Least-Squares Optimization of Fault Surfaces Using the Rigid Block Approximation

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SUMMARY

In the general context of macro-model determination by inversion techniques, we propose a specific least-squares criterion to constrain the shape of faults so as to be compatible with the rigid block approximation, i.e., the two blocks remain in contact and slip on each other without strain. If this condition is met, the fault surface is a *thread*, i.e., a surface that is everywhere tangent to a nonzero twistor vector field. Besides, we constrain the surface to be close to given points (well data or pickings on a depth-migrated seismic 3D cube), and to be smooth (principal curvature minimization). With the help of these curvature and proximity criteria, the "rigid block" criterion results in a smoothing method dedicated to fault surfaces. The rigid block criterion yields more plausible results than conventional approximation techniques based on second derivatives or curvature minimization, and it also predicts the direction of striae.

INTRODUCTION

The geological knowledge introduced in macro-model inversion techniques may concern the velocity field or the layer geometry, but in this paper we focus our attention on the shape of faults. Sometimes faults separate blocks in which only small internal strain occurred during the history of the structure; then the rigid block approximation is acceptable in such cases.

We formulate in least-squares terms this geological knowledge: "a fault surface separates two approximately rigid blocks" and we optimize the surface with respect to that criterion.

First, we translate the rigid block approximation into geometrical terms. Second, we present the problem we will solve numerically. Next, we detail the six objective functions to be minimized and we present the inverse problem. Lastly we present numerical results for a simple case.

RIGID BLOCKS AND *THREAD* SURFACES

A fault is a surface S, which we represent parametrically:

$$(u,v) \in U \subset R^2 \longrightarrow \Phi(u,v) = \begin{pmatrix} x(u,v) \\ y(u,v) \\ z(u,v) \end{pmatrix} \in S \subset R^3.$$

We discretize map Φ by using B-spline tensor products.

Assuming that both blocks are rigid and remain in contact during faulting. The fault surface necessarily belongs to a specific kind of surface, which we define as a *thread* surface, by analogy with a screw and a nut. Planes, spheres, cylinders or surfaces of revolution are typical examples of threads. However, surfaces such as an egg-box are not threads. From a geological viewpoint, the rigid block approximation may sometimes be very coarse, because of compaction or internal strain in the blocks.

The above definition is equivalent to the following characteristic property [C. Marle, Paris VI University]. There is at least one nonzero twistor W such that $W(P) \in T_P S$ at any point P on surface S, with $T_P S$ being the tangent plane to S at P. A twistor W is a vector field such that $W(B) = W(A) + \Omega \wedge AB$ for any points A and B in the Euclidean physical space R^3. Twistor W is an auxilliary unknown (Figure 1). Twistor W needs to be nonzero because any surface would be a thread without this condition. The lines that are everywhere tangent to a twistor are helices with the same axis and the same period.

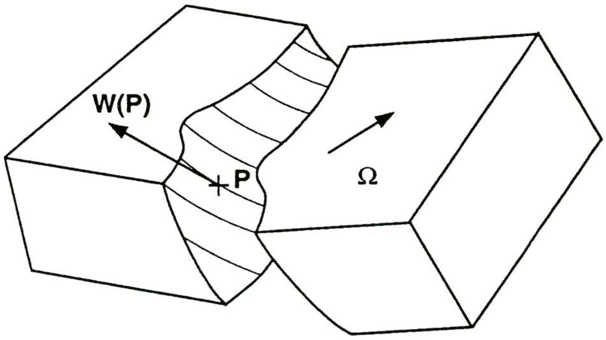

Figure 1. *Thread surfaces are such that there is a nonzero twistor everywhere tangent to it. We measure the deviation from a thread by the RMS value of the (twistor-tangent plane) angle.*

THE PHYSICAL PROBLEM

We consider a fault surface that is constrained by the following information:

1. The surface is as close as possible to given points, which represent well data or pickings on a depth-domain seismic cube.
2. The surface is as smooth as possible. The principal curvatures measure how the surface is not smooth.
3. The surface is as close as possible to a thread. We measure the deviation from a thread by the RMS value of the (twistor-tangent plane) angle.

Now we detail the formulation of this knowledge in terms of least-squares criteria.

