Summaries of the talks

Séminaire Lotharingien de Combinatoire 72

Invited Lectures

Christophe Hohlweg (UQAM, Montréal)

Words and Roots in Infinite Coxeter Groups

This series of lectures is an invitation to the study of the combinatorics of words and roots in infinite Coxeter groups. The combinatorial interplay between words and roots plays a fundamental role in the study of these groups. They are for instance at the heart of the proof by Brink and Howlett that Coxeter groups are automatic.

We begin with an introduction to Coxeter groups, with particular emphasis on the relationship between reduced words and roots illustrated with classic examples.

In a second part, our focus will be the "weak order", which plays an important role in the study of reduced words. In finite Coxeter groups, it is a lattice and an orientation of the 1-skeleton of the permutahedron that provides a nice framework to construct generalized associahedra (via N. Reading's Cambrian lattices). We will discuss also a conjecture of Matthew Dyer that proposes a generalization of the framework weak order/reduced words to infinite Coxeter groups via certain generalizations of inversion sets.

Finally, we will discuss the case of infinite Coxeter groups in which the combinatorics of words and roots is mostly uncharted territory. On the way we will present a new framework for studying words and root systems that involves limits of roots and tilings of their convex hull. This last part will be based on joint works with M. Dyer, J.P. Labbé and V. Ripoll.

Tom Koornwinder (University of Amsterdam)

BC_n-type Orthogonal Polynomials: Old and New Developments

The three lectures will survey by now classical results for BC type Jacobi polynomials and Macdonald-Koornwinder polynomials.Some new results and some directions for research suggested by low rank cases will also be given.

Here is a list of topics, on the one hand only tentative, on the other hand not necessarily exhaustive:

- Jacobi polynomials associated with root systems: the Heckman-Cherednik approach using Dunkl type operators;

- Opdam's shift operators;

- Jack polynomials, shifted Jack polynomials and generalized binomial coefficients;

- Expansion of BC type Jacobi polynomials in terms of Jack polynomials;
- Explicit results for BC_2 type Jacobi polynomials and their possible extensions to BC_n;
- Macdonald polynomials associated with root systems and the Macdonald-Koornwinder (MK) case;
- Double affine Hecke algebra (DAHA) and non-symmetric MK polynomials;
- Limits of MK and its DAHA to $q = 1$;
- Explicit results in the rank 1 and 2 MK case and possible generalizations to general rank;
- The $q = 0$ limit (BC_n Hall-Littlewood);
- Elliptic analogue of MK polynomials.

Contributed Talks

Olga Azenhas *Growth diagrams and non-symmetric Cauchy identities over near staircases*

Fomin's growth diagrams for Robinson-Schensted-Knuth correspondences are used to give, on the one hand, a formulation, via reverse RSK, of an analogue of RSK, due to Mason, and, on the other hand, an interpretation of the action of crystal operators on multisets of cells in a Ferrers shape. In particular, for multisets of cells in a near staircase, a staircase with some boxes sited on the stairs, the right keys for the pairs of tableaux, are characterized.

Then this characterization allows to explicit the tableaux in a non-symmetric Cauchy kernel expansion, over near staircases, due to Lascoux, on the basis of Demazure characters and the basis of Demazure atoms, under the action of appropriate Demazure operators, one for each box sited on the stairs of the staircase. This is a joint work with Aram Emami.

Olivier Bouillot *Multiple Bernoulli Polynomials and Numbers*

The main goal of the talk is to introduce the notion of multiple Bernoulli polynomials and numbers. It turns out that, currently, there is no uniqueness of the notion. We will describe some necessary conditions that a family of polynomials has to satisfy in order to be called "multiple Bernoulli polynomials". Then, we will construct an explicit and satisfactory family of polynomials with these conditions.

Grégory Châtel *A bijection between intervals of the Tamari order and flows on rooted trees*

We will give a bijection between intervals of the Tamari order, which is an order on Catalan objects and flows on rooted trees, which are objects appearing in the study of the pre-Lie operad. To compute this bijection, we will use an object called interval-poset which has been introduced recently to work with intervals of the Tamari order.

