A Local Multi-grid X-FEM approach applied to crack growth simulations

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CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

OUTLINE

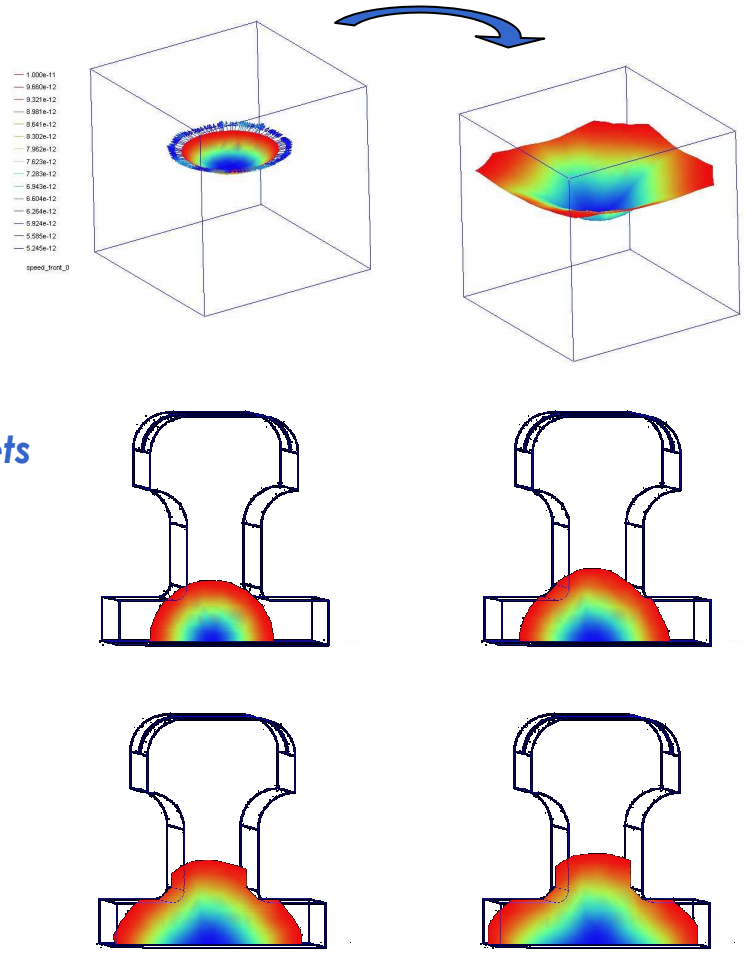
1. *Introduction of X-FEM+level sets for crack modeling*
2. *An example of multi-scale approach coupled with X-FEM*
3. *Conclusions & perspectives*

Introduction

- *eXtended Finite Element Method + level sets*

- *Advantages:*
  - *Similar to FEM*
  - *No remeshing*
  - *No field interpolation*
  - *Good topologic properties*
  - *Flexibility in the initialization of level sets*

- *Drawbacks:*
  - *Specific numerical integration and preconditioning*
  - *Post-treatment*
  - *Specific strategies of enrichment for time-dependent problems*



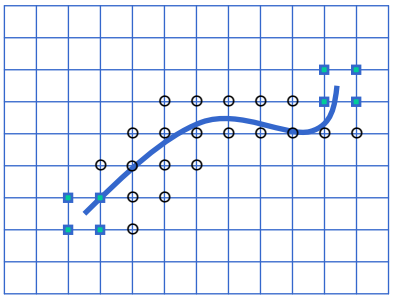
Introduction

- *Local partition of unity (two scale strategy):*

- *Discontinuous and asymptotic enrichment of the displacement field*

$$U(x) = \sum_{i \in N} N_i(x)\, u_i + \sum_{i \in N_H} N_i(x)\, H(x)\, a_i + \sum_{i \in N_F} \sum_{j} N_i(x)\, F_j(x)\, b_i^j$$

$$\{F_j\} = \left\{ \sqrt{r}\sin\frac{\theta}{2},\ \sqrt{r}\cos\frac{\theta}{2},\ \sqrt{r}\sin\frac{\theta}{2}\sin\theta,\ \sqrt{r}\cos\frac{\theta}{2}\sin\theta \right\}$$



