

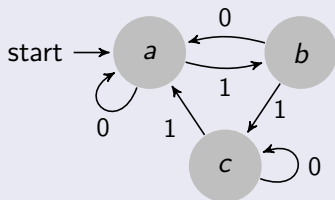
Automatic sequences corresponding to synchronized automata fulfill the Sarnak conjecture

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Number Theory, Numeration Systems and Ergodic Theory
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Overview

Automaton



Transition: $\delta(a, 010) = a$.

Synchronizing Automata

$\exists \mathbf{w}_0 : \delta(q, \mathbf{w}_0) = a \quad \forall q.$

e.g. $\mathbf{w}_0 = 010$.

Automatic Sequence

$a_n = \delta(a, (n)_2)$

e.g. *bacabcaabaccaababacab...*

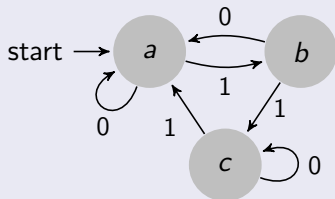
Theorem (Deshouillers, Drmota and Müllner, 2015+)

Let (X, T) be the dynamical system related to such a sequence. Then all sequences $\xi(n) = f(T^n(x))$ ($x \in X$) are orthogonal to the Möbius function, that is, the full Sarnak Conjecture holds in that case.

Remark: Almost all automata are synchronizing. Thus, the Sarnak conjecture holds for almost all automata.

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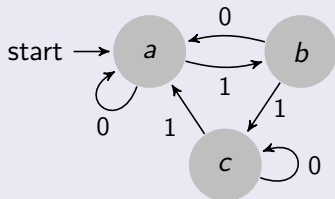
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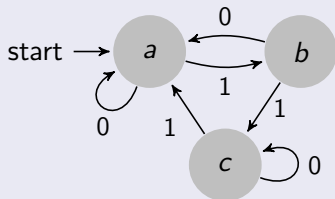
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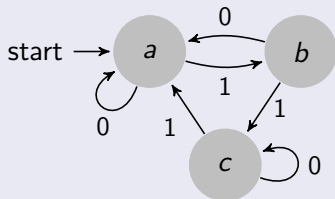
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