Inversion of 3-D Geological Structures Using Parallelism, Developability, and Smoothness Least-Squares Criteria

Michel Léger, Institut Français du Pétrole (IFP); Jean-Marie Morvan, Univ. Claude Bernard Lyon I; and Hery Rakotoarisoa, IFP, France*

SUMMARY

Quantitative estimation of complex structures is a difficult problem which can be dealt with using back and forth geophysical techniques such as tomography or geological methods such as cross-section balancing. This means tedious syntheses and suggests they should be quantitatively integrated into a single consistent frame. Inverse problem techniques allow integration via multicriteria optimization. We model a geological structure as a set of jointed foliations, so as to cope with faults and unconformities, and we represent these foliations parametrically. We propose three kinds of geological data: the deviation from parallelism of a foliation, the total curvature (developability) and the mean curvature (smoothness) of its leaves. The choice of a L^2 norm on these vector or scalar fields results in least-squares criteria. Assuming one leaf of a foliation and well trajectories to be known, we optimize the foliation with respect to these criteria. Properly balancing the criteria allows easy control of the shape of the foliation as it results from numerical implementation.

INTRODUCTION

Tomographic inversion is an increasingly popular technique to determine geological structure (Bishop et al., 1985, Farra & Madariaga, 1988, Haas & Viallix, 1989, for instance). Besides, introducing a priori information is known to improve the results of inversion algorithms (Jackson, 1979, Tarantola, 1987).

The a priori information can be designed to have geological significance, and hence we propose constraining structure geometry with three least-squares criteria based on geological considerations: layer parallelism, developability and smoothness of folded interfaces.

First, we present which geometrical modeling technique we use to describe complex geological structures. Second, we present a geological argument that leads to these criteria, and lastly we present numerical results in a simple case and discuss the respective effects of these criteria.

GEOMETRICAL MODELING

The unknown is the geological structure. We model a structure locally with the geometrical concept of foliation, i.e., each point of a space domain belongs to one and only one surface, called a leaf, which represents a deposition isochron. This model becomes inadequate where faults and unconformities occur. For this reason, we define a geological model globally as a set of jointed foliations separated by faults and unconformities. Physical properties like velocity or density can be assigned to each point for geophysical purposes.

Numerical application requires parametric representation of these foliations. Once a set $\vec{u} = (u^1, u^2, u^3)$ of curvilinear coordinates and a set $\vec{x} = (x^1, x^2, x^3)$ of Cartesian coordinates in physical space are chosen, a foliation $\mathcal{F}$ can be defined by a parametric representation Φ as follows:

$$\vec{u} \in U \subset R^3 \longrightarrow \vec{x} = \Phi(\vec{u}) = \begin{pmatrix} x^1(u^1,u^2,u^3) \\ x^2(u^1,u^2,u^3) \\ x^3(u^1,u^2,u^3) \end{pmatrix} \in \mathcal{F} \subset R^3$$

The coordinates u^1, u^2 are the same as for a surface parametric representation. The coordinate u^3 numbers the leaves of the foliation (Figure 1).

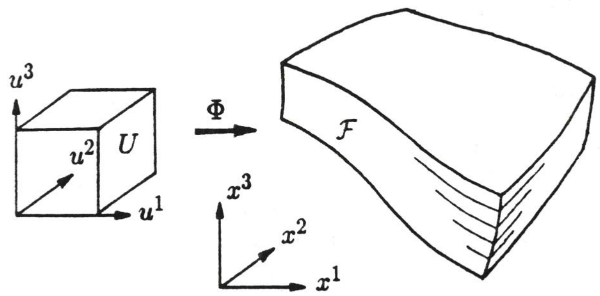

Figure 1. *Parametric representation of a foliation. The curvilinear coordinate u^3 numbers the leaves of the foliation, which represent deposition isochrons. The coordinates u^1 and u^2 number the points of a leaf.*

Numerical implementation of foliation parametric representations requires their discretization. To do this, we use B-spline tensor-products, so that U is a rectangular parallelepiped.