THE LEAST-SQUARES CRITERIA

Proximity to data points

For the sake of simplicity, we chose *a priori* several points Φ_i on the surface and we specify them by their curvilinear coordinates u_i and v_i: $\Phi_i = \Phi(M_i)$ with $M_i = (u_i, v_i)$. We measure the proximity of points Φ_i to given points P_i by the normal part $(P_i, \Phi_i)_\perp$ of vector (P_i, Φ_i). Figure 2 shows that $(P_i, \Phi_i)_\perp$ is close to the distance between P_i and surface S if this distance is much shorter than the radii of curvature of S.

The physical objective function related to the proximity of the surface to the data points is then

$$Q_{P\varphi} = \sum_i \|(P_i, \Phi_i)_\perp\|^2.$$

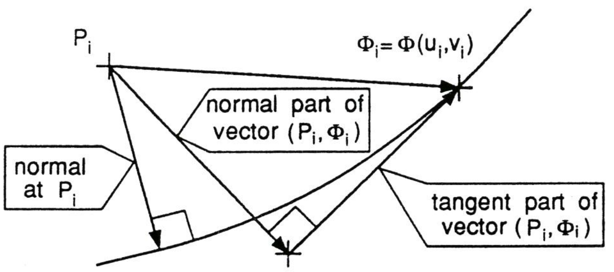

Figure 2. *For the i-th data point P_i, vector (P_i, Φ_i) is split into its normal and tangent parts, with $\Phi_i = \Phi(u_i, v_i)$. We consider the normal part to be physical, and the tangent part unphysical.*

Principal curvatures

We smooth a fault surface by minimizing the objective function

$$Q_C = \int_S (\lambda_1^2 + \lambda_2^2) dS.$$

At each point on surface S, the principal curvatures λ_1 and λ_2 are the eigenvalues of the curvature matrix

$$C = \begin{pmatrix} <D_X N, X> & <D_X N, Y> \\ <D_Y N, X> & <D_Y N, Y> \end{pmatrix}$$

where $<,>$ denotes the scalar product, X and Y are orthogonal unit vector fields everywhere tangent to S and N is the unit vector field normal to S. $D_X A$ denotes the directional derivative of vector field A in the direction of vector field X:

$$(D_X A)_m = \sum_{i,j} (X^i)_m \left(\frac{\partial Y^j}{dx^i}\right)_m e_j$$

where x^1, x^2 and x^3 are the Cartesian coordinates related to basis vectors (e_1, e_2, e_3).

A curvature-based criterion is more satisfactory than a second derivative-based criterion because the former is insensitive to parametrization, whereas the latter is not. We illustrate this fact in the section "Curvature versus second derivatives".

Thread property

We measure the difference between a surface and a thread by the objective function

$$Q_T = \int_S < N(P), W(P) >^2 dS.$$

$< N(P), W(P) >$ is the scalar product at P of unit normal vector N and twistor W. Clearly, the optimization of Q_T constrains the surface to be almost parallel to twistor W.

The weighted addition of these objective functions results in the physical objective function Q_φ:

$$Q_\varphi = w_{P\varphi} Q_{P\varphi} + w_C Q_C + w_T Q_T.$$

THE INVERSE PROBLEM

Since many parametrizations may describe the same surface, there are an infinite number of parametrizations that optimize objective function Q_φ, even if a single surface minimizes it. In other words, the inverse problem is ill-posed. To make it well-posed, we introduce a weighted additional objective function:

$$Q_\alpha = w_{P\alpha} Q_{P\alpha} + w_D Q_D + w_N Q_N$$

This function is designed to be unphysical; this means that a parametrization that minimizes $Q_\varphi + Q_\alpha$ does represent a surface that minimizes Q_φ. Objective function $Q_{P\alpha}$ represents the tangential part of the proximity term of the surface to the data points (Figure 2):

$$Q_{P\alpha} = \sum_i \|(P_i, \Phi_i)_{tang}\|^2.$$

Objective function Q_D is a modified regularisation based on the second derivatives of parametrization Φ. This technique is described in [Léger et al., 1991b].

We have mentioned that twistor W should be nonzero. Therefore we introduce the normalization objective function

$$Q_N = \left(\int_S \|W\|^2 dS - \int_S dS\right)^2.$$

The minimization of Q_N constrains the RMS value of twistor W to be close to 1, and therefore, we interpret the $<N(P), W(P)>$ term in the previous section as an angle.

We minimize overall objective function $Q = Q_\varphi + Q_\alpha$ by using a Gauss-Newton procedure.

