Appendix to "Production of faces of the Kronecker cone containing only stable triples"
All possible order matrices of size 3x3

There are 36 possible order matrices (i.e. 36 types of additive matrices) of size 3x3:

$$\textcircled{1}\begin{pmatrix} \textcircled{1} & \textcircled{2} & \textcircled{3} \\ \textcircled{4} & \textcircled{5} & \textcircled{6} \\ \textcircled{7} & \textcircled{8} & \textcircled{9} \end{pmatrix} \quad \textcircled{2}\begin{pmatrix} \textcircled{1} & \textcircled{2} & \textcircled{3} \\ \textcircled{4} & \textcircled{5} & \textcircled{7} \\ \textcircled{6} & \textcircled{8} & \textcircled{9} \end{pmatrix} \quad \textcircled{3}\begin{pmatrix} \textcircled{1} & \textcircled{2} & \textcircled{3} \\ \textcircled{4} & \textcircled{5} & \textcircled{8} \\ \textcircled{6} & \textcircled{7} & \textcircled{9} \end{pmatrix} \quad \textcircled{4}\begin{pmatrix} \textcircled{1} & \textcircled{2} & \textcircled{3} \\ \textcircled{4} & \textcircled{6} & \textcircled{7} \\ \textcircled{5} & \textcircled{8} & \textcircled{9} \end{pmatrix}$$

$$\textcircled{5}\begin{pmatrix} \textcircled{1} & \textcircled{2} & \textcircled{3} \\ \textcircled{4} & \textcircled{6} & \textcircled{8} \\ \textcircled{5} & \textcircled{7} & \textcircled{9} \end{pmatrix} \quad \textcircled{6}\begin{pmatrix} \textcircled{1} & \textcircled{2} & \textcircled{4} \\ \textcircled{3} & \textcircled{5} & \textcircled{6} \\ \textcircled{7} & \textcircled{8} & \textcircled{9} \end{pmatrix} \quad \textcircled{7}\begin{pmatrix} \textcircled{1} & \textcircled{2} & \textcircled{4} \\ \textcircled{3} & \textcircled{5} & \textcircled{7} \\ \textcircled{6} & \textcircled{8} & \textcircled{9} \end{pmatrix} \quad \textcircled{8}\begin{pmatrix} \textcircled{1} & \textcircled{2} & \textcircled{4} \\ \textcircled{3} & \textcircled{5} & \textcircled{8} \\ \textcircled{6} & \textcircled{7} & \textcircled{9} \end{pmatrix}$$

$$\textcircled{9}\begin{pmatrix} \textcircled{1} & \textcircled{2} & \textcircled{4} \\ \textcircled{3} & \textcircled{6} & \textcircled{7} \\ \textcircled{5} & \textcircled{8} & \textcircled{9} \end{pmatrix} \quad \textcircled{10}\begin{pmatrix} \textcircled{1} & \textcircled{2} & \textcircled{4} \\ \textcircled{3} & \textcircled{6} & \textcircled{8} \\ \textcircled{5} & \textcircled{7} & \textcircled{9} \end{pmatrix} \quad \textcircled{11}\begin{pmatrix} \textcircled{1} & \textcircled{2} & \textcircled{5} \\ \textcircled{3} & \textcircled{4} & \textcircled{6} \\ \textcircled{7} & \textcircled{8} & \textcircled{9} \end{pmatrix} \quad \textcircled{12}\begin{pmatrix} \textcircled{1} & \textcircled{2} & \textcircled{5} \\ \textcircled{3} & \textcircled{4} & \textcircled{7} \\ \textcircled{6} & \textcircled{8} & \textcircled{9} \end{pmatrix}$$