GEOLOGICAL DATA

To determine this model, we consider geological information. From available knowledge about sedimentology and tectonics, geometrical consequences can be deduced. First, sedimentologists tell us:

(S1). Deposition isochrons are almost parallel.

(S2). These isochrons are almost plane.

Hypothesis (S1) means that the convergence vector $\vec{\gamma}$ (see below), which measures the deviation from parallelism at any point of a foliation, is more or less close to zero. (S2) means that total curvature K, which measures the deviation from developability, and mean curvature H (see below), are close to zero everywhere on any leaf of a foliation. Moreover, tectonicians tell us that sedimentary rocks are often folded like a paper-bound book, i.e.:

(T1). The volume of rocks is preserved.

(T2). Lengths are preserved on isochrons.

(T3). Folding is not intense.

Hypotheses (T1) and (T2) correspond to the interbedding slip phenomenon during folding. Geometers tell us that, under these physical assumptions, the following consequences can be derived:

(G1). If (S1), (T1) and (T2) are verified, then parallelism is preserved. Moreover, the convergence vector is also preserved.

(G2). If (S2) and (T2) are verified, then $K \simeq 0$ at each point of the present structure. The leaves of such a foliation are called developable surfaces.

(G3). If (S2) and (T3) are verified, then $H \simeq 0$.

Of course, the above hypotheses are only approximations to reality, first because layers are never exactly flat and parallel just after deposition, second because compaction and internal strain during deformation always occur to some extent. Therefore, we will consider parallelism, developability and zero mean curvature as uncertain data: $\vec{\gamma} = \vec{0} \pm \Delta\vec{\gamma}$, $K = 0 \pm \Delta K$ and $H = 0 \pm \Delta H$, and we will formulate this data in least-squares terms.

FORWARD PROBLEM AND GEOLOGICAL CRITERIA

The forward problem consists in computing the synthetic data as a function of the unknowns. Thereafter, the choice of a L^2 norm on the field of the deviations between synthetic and available data results in least-squares criteria.

The synthetic data, convergence vector $\vec{\gamma}$, total curvature K and mean curvature H, are computed using the field of the unit vector $\vec{n}$ normal to the local leaf at any point of a foliation.

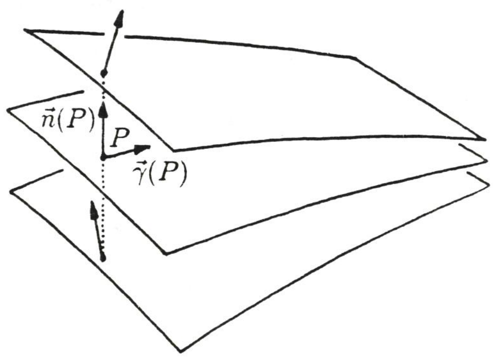

Figure 2. *The convergence vector $\vec{\gamma}$ is the normal derivative of the unit vector $\vec{n}$ normal to the local leaf. It measures the deviation from parallelism at any point of a foliation. Its L^2 norm results in a parallelism least-squares criterion.*

The convergence vector $\vec{\gamma}$ (Léger & Rakotoarisoa, 1990) is the directional derivative of $\vec{n}$ in the direction $\vec{n}$ itself (Figure 2).

On the other hand, K and H are related to the behaviour of $\vec{n}$ in directions lying in the tangent plane to the local leaf. Gauss's mapping associates its normal unit vector $\vec{n}$ to a point P of the foliation (Figure 3). The linearization of Gauss's mapping around a point P_0 yields the expression $\vec{dn} = C(P).\vec{dP}$, where $\vec{dn} = \vec{n}(P) - \vec{n}(P_0)$ and $\vec{dP} = P - P_0$. $\vec{dP}$ and $\vec{dn}$ belong to the tangent plane. The (2×2) matrix $C(P)$ is the curvature matrix at P of the leaf where P is located. The total curvature $K(P)$ at P is the determinant of the matrix $C(P)$ and the mean curvature H is defined using its trace, $H(P) = -\frac{1}{2}\text{trace}(C(P))$ (Spivak, 1979).