CURVATURES *versus* SECOND DERIVATIVES

In this section, we describe optimization results obtained without thread term Q_T and without normalization term Q_N. We compare smoothing by curvature minimization and smoothing by second derivative minimization. Figure 3 locates data points P_i and characterizes points Φ_i of the surface that are linked to data points by the proximity terms $Q_{P\varphi}$ and $Q_{P\alpha}$. We have performed the inversion with several sets of curvilinear coordinates for points Φ_i. Curvature minimization always yields the result shown in Figure 4. If curvilinear coordinates of points Φ_i are chosen to be the same as the Cartesian coordinates of corresponding given points P_i, the minimization of curvature or second derivatives yields the same result.

On the contrary, in the case depicted in Figure 3, the minimization of the second derivatives shows an anisotropic smoothing effect. The larger the Jacobian matrix $d\Phi$ of Φ is in a direction (i.e., the larger $\|d\Phi(V)\|$ for $\|V\| = 1$ is), the stronger the smoothing effect is in that direction (i.e., in the direction of $d\Phi(V)$).

Therefore, we have chosen to use curvature as a smoothing tool rather than second derivatives, despite the fact that partial derivatives needed at the optimization step are harder to compute.

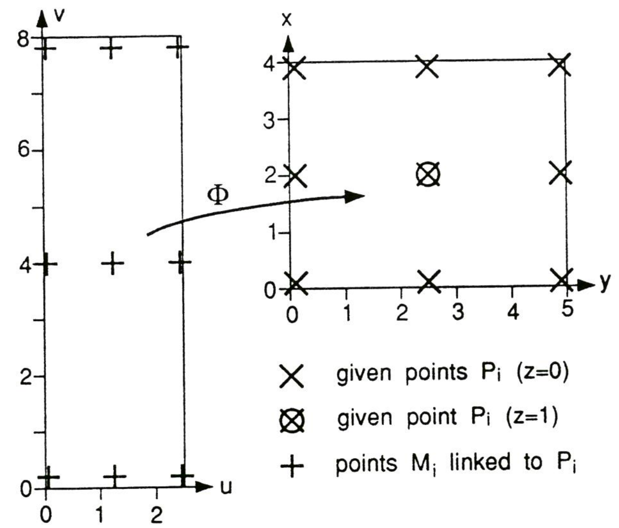

Figure 3. *Cartesian and curvilinear coordinates of the data points used in Figures 4 and 5.*

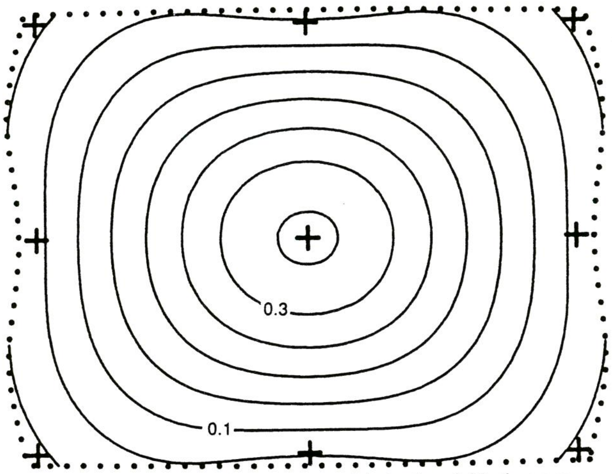

Figure 4. *Smoothing by curvature minimization yields the same result whatever the curvilinear coordinates of points M_i may be.*

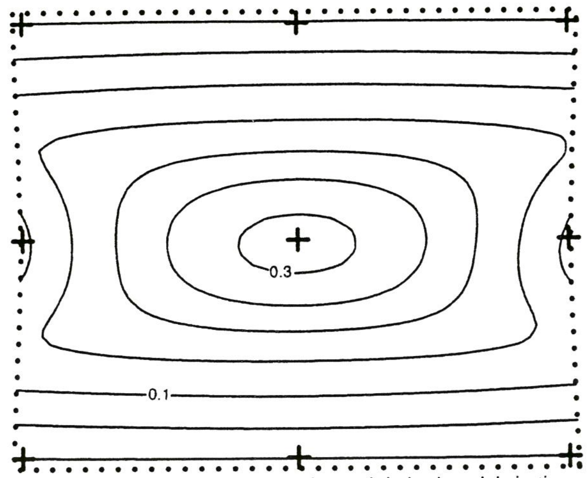

Figure 5. *The smoothing effect of second derivative minimization depends on the curvilinear coordinates of points M_i.*

INVERSION OF THREAD SURFACES

We give numerical results concerning the inversion of fault surfaces for proximity to given points, curvature and thread criteria.