$$\textcircled{13}\begin{pmatrix} \textcircled{1} & \textcircled{2} & \textcircled{5} \\ \textcircled{3} & \textcircled{4} & \textcircled{8} \\ \textcircled{6} & \textcircled{7} & \textcircled{9} \end{pmatrix} \quad \textcircled{14}\begin{pmatrix} \textcircled{1} & \textcircled{2} & \textcircled{5} \\ \textcircled{3} & \textcircled{6} & \textcircled{8} \\ \textcircled{4} & \textcircled{7} & \textcircled{9} \end{pmatrix} \quad \textcircled{15}\begin{pmatrix} \textcircled{1} & \textcircled{2} & \textcircled{6} \\ \textcircled{3} & \textcircled{4} & \textcircled{8} \\ \textcircled{5} & \textcircled{7} & \textcircled{9} \end{pmatrix} \quad \textcircled{16}\begin{pmatrix} \textcircled{1} & \textcircled{2} & \textcircled{6} \\ \textcircled{3} & \textcircled{5} & \textcircled{8} \\ \textcircled{4} & \textcircled{7} & \textcircled{9} \end{pmatrix}$$

$$\textcircled{17}\begin{pmatrix} \textcircled{1} & \textcircled{2} & \textcircled{7} \\ \textcircled{3} & \textcircled{4} & \textcircled{8} \\ \textcircled{5} & \textcircled{6} & \textcircled{9} \end{pmatrix} \quad \textcircled{18}\begin{pmatrix} \textcircled{1} & \textcircled{2} & \textcircled{7} \\ \textcircled{3} & \textcircled{5} & \textcircled{8} \\ \textcircled{4} & \textcircled{6} & \textcircled{9} \end{pmatrix}$$

and exactly all the transposed matrices of these ones (that we number in the same way: the transposed matrix of Matrix $\textcircled{k}$ has number 18+k). Respectively associated dominant, regular, $\hat{G}$-regular one-parameter subgroups are for instance:

$\tau_1 = [6,3,0|2,1,0]$ $\quad \tau_2 = [10,4,0|5,2,0]$ $\quad \tau_3 = [8,2,0|4,3,0]$ $\quad \tau_4 = [10,3,0|6,2,0]$

$\tau_5 = [7,1,0|4,2,0]$ $\quad \tau_6 = [8,5,0|4,2,0]$ $\quad \tau_7 = [6,3,0|4,2,0]$ $\quad \tau_8 = [12,5,0|10,6,0]$

$\tau_9 = [12,5,0|10,4,0]$ $\quad \tau_{10} = [9,2,0|8,4,0]$ $\quad \tau_{11} = [7,5,0|4,3,0]$ $\quad \tau_{12} = [12,8,0|9,6,0]$

$\tau_{13} = [10,6,0|8,7,0]$ $\quad \tau_{14} = [8,2,0|9,4,0]$ $\quad \tau_{15} = [8,4,0|9,6,0]$ $\quad \tau_{16} = [8,2,0|10,7,0]$

$\tau_{17} = [4,2,0|6,5,0]$ $\quad \tau_{18} = [4,1,0|7,5,0]$

(for the transposed matrices, one simply has to exchange the roles of V_1 and V_2).

Then each one of these matrices gives exactly one "additive face" of PKron_{n_1,n_2} by the result of Manivel and Vallejo, i.e. one well-covering pair. Moreover, by Theorem

3.17, they also give other such pairs. Here are the numbers of "new" well-covering pairs that each one gives by this theorem:

① 6 ② 4 ③ 5 ④ 5 ⑤ 5 ⑥ 4 ⑦ 2 ⑧ 3 ⑨ 3

⑩ 3 ⑪ 5 ⑫ 3 ⑬ 4 ⑭ 4 ⑮ 3 ⑯ 3 ⑰ 5 ⑱ 5

Each transposed matrix gives furthermore by Theorem 3.17 the same number of well-covering pairs as the original one. As a consequence that makes a total of 144 "new"[1] well-covering pairs.

In addition Theorem 3.18 provides from these 36 order matrices a certain number of dominant pairs. Among them, some of them are probably well-covering while others do not in fact define a new face of $\mathrm{PKron}_{3,3}$. Here are the numbers of "new" dominant pairs that each order matrix gives (for the transposed matrix, it will be the same):

① 6 ② 4 ③ 9 ④ 9 ⑤ 12 ⑥ 4 ⑦ 2 ⑧ 5 ⑨ 3

⑩ 6 ⑪ 9 ⑫ 3 ⑬ 6 ⑭ 6 ⑮ 6 ⑯ 5 ⑰ 12 ⑱ 9

(232 dominant pairs in total.)

[1]one would have to check that they are pairwise distinct