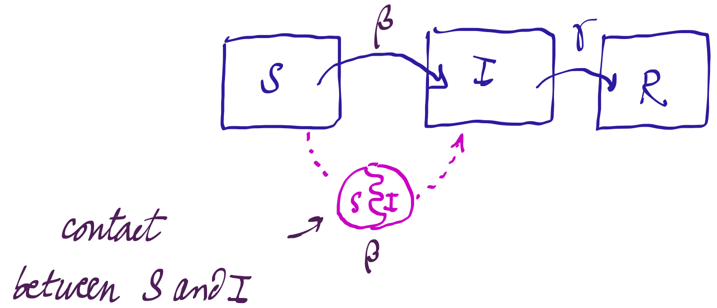
BS-3COMATH1-S1 - Modeling biological dynamics with ordinary differential equations

Introduction to modeling skills 2/2

Exercise 1:

1. a. $$\begin{cases} S' = -\beta S I \\ I' = \beta S I - \gamma I \\ R' = \gamma I \end{cases}$$



b. $S' + I' + R' = 0 \quad (*)$

Note $S + I + R = N$ (total population)

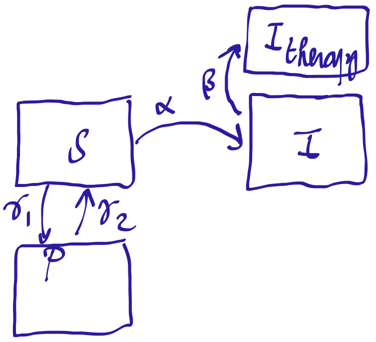
$(*) \iff N' = 0$ that is $N = c^{st}$: constant population

c. $S(0) = S_0 > 0$

$I(0) = I_0 > 0$

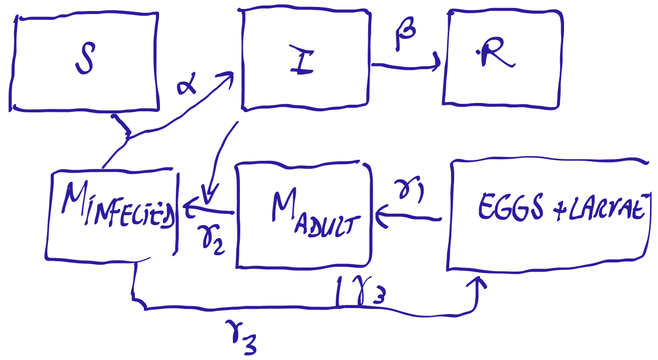
$R(0) = R_0$ can be >0 or $=0$

2. a.



b.
$$\begin{cases} S' = \gamma_2 P - \alpha S I - \gamma_1 S \\ I' = \alpha S I - \beta I \\ P' = \gamma_1 S - \gamma_2 P \end{cases}$$

3. a.



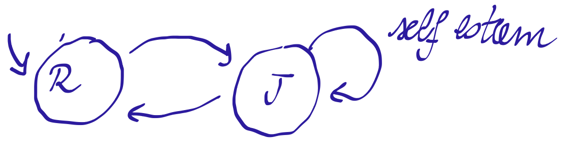
b.
$$\begin{cases} S' = -\alpha I M_{INF} \\ I' = \alpha I M_{INF} - \beta I \\ M'_{EGG} = -\gamma_1 M_{EGG} + \gamma_3 (M_{AD} + M_{INF}) \\ M'_{AD} = \gamma_1 M_{EGG} - \gamma_2 M_{AD} \cdot I \\ M'_{INF} = \gamma_2 M_{AD} I \end{cases}$$

+ death rates

Exercise 2 :

1. a. self esteem

self esteem



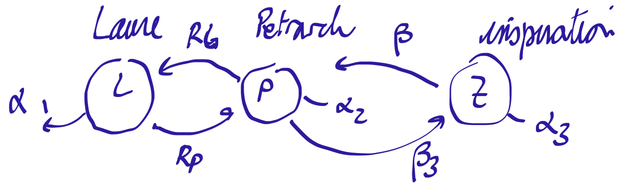
b. $\begin{cases} R' = aR + bJ \\ J' = cR + dJ \end{cases}$

c. Lose confidence on feelings: negative term

$$R' = aR + bJ - a_1 R - b_1 J$$

or $$= aR\left(1 - \frac{R}{K_L}\right) + bJ\left(1 - \frac{J}{K_b}\right) - \text{negative term}$$

2.



L: love of Laura for the Petrarch

P: " " Petrarch " Laura

Z: inspiration

R_L and R_P: reaction functions

A_P [A_L]: appeal of Petrarch [Laura] (appeal: physical, social, intellectual)

$$L' = \underbrace{-\alpha_1 L}_{\text{forgetting process}} + \underbrace{R_L(P)}_{\text{reaction of Laura to the love of Petrarch}} + \underbrace{\beta_1 A_P}_{\text{response to his appeal}}$$

love of Petrarch

$$P' = -\alpha_2 P + R_p(L) + \beta_2 \frac{A_L}{1+\delta Z(t)}$$

$-\alpha_2 P$: forgetting process

$R_p(L)$: reaction of Petrarch to the love of Laura

$\beta_2 \frac{A_L}{1+\delta Z(t)}$: response to her appeal + to the inspiration (the more he is inspired the less he is attracted)

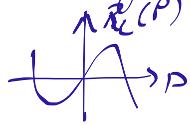
$$Z' = -\alpha_3 Z + \beta_3 P$$

$-\alpha_3 Z$: inspiration decays

$\beta_3 P$: sustained inspiration by love

6. $R_p(L) = \beta_2 L$: the more Laura loves Petrarch the more he loves her and viceversa

$$R_L(P) = \beta_1 P\left(1-\left(\frac{P}{\gamma}\right)^2\right) :$$



if the love of Petrarch is too big then Laura is scared and if it fade away she is back to love

3. a. $P_1' = \alpha P_1\left(1-\frac{P_1}{K}\right)(P_1 - M) - \epsilon P_1$

$E_1(P_2)$

(coupled:
$$\begin{cases} P_1' = \alpha(P_2) P_1 \left(1 - \frac{P_1}{K_2(P_2)}\right)\left(P_1 - M(P_2)\right) - E P_1 \\ P_2' = \alpha_2(P_1) P_2 \left(1 - \frac{P_2}{K_1(P_1)}\right)\left(P_2 - M_2(P_1)\right) - E_2 P_2 \end{cases}$$

$E_2(P_1)$

6. Perturbations can be added in the initial conditions (pulses) or in stochastic terms in the equations

Exercise 3:

1.

Rabbit grass sheep

2. $$x' = \alpha_1 x\left(1 - \frac{x}{K_1}\right)$$
$$y' = \alpha_2 y\left(1 - \frac{x}{K_2}\right)$$

$\alpha_1 \gg \alpha_2$, $K_1 \gg K_2$

3. $$\begin{cases} x' = x(3 - x - 2y) & \to \text{Rabbits} \\ y' = y(2 - x - y) & \to \text{sheep} \end{cases}$$