- *Crack shape modeling by two level sets:*

$\varphi(x)$ $\psi(x)$ *crack*

$\varphi(x)$ $\psi_1(x)$ *Crack front*

$\varphi(x)$ $\psi_2(x)$ *Crack virtual extension*

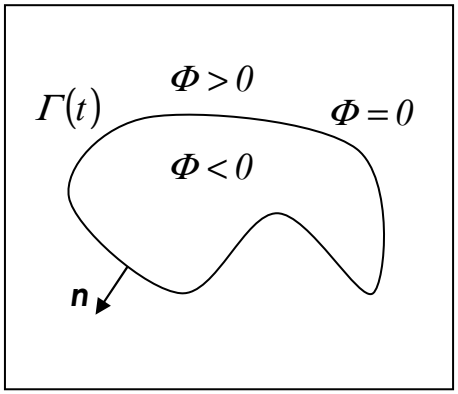
Local orthogonality

- *Level set update* $\varphi^{n+1} = \varphi^{n} + V_\varphi \Delta t$

[Moës 1999, Stolarska 2001, Duflot 2005, Béchet 2005, Sukumar 2007] [Sethian 1997]

[Gravouil A., Moës N., Belytschko T., IJNME, 2002]

Level sets



Signed distance:

$$\begin{cases} \Phi(\mathbf{x},t) < 0 & in\ \Omega^-(t) \\ \Phi(\mathbf{x},t) = 0 & on\ \Gamma(t) \\ \Phi(\mathbf{x},t) > 0 & in\ \Omega^+(t) \end{cases}$$

→ *Allows to model implicitly moving interfaces*

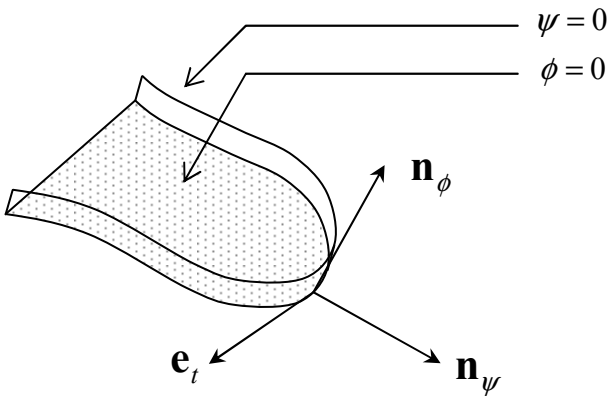
- **Governing equation:** $\frac{\partial \Phi}{\partial t} + V|\nabla \Phi| = 0$
- **Normal vector:** $\mathbf{n} = \frac{\nabla \Phi}{|\nabla \Phi|}$ **curvature:** $\kappa = \nabla . \frac{\nabla \Phi}{|\nabla \Phi|}$
- **Hausdorff measure:** $|\Gamma(t)| = \int \delta(\Phi)|\nabla \Phi| dx$

[Osher and Sethian 1988]

- **Lebesgue measure:** $|\Omega^-(t)| = \int H(-\Phi) dx$

Non-planar crack modeling

- *Local basis linked to the level sets*



$$\mathbf{n}_\psi = \nabla \psi$$

$$\mathbf{n}_\phi = \nabla \phi$$

$$\mathbf{e}_t = \mathbf{n}_\psi \times \mathbf{n}_\phi$$

- *Component velocity on the local basis of the crack front* V_ϕ *and* V_ψ

$$\mathbf{V} = V_\psi \mathbf{n}_\psi + V_\phi \mathbf{n}_\phi$$

Non-planar crack modeling

- **Initialization and re-initialization** $|\nabla\psi| = 1$ $\quad \frac{\partial\psi}{\partial\tau} + sign(\psi)(|\nabla\psi| - 1) = 0$

- **Extension of the velocity field** $\nabla\phi.\nabla V_\phi = 0$ **[Peng and al. 1999]**

$$\frac{\partial V_\phi}{\partial\tau} + sign(\phi)\frac{\nabla\phi}{|\nabla\phi|}.\nabla V_\phi = 0 \qquad \frac{\partial V_\phi}{\partial\tau} + sign(\psi)\frac{\nabla\psi}{|\nabla\psi|}.\nabla V_\phi = 0$$

$$\mathbf{n}_\psi.\nabla V_\phi = 0 \qquad \mathbf{n}_\phi.\nabla V_\phi = 0$$

- **Update of the level sets** $\frac{\partial\phi}{\partial t} + V_\phi|\nabla\phi| = 0$ $\quad \frac{\partial\psi}{\partial t} + V_\psi|\nabla\psi| = 0$

