BERRARD
Josephine

TB

DM.

Très bonne présentation !

Exercice 5 :

$$\begin{vmatrix} 1 & 2 & 3 & -5 \\ 0 & 1 & 0 & 4 \\ 7 & 3 & -2 & 1 \\ -3 & -5 & 1 & 2 \end{vmatrix} = \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4-4C_2 \end{array} \\ \begin{vmatrix} 1 & 2 & 3 & -13 \\ 0 & 1 & 0 & 0 \\ 7 & 3 & -2 & -11 \\ -3 & -5 & 1 & 22 \end{vmatrix} \end{array} = (-1)^4 \times \begin{vmatrix} 1 & 3 & -13 \\ 7 & -2 & -11 \\ -3 & 1 & 22 \end{vmatrix} = \begin{array}{c} \begin{array}{ccc} C_1 & C_2-3C_3 & C_3+13C_1 \end{array} \\ \begin{vmatrix} 1 & 0 & 0 \\ 7 & -23 & 80 \\ -3 & 10 & -17 \end{vmatrix} \end{array}$$

$$= (-1)^2 \times \begin{vmatrix} -23 & 80 \\ 10 & -17 \end{vmatrix} = -23 \times (-17) - 80 \times 10 = -409$$ ✓

Exercice 9 :

$$B = \begin{pmatrix} 1 & n & \cdots & n \\ n & 2 & n \cdots & n \\ \vdots & & \ddots & \vdots \\ n & \cdots & n & n \end{pmatrix}$$

$$\det B = \begin{array}{c} L_1-L_n \\ L_2-L_n \\ L_3-L_n \\ \vdots \\ L_{n-1}-L_n \\ L_n \end{array} \begin{vmatrix} 1-n & 0 & \cdots & \cdots & 0 \\ 0 & 2-n & 0 & \cdots & 0 \\ 0 & 0 & 3-n & 0 \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & \cdots & 0 & (n-1-n) & 0 \\ n & n & \cdots & \cdots & n \end{vmatrix}$$

donc $\det B = (1-n) \times (2-n) \times \cdots \times (-1) \times n$ (car on a une matrice triangulaire) ✓

$$= n \times \prod_{i=1}^{n-1} (i-n)$$

$$= n \times (-1)^{n-1} \prod_{i=1}^{n-1} (n-i)$$

$$= n \times (-1)^{n-1} \times (n-1)!$$

$$\det B = (-1)^{n-1} \times n!$$ ✓