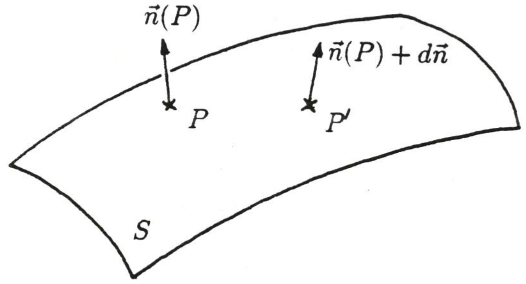

Figure 3. *The variation of the unit normal vector field $\vec{n}(P)$ around a point P in a surface S allows definition of the total and mean curvatures of S at P. A L^2 norm on these scalar fields results in least-squares criteria.*

As stated above, the available data is the nullity of $\vec{\gamma}$, K and H. Therefore, the parallelism, total curvature and mean curvature least-squares criteria will simply be the minimization of a norm on these fields.

$$Q_\gamma = \int_{\mathcal{F}} \|\vec{\gamma}\|_\gamma^2 \, d\mathcal{F} \; ; \; Q_K = \int_{\mathcal{F}} \|K\|_K^2 \, d\mathcal{F} \; ; \; Q_H = \int_{\mathcal{F}} \|H\|_H^2 \, d\mathcal{F}$$

where $\mathcal{F}$ is the foliation under study. This is the continuous geometrical expression of these least-squares criteria.

Since a foliation is dealt with in practice via a parametric representation, we introduce these functional continuous expressions:

$$Q_\gamma = \int_U \|\vec{\gamma} \circ \Phi\|_\gamma^2 \, |J_\Phi| \, dU$$

$$Q_K = \int_U \|K \circ \Phi\|_K^2 \, |J_\Phi| \, dU$$

$$Q_H = \int_U \|H \circ \Phi\|_H^2 \, |J_\Phi| \, dU$$

where Φ is the parametric representation of $\mathcal{F}$, $|J_\Phi|$ its Jacobian and U the space of curvilinear coordinates.

These least-squares criteria are finally discretized using a regular grid G_U of points $\vec{u}_i$ in U, so that the previous integrals become sums:

$$Q_\gamma = \sum_{\vec{u}_i \in G_U} \|\vec{\gamma} \circ \Phi(\vec{u}_i)\|_\gamma^2 \, |J_\Phi(\vec{u}_i)| \, U_i$$

$$Q_K = \sum_{\vec{u}_i \in G_U} \|K \circ \Phi(\vec{u}_i)\|_K^2 \, |J_\Phi(\vec{u}_i)| \, U_i$$

$$Q_H = \sum_{\vec{u}_i \in G_U} \|H \circ \Phi(\vec{u}_i)\|_H^2 \, |J_\Phi(\vec{u}_i)| \, U_i$$

where U_i is the volume of the mesh centered at $\vec{u}_i$.

The norms $\|.\|_\gamma$, $\|.\|_K$ and $\|.\|_H$ should account for the uncertainties as evaluated by geostatistical studies for instance. However, to our knowledge, such results are not yet available as far as these quantities are concerned. Hence, we simply define the norms $\|.\|_\gamma$, $\|.\|_K$ and $\|.\|_H$ as weighted L^2 norms so that the global physical objective function Q_φ is:

$$Q_\varphi = Q_\gamma + Q_K + Q_H = C_\gamma Q_\gamma^{L^2} + C_K Q_K^{L^2} + C_H Q_H^{L^2}$$

where C_γ, C_K, C_H are weighting coefficients related to the uncertainties about $\vec{\gamma}$, K and H.

AN EXTRAPOLATION PROBLEM

These criteria clearly do not suffice to determine the location of a foliation. Therefore, in order to have a well-posed problem and to illustrate the effect of these criteria, we build a simple extrapolation problem in which some extra information is introduced. This problem consists in determining a foliation such that:

— One of its leaves is known and described by its parametric representation.