- **Re-orthogonalization:**

$$\frac{\partial\psi}{\partial\tau} + sign(\phi)\frac{\nabla\phi}{|\nabla\phi|}.\nabla\psi = 0 \quad \Rightarrow \quad \nabla\phi.\nabla\psi = 0$$

Non-planar crack modeling

- **Time and space discretization for structured meshes**

$$\phi_{ij}^{n+1} = \phi_{ij}^{n} - \Delta t \left\{ \begin{array}{l} \left(s_{ij} n_{ij}^{x}\right)^{+} \dfrac{\phi_{ij} - \phi_{i-1\,j}}{\Delta x} + \left(s_{ij} n_{ij}^{x}\right)^{-} \dfrac{\phi_{i+1\,j} - \phi_{ij}}{\Delta x} \\ + \left(s_{ij} n_{ij}^{y}\right)^{+} \dfrac{\phi_{ij} - \phi_{ij-1}}{\Delta y} + \left(s_{ij} n_{ij}^{y}\right)^{-} \dfrac{\phi_{ij+1} - \phi_{ij}}{\Delta y} \end{array} \right\}$$

$$\left\{ \begin{array}{l} \widetilde{\Phi}^{n+1} = \Phi^{n} - \Delta t H\left(\Phi^{n}\right) \\ \Phi^{n+1} = \frac{\left(\Phi^{n} + \widetilde{\Phi}^{n+1}\right)}{2} - \frac{\Delta t}{2} H\left(\widetilde{\Phi}^{n+1}\right) \end{array} \right.$$

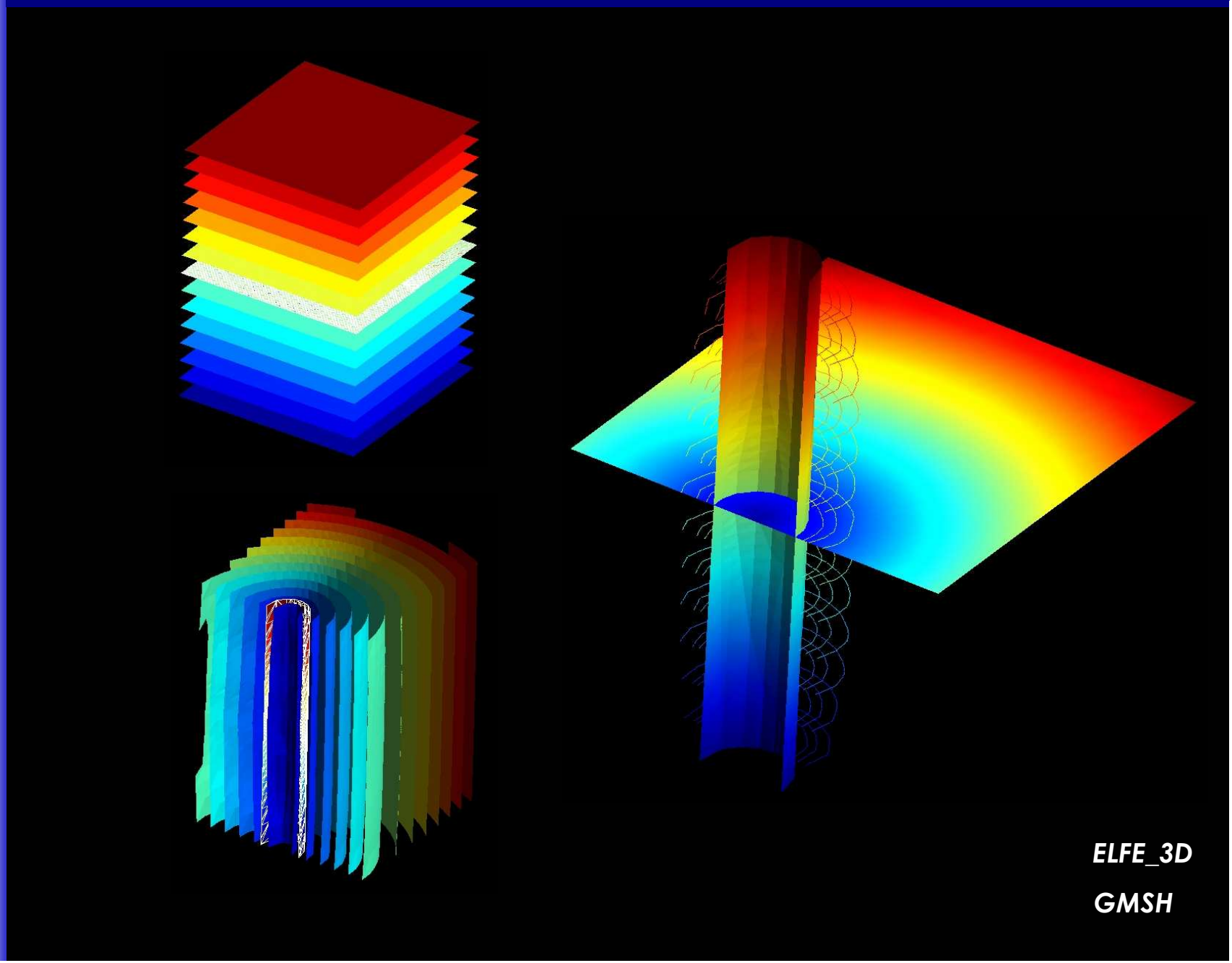
$$(x)^{+} = \max(x,0) \qquad (x)^{-} = \min(x,0) \qquad s_{ij} = \frac{\Phi_{ij}}{\sqrt{\Phi_{ij}^{2} + \Delta x^{2}}}$$

