \frac{2\pi G}{r}\int_0^{R_g}[\ {}_2F_1([3/2,1/2],[1],u^2/r^2) - 1]\,u\,\Sigma(u)\ du.$$

But the function $f(u^2/r^2) = \ {}_2F_1([3/2,1/2],[1],u^2/r^2) - 1$ has the two following properties :

if $r \geq R_g$ then $\frac{3}{4}\frac{u^2}{r^2} + \frac{2}{3}\frac{u^4}{r^4} \leq f(u^2/r^2)$ and

if $r \geq 2R_g$ then $f(u^2/r^2) \leq \frac{3}{4}\frac{u^2}{r^2} + \frac{u^4}{r^4}$. Denoting $M_n = 2\pi\int_0^{R_g} u^n \Sigma(u)\ du$ the momentum of order n of the surface density, we thus obtain the following estimates :

for $r \geq R_g$,

$$\frac{3}{4}\frac{GM_3}{r^3} + \frac{2}{3}\frac{GM_5}{r^5} \leq v^2(r) - v_k^2(r),$$

and for $r \geq 2R_g$

$$v^2(r) - v_k^2(r) \leq \frac{3}{4}\frac{GM_3}{r^3} + \frac{GM_5}{r^5}.$$

This inequality proves indeed the accuracy of our method : let v_{sup} and v_{inf} be the rotation curves associated with the extremal surface densities. Taking into account the above relations, we get :

$$\frac{M_{sup} - M_{inf}}{M} \leq \frac{4}{\pi}\frac{\delta v_o}{v_o} + \frac{2}{3\pi R_g^2}\frac{M_{sup,3} - M_{inf,3}}{M}$$

where $M_{sup,3}$ and $M_{inf,3}$ are the three momentum of the surface densities associated respectively to the M_{sup} and M_{inf} configurations ; and $\delta v_o = \sup_{R_g \leq r \leq 2R_g}(v_{sup}(r) - v_{inf}(r))$. The LHS of this inequality is expected to be small, because the rotation curves are both near the Keplerian curve, and the momentum of order 3 are small.