INSA Lyon
Fall semester
2022-2023

BS-3-COMATH1-S1 - Biomathematics 1 :
Modelling biological dynamics
with ordinary differential equation

Introduction to modeling skills (2 h)

Part 2/2

Objectives :
The goal of this exercise sheet is to understand how mathematical models can be designed from biological observations and simplifying assumptions. This part is dedicated to the practice of illustrate with a scheme a biological problem from equations. And reversely, to writhe equations from a diagram.

Exercice 1. *Epidemiology problem*

1. We consider one of the most famous epidemiological problem : the SIR model, where S stands for the susceptibles, I infected and R the recovered.
   The classic system describing the problem is the following :
$$\begin{cases} \dfrac{dS}{dt} &= -\beta SI, \\ \dfrac{dI}{dt} &= \beta SI - \gamma I, \\ \dfrac{dR}{dt} &= \gamma I. \end{cases} \tag{1}$$
   (a) Illustrate this system with a scheme.
   (b) If you add all the lines, what do you obtain? Does it seem biologically relevant?
   (c) What should be the initial condition?
2. Consider now a HIV model with a preventive strategy to protect the most exposed population (PrEP (Pre-Exposure Prohylaxis)). In other words.
   (a) Illustrate with a figure taking the susceptibles, infected but under therapy population, infected with no therapy (HIV not detected yet), the PrEP users.
   (b) Write a system of equations to illustrate this problem.
3. Same question with dengue. This time, the way of infection is through mosquitoes. We call them vector-born diseases.
   (a) Draw a figure taking into account the mosquito life.
   (b) Try to guess a simple system of equations describing this problem

Exercice 2. *Love relationship*

1. In 1988, an Steven Strogatz wrote a simple model of describing love relationship. The system was called the Romeo and Juliet model. The paper was called “Love Affairs and Differential Equations”.
   It was based on simple assumptions : Juliet is in love with Romeo. But in this version of the story, Romeo is a fickle lover. The more Juliet loves him, the more he begins to dislike her. But when she loses interest, his feelings for her warm up. She on the other hand, tends to echo him. Her love grows when she loves her, and turns to hate when he hates her.
   (a) Illustrate with a diagram.
   (b) Write a simple model for this relationship.
   (c) How could we write a more complex model when Romeo and Juliet gain or lose confidence on their feelings ?
2. In 1998, Sergio Rinaldi wrote a love model between Laura and Petrarch. To complete the model he added inspiration in the equations. The system was the following :
$$\left\{\begin{array}{rcl} \dfrac{dL}{dt} &=& -\alpha_1 L + R_L(P) + \beta_1 A_P, \\ \dfrac{dP}{dt} &=& -\alpha_2 L + R_P(L) + \beta_2 \dfrac{A_L}{1+\delta Z}, \\ \dfrac{dZ}{dt} &=& -\alpha_3 Z + \beta_3 P, \end{array}\right. \tag{2}$$
   where L and P are respectively the feelings of Laura toward Petrarch and inversely the feeling of Petrarch toward Laura.
   (a) Give an interpretation of the model.
   (b) Rinaldi assumed that $R_P(L) = \beta_2 L$ and $R_L(P) = \beta_1 P(1-(\frac{P}{\gamma})^2)$. Explain.
3. Let us assume a relationship like an ecological model (Allee effect).
   (a) How would you write such a model ?
   (b) How would you add perturbations like arguments or fights ?

Exercice 3. *Sheep vs Rabbits*

Suppose that sheep and rabbits are competing for the same food supply (grass) and the amount available is limited. Furthermore, ignore all other complications (no predators, no seasonal effects, no other sources of food).
Two main effect should be considered.
1- each species would grow in a logistic way, and grow to its carrying capacity in the absence of the other. Remember that rabbits reproduce faster than sheep.
2- when rabbits and sheep encounter each other, trouble starts. Sometimes the rabbit gets to eat, but more usually the sheep nudges the rabbit aside and starts nibbling. We assume that these conflicts occur at a rate proportional to the size of each population. Furthermore, we assume that the conflicts reduce the growth rate for each species but the effect is more sever for the rabbits.

1. Draw a diagram to illustrate this.
2. Write the model when the two populations are separated.
3. Write a model when the two populations are mixed.