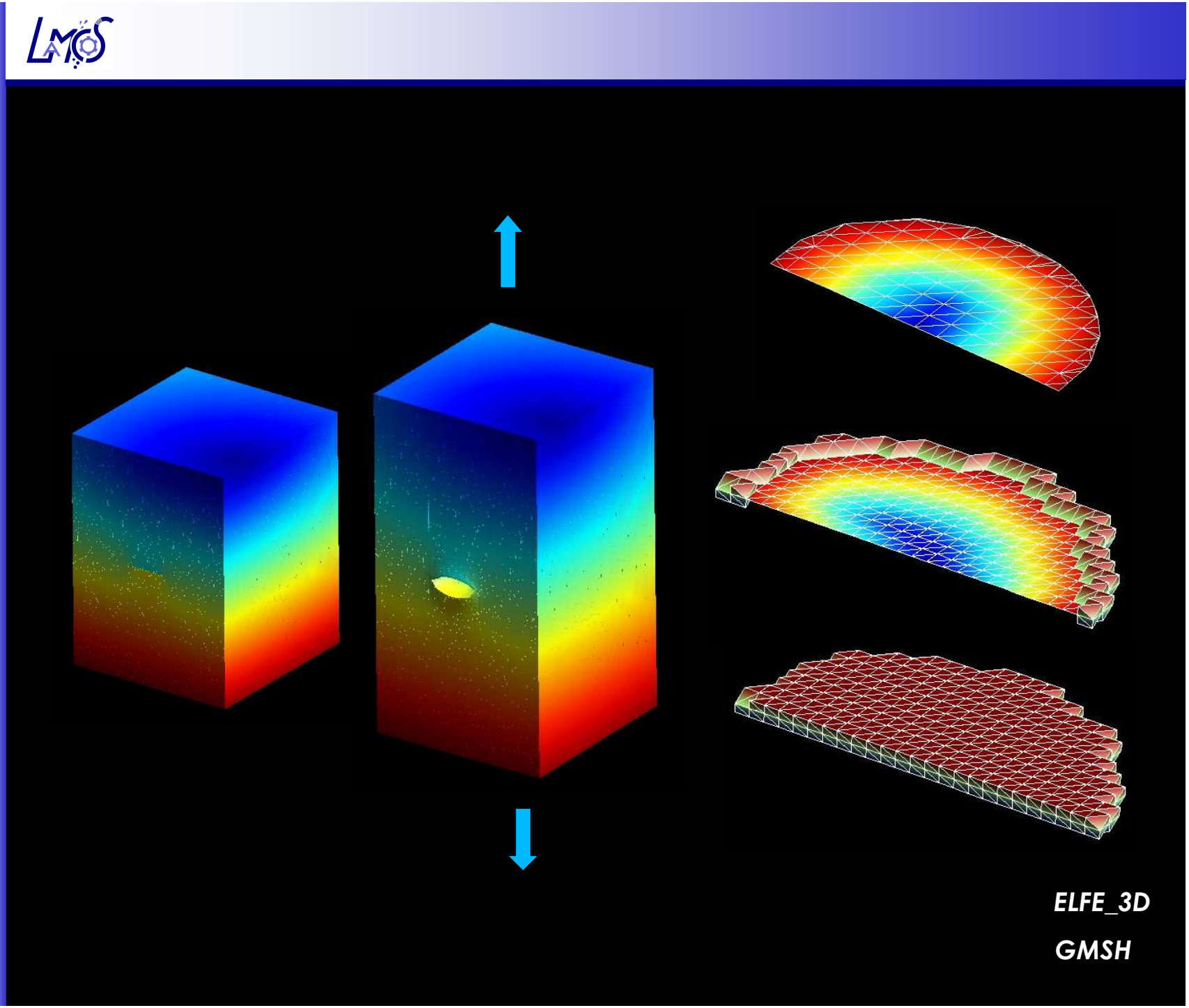
- **Time and space discretization for non-structured meshes**