Whereas the most convenient initial surface is simply a piece of plane if we disregard the thread criterion, a more elaborate initial model is required if we introduce this term in the overall objective function. This is due to the fact that a plane (as well as a sphere or a circular cylinder) is a surface that is invariant under a two parameter-family of moves. Since we have introduced a twistor normalization criterion, this two-parameter family becomes a one-parameter family of twistors. This means that there is (at least) one direction in the (surface + twistor) parameter space in which the overall objective function is constant. Hence it is not coercive and the problem is ill-posed.

To overcome this difficulty, we could introduce another criterion related to a priori knowledge about the twistor. We preferred to begin the inversion without the thread criterion, and to introduce it after several iterations, once the difficulty has been avoided.

Figure 6 gives information about the data points used in inversions of Figures 7 to 10.

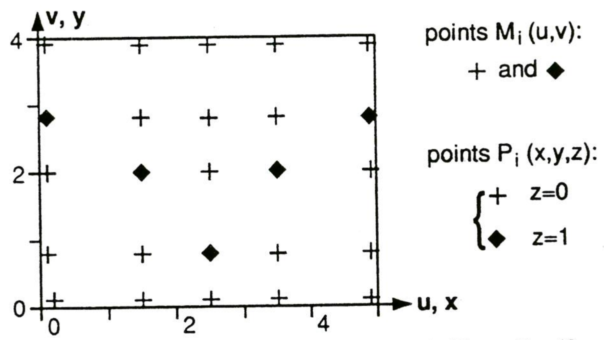

Figure 6. *Data points used in inversions shown in Figures 7 to 10.*

Figure 7 shows the result of a primary inversion in which we have optimized curvature and proximity to data points. This surface is clearly not a thread. Striae suggest the initial twistor. (Striae are the projection on the surface of the helices everywhere tangent to the twistor.) The best possible fit with the surface is clearly not achieved.

Figure 8 shows the twistor (via striae on the surface) as it results from the optimization of the thread and normalization criteria. The surface is fixed (same as Figure 7).

In Figure 9, the twistor is such that $W(P)=(1,0,0)$ at any P. Consequently, the unknown surface almost becomes a cylinder.

In Figure 10, the twistor and the surface are both unknown. The result is close to a surface of revolution. The RMS value of the (twistor-tangent plane) angle is 0.15 degree, whereas it was 0.11 degree for Figure 9 and 2.6 for Figure 8. The RMS radius of curvature $([\int (\lambda_1^2+\lambda_2^2)dS/\int dS]^{-\frac{1}{2}})$ is 21, 34 and 27, and the RMS distance to the data points is 0.32, 0.35 and 0.34 for Figures 8, 9 and 10 respectively. A slight downgrade of the proximity and curvature criteria is compatible with a great improvement of the thread criterion.

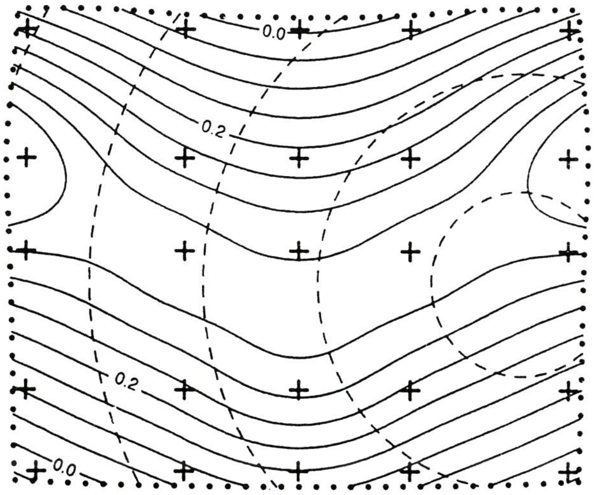

Figure 7. *Initial model of the inversion process. The striae (dashed lines) represent the initial inappropriate twistor.*