— A few curves representing wells are known. They cross all the leaves once and only once. The curves are parameterized using the index u^3 of the leaves, which means that borehole correlations are available.

— It minimizes the geological objective function Q_φ.

— The precise location and shape of the lateral edges of the foliations are considered to have no particular significance in this simple example.

A foliation can be described by an infinite number of parametric representations. To overcome this problem we proceed in two steps. First, assuming the problem to be well-posed in terms of foliations, we define a criterion which is designed to make it well-posed in terms of parametric representations as well. This criterion simply consists in smoothing the mesh associated with the parametric representation. Second, we modify this criterion so that it does not interfere with the physical problem. We present details about this technique in a joint paper (Léger et al., 1991b).

The given surface and well data are modeled using linear equality constraints. Discretization in parameter space is carried out using B-spline tensor products. Q_φ is minimized using a Gauss-Newton algorithm.

PRACTICAL EFFECT OF GEOLOGICAL CRITERIA

Figures 4 to 7 show solutions of the extrapolation problem. The top surface is plane and rectangular and the two wells are orthogonal to it. The vertical thickness increase is 30%. The foliation is discretized using 10 (resp. 6, 4) parameters in u^1 (resp. u^2, u^3) direction for each Cartesian coordinate. This results in 720 parameters and 198 equality constraints. In all cases, the initial model is a parallelepiped made up of parallel rectangles.

These figures show that using only the parallelism criterion may yield unlikely results since the foliation leaves may be sharply curved in the vicinity of the wells (Figure 4). Introducing the total curvature criterion makes the surfaces more developable, and even cylindrical in the case of Figure 5. The effect of the mean curvature criterion is to smooth the folds (Figure 6). The surfaces become almost plane if the two curvature criteria are heavily weighted (Figure 7).

The last numerical experiment (Figure 8) shows that optimizing adequately weighted parallelism, developability and smoothness criteria may yield a plausible foliation as long as a sufficent number of wells is given, even if none of the foliation leaves is given. Five wells cross the foliation thoroughly and three just cross the top surface.

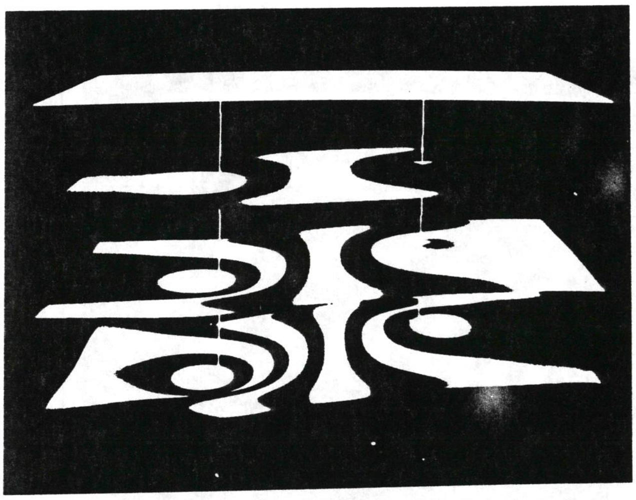

Figure 4. $Q_\varphi = Q_\gamma^{L^2}$. *Optimizing only the parallelism criterion only may yield highly curved surfaces in the vicinity of the wells. Gray intensity is a sine function of depth, hence light and dark zones suggest depth contours.*

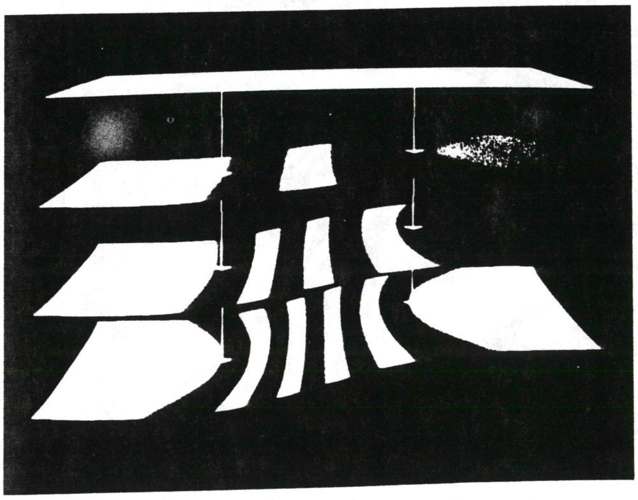

Figure 5. $Q_\varphi = Q_\gamma^{L^2} + 70Q_K^{L^2}$. *The total curvature criterion makes the surfaces more cylindrical (more developable in general).*