Ali Chouria *Bell polynomials in combinatorial Hopf algebras*

Partial multivariate Bell polynomials have been defined by E.T. Bell in 1934. These polynomials have numerous applications in Combinatorics, Analysis, Algebra, Probabilities etc. Many of the formulæ on Bell polynomials involve combinatorial objects (set partitions, set partitions in lists, permutations etc). So it seems natural to investigate analogous formulæ in some combinatorial Hopf algebras with bases indexed by these objects. During this talk, we show that most of the results on Bell polynomials can be written in terms of symmetric functions (for example in the algebra of symmetric functions, we give an expression of the Bell polynomials in terms of complete symmetric functions). Then, we show that these results are clearer when stated in other Hopf algebras (this means that the combinatorial objects appear explicitly in the formulæ).

Wenjie Fang *Bijective proofs of character evaluations using trace forest of the jeu de taquin*

Irreducible characters in the symmetric group are of special interest in combinatorics. They can be expressed either combinatorially with ribbon tableaux, or algebraically with contents. In our work, we try to relate these two expressions in a combinatorial way. We first introduce a fine structure in the famous jeu de taquin called "trace forest", with which we are able to count certain types of ribbon tableaux, leading to a simple bijective proof of a character evaluation formula in terms of contents that dates back to Frobenius (1901). Inspired by this proof, we give an inductive scheme that gives combinatorial proofs to more complicated formulae for characters in terms of contents. This is a work in progress.

Thomas Gerber *An analogue of the Robinson-Schensted-Knuth correspondence for affine type A*

Schensted's bumping algorithm has a natural interpretation in terms of crystals for the quantum algebra of type A. After explaining the features of the affine case by introducing the notion of Fock space and its crystal, I determine an affine analogue of this algorithm. This extends to an analogue of the whole Robinson-Schensted-Knuth correspondence.

Cédric Lecouvey *Induced modules and Dynkin diagram automorphims*

Consider g a Lie algebra and l a Levi subalgebra. It is quite easy to prove that two finite-dimensional l-modules have isomorphic inductions to g when their highest weights are conjugate under the action of a Weyl group element of g which is also a Dynkin diagram automorphism of l. In a joint work with J. Guilhot, we conjecture the converse is true and prove this conjecture holds in various cases. If time permits, I will also explain how analogous problems also appear in combinatorial representation theory when the branching coefficients are replaced by tensor product multiplicities or décomposition numbers.

Ana F. Loureiro *q-Jacobi-Stirling numbers and q-differential equations for q-classical polynomials*

This talk concerns with q-differential equations of arbitrary even order fulfilled by the q-classical polynomials. These can be characterised as eigenfunctions of a certain 2nd order q-differential operator, L say, with polynomial coefficients. We will explain how the result can be extended to higher order q-differential operators that will be explicitly given. The latter operators and any integer composite power of L are bridged via the q-Stirling numbers together with another new pair of numbers that we introduce – the so-called q-Jacobi Stirling numbers. Alongside, we will give a combinatorial interpretation and discuss the foremost important properties of this new pair of numbers.

Thibault Manneville *Graph Properties of Graph Associahedra*

Associahedra are polytopes which encodes geometrically the combinatorics of the dissections of a convex polygon. Different constructions of associahedra lead to different generalizations. Here we the associahedron as a simplicial complex defined by some compatibility relation on subpaths of a given path. This definition admits a natural generalization to arbitrary finite graphs. It is then known that the complex is still polytopal, and the resulting polytopes are called graph associahedra. One can obtain classical polytopes in this framework : the permutahedron is a complete graph associahedron and the cyclohedron is a cycle associahedron. We are interested in graph properties of graph associahedra, that is in combinatorial properties of their flip graph. We present results on their diameter, which was known for (classic, thus path) associahedra and permutahera, and on their hamiltonicity, which also has been proved for these two examples.

Henri Mühle *On m-Cover Posets*

In this talk, we define a certain subposet of the m-fold direct product of a bounded poset, the so-called m-cover poset. We first present some general results of the m-cover posets, in particular, we determine their cardinality, their irreducibles, and we characterize the bounded posets, for which the m-cover poset is a lattice for all m. Subsequently, we consider the m-cover poset of the Tamari lattice of parameter n, and show that its Dedekind-MacNeille completion is isomorphic to the m-Tamari lattice of parameter n, defined by Bergeron and Préville-Ratelle. For proving this result, we introduce a new decomposition of m-Dyck paths of height n into m-tuples of Dyck paths of height n. Finally, we introduce a family of "m-Tamari like" lattices associated with the dihedral groups, and discuss possible generalizations to other Coxeter groups.