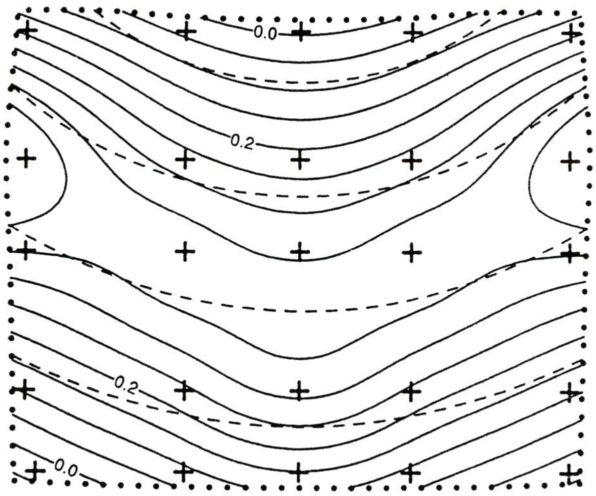

Figure 8. *The twistor suggested by the striae fits at best the thread and normalisation criteria. The surface itself is not inverted (same as Figure 7).*

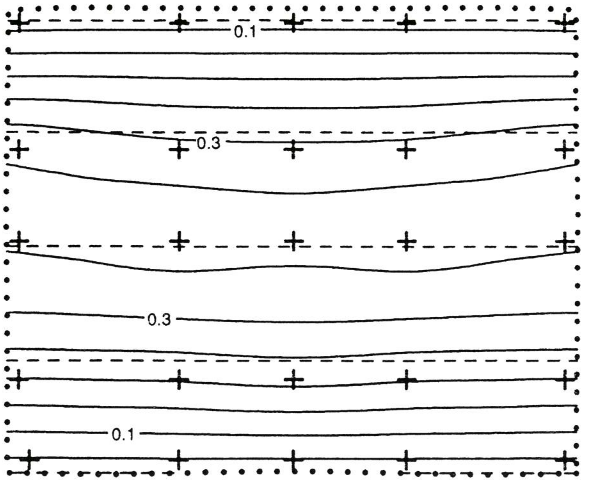

Figure 9. *If the twistor is such that $W(P) = (1,0,0)$, $\forall P$, the thread almost becomes a cylinder and the striae are almost straight lines.*

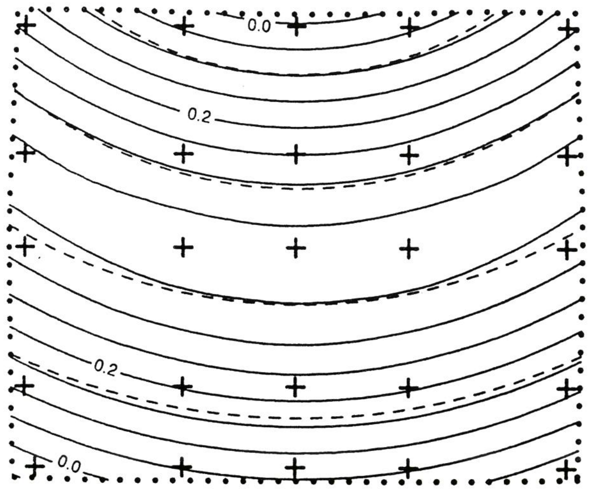

Figure 10. *This surface and the twistor suggested by the striae result from the optimization of the model shown in Figure 7 for proximity to data points, curvature and thread property least-squares criteria.*

CONCLUSIONS

Our results demonstrate that it is possible to constrain the shape of faults with more elaborate geological knowledge than only smoothness and proximity to data points. The least-squares "thread criterion", which derives from the rigid block approximation, improves predictions about the shape of faults, especially if a few wells cross it or if the seimic image of that fault is vague or inaccurately depth-migrated, depending on the confidence of the geologist in the rigid block approximation. In the general case, the "thread criterion" yields the direction of the striae as a by-product. The method is flexible with respect to the available knowledge about the twistor which represents the relative move between the two blocks. This move can easily be said to be a pure rotation of known axis, a pure translation, of known direction or not, either completely known or completely unknown.

Besides, if a surface is represented parametrically, its smoothing by the minimization of principal curvatures leads to more satisfying results than by the minimization of second derivatives.

Measurements concerning striae direction could be used to formulate another criterion. We plan to test our method on field data in the near future.

ACKNOWLEDGMENTS

We thank J. Brac for his fast and convenient B-spline routines and L. Nguyen Luc for his help in graphic matters. We thank B. Colletta, J.-P. Gratier and J. Letouzey for many discussions on the validity of the rigid block *approximation*.

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