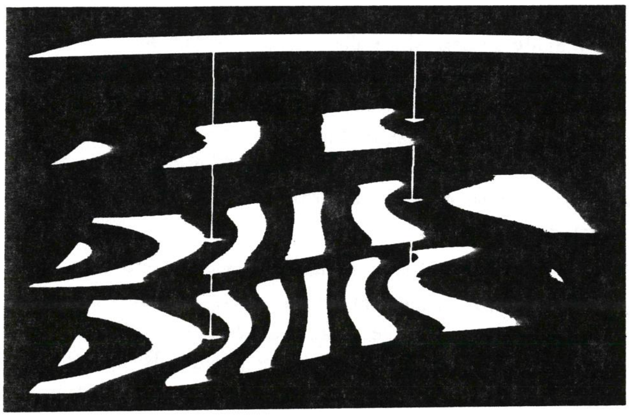

Figure 6. $Q_\varphi = Q_\gamma^{L^2} + Q_K^{L^2} + Q_H^{L^2}$. *The mean curvature criterion smooths the folds.*

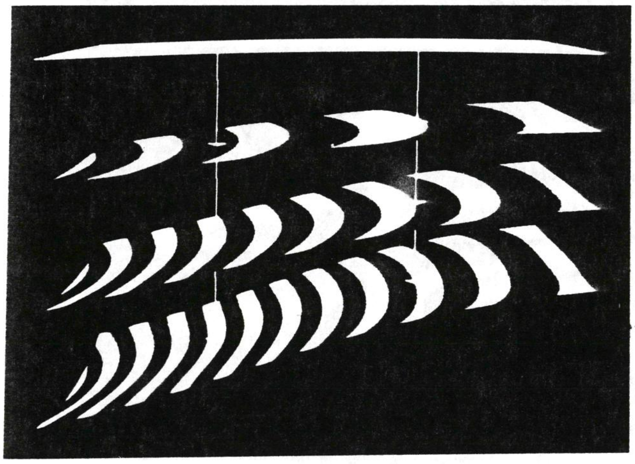

Figure 7. $Q_\varphi = Q_\gamma^{L^2} + 70Q_K^{L^2} + 50Q_H^{L^2}$. *If heavily weighted, the curvature criteria make the foliation leaves almost plane.*

Figure 8. $Q_\varphi = Q_\gamma^{L^2} + 100Q_K^{L^2} + 0.01Q_H^{L^2}$. *In this experiment, five wells are given through the foliation and three more wells cross the top surface. It shows that these criteria may yield a plausible interpolation foliation between wells, if properly weighted.*

CONCLUSIONS

Foliation has proven to be a good local geometric modeling tool for geological structures in the oil exploration context. It allows simple and reliable definition of deviation from parallelism. Moreover, it would be easy to correlate the shape of the velocity or density heterogeneities with the orientation of local dip, since dip exists everywhere.

Simple but quantitative geological information such as layer parallelism and fold developability or smoothness can be translated into geometrical terms, i.e., convergence vector, total curvature and mean curvature.

Since this information is only an approximation to reality, least-squares formulation is adequate to take uncertainties into account. Assuming one leaf of a foliation and several curves representing wells to be known, we implement optimization of these criteria and demonstrate their effectiveness.

We plan to improve this toolkit of geological least-squares criteria by adding other criteria that would constrain the shape of the faults, for instance. We believe that upgrading this geological toolkit with geophysical criteria (typically traveltimes) will yield more accurate estimates of complex geological structures in the context of intensive oil exploration.

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