Bérénice Oger *A Bialgebra on Hypertree and Partition posets*

The aim of the talk is to adapt the computation of characters on incidence Hopf algebras introduced by W. Schmitt in the 1990s to hypertree and partition posets. First, we will describe how we can construct an incidence Hopf algebra and a smaller bialgebra on this family of posets. Then we will compute the coproduct on the bialgebra, which is linked with the number of hypertrees with fixed valency set and edge sizes set. Finally we will apply the results to recover the Moebius numbers of the hypertree posets.

Soichi Okada *Greatest common divisors of specialized Schur functions*

Given two positive integers n and k, we find the greatest common divisor of the special values $s_\lambda(1^k)$ of Schur functions, where λ runs over all partitions of n. As an application, we determine when generalized parking spaces exists for the symmetric groups. Also we discuss the greatest

common divisor of the principal specializations of Schur functions.
This talk is based on a joint work with Yusuke ITO.

Mathias Pétréolle *Cyclically fully commutative elements in affine Coxeter groups*

An element w of a Coxeter group W is fully commutative if any two of its reduced decompositions are related by a series of transpositions of adjacent commuting generators. The element w is cyclically fully commutative if any of its cyclic shifts remains fully commutative. These elements were recently studied by Boothby et al., who characterized Coxeter groups in which they are in finite number and enumerated them in these cases. In this talk we will show how to characterize and enumerate cyclically fully commutative elements according to their Coxeter length in all affine Coxeter groups by using a new operation on heaps, the cylindric transformation. We will also explain how this yields refinements of the known results in finite types.

Jean-Baptiste Priez *Hopf algebras and polynomial realization*

We give a framework to define easily Hopf algebras from polynomial realizations. As a result, we show a family of Hopf algebras indexed by some generalized parking functions.

Paolo Sentinelli *Isomorphisms of Hecke Modules and Parabolic Kazhdan-Lusztig Polynomials*

For any Coxeter system we define two families of modules over the Hecke algebra of any parabolic subgroup. There are isomorphisms of these modules which lead to equalities between parabolic Kazhdan-Lusztig and R-polynomials in different Coxeter systems.

Omar Tout *Polynomiality of the structure coefficients of double-class algebras*

We consider a sequence of pairs (G_n, K_n) where G_n is a group and K_n is a sub-group of G_n for every n. We are interested in the structure coefficients of the K_n-double-classes algebra. In particular cases such as $(S_n \times S_n^{opp}, diag(S_n))$ and (S_{2n}, H_n), we have a polynomiality property for these coefficients while an explicit formula does not exist. We show a general framework in which we can re-obtain these polynomiality properties.

François Viard *Type of a tableau, definition and properties*

In order to enumerate the set of the reduced decompositions of a permutation in the symmetric group, Edelman and Greene introduced the family of balanced tableaux. It turns out that there is the same number of balanced and standard Young tableaux of the same shape. By the way, the proof of this fact is very technical.

In this talk we explain how to give a new and simpler proof of this result by generalizing the concept of balanced tableaux. More precisely, we will introduce a new notion of type of a tableau. This will allows us to provide also a new combinatorial interpretation of the set of reduced decompositions of a given permutation.

Vincent Vong *Non commutative Gandhi polynomials and surjective pistols*

In this talk, we use non commutative analog of the finite difference operator in order to build non commutative Gandhi polynomials. From this point of view, surjective pistols arise naturally. Then, we find back some results about some generalization of Gandhi polynomials as The Dumont-Foata polynomials or the q-analogs defined by Han and Zeng.

Meesue Yoo *Schur coefficients of the integral form Macdonald polynomials*

In this talk, we consider the Schur coefficients of the integral form of the Macdonald polynomials. As an attempt to prove Haglund's conjecture that $\left\langle \frac{J_\lambda[X;q,q^k]}{(1-q)^n}, s_\mu(X) \right\rangle \in \mathbb{N}[q]$ has positive coefficients, we have found explicit combinatorial formula for the Schur coefficients in one row case, two column case and certain hook shape cases. A result of Egge-Loehr-Warrington ('2010) gives a combinatorial way of getting Schur expansion of symmetric functions when the expansion of the function in terms of Gessel's fundamental quasi symmetric functions is known. We apply this result to the combinatorial formula for the integral form Macdonlad polynomials of Haglund in quasi symmetric functions to prove the Haglund's conjecture in general cases.