$$\left\{ \begin{array}{l} \dfrac{\partial \phi}{\partial t} + \mathrm{H}\left(\nabla \phi, \mathbf{x}, t\right) = 0 \\ \phi(\mathbf{x},0) = \phi_{0}(\mathbf{x}) \end{array} \right.$$

Space: [Barth and Sethian 1998]
Time: Runge Kutta

→ *Numerical schemes stable, accurate and convergent. However, finite difference approaches are more accurate for an equivalent size element mesh*

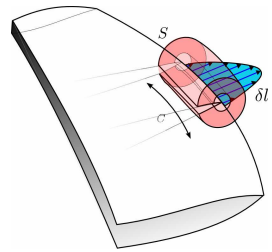

ELFE_3D
GMSH


ELFE_3D
GMSH

Stress Intensity Factors calculation

- *Dissipated energy for a virtual crack extension*

$$W_{diss} = \int_c G(s)\,\sigma_{ij}(\quad)\,\varepsilon_{ij}(\quad)\,ds \qquad \int_c G(s)\,\sigma_{ij}\,\varepsilon^{aux}_{ij}\,ds \qquad P = W_{\quad}(\quad)$$

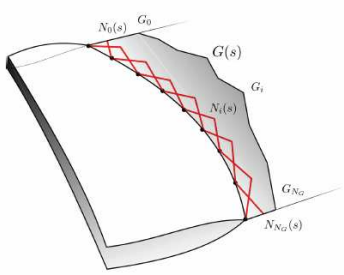


- *G(s) is discretized along the front with curvilinear shape functions:*

$$G(\quad) = \sum_{i=1}^{N_G} G_i\,N_i(\quad) \qquad A_{ij} = \int_c G(s)\,N_i(\quad)\,ds$$

$$\int_V \sigma_{ij}\,\varepsilon^{aux}_{ij}\,dV$$

[Parks et al., 2000]



- *Diagonal matrix:*

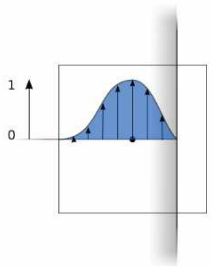
$$A^{lump}_{ii} = \sum_{j=1}^{N_G} A_{ij} = \int_c \sum_{j=1}^{N_G} N_i(\quad)\,N_j(\quad)\,ds = \int_c N_i(\quad)\,ds$$

- *Irwin formula*

$$G = \frac{E}{\quad}\left[K_I^2(\quad) + K_{II}^2(\quad)\right] + \frac{1}{\quad}K_{III}^2(\quad)$$

- *Interaction integral*

$$I_{int} = \int_V q_{i,i}\left[\sigma_{kl}\varepsilon^{aux}_{kl}\,\delta_{ij} - \sigma_{kj}\frac{\partial u^{aux}_k}{\partial x_i} - \sigma^{aux}_{kj}\frac{\partial u_k}{\partial x_i}\right]dV$$

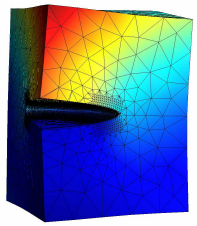


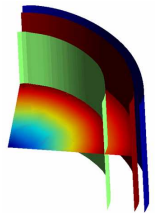
[Combescure, Suo, 1986] [Moës N., Gravouil A., Belytschko T. 2002]

[Gosz & al. 1997,2002, Béchet 2005, Réthoré 2005, Elguedj 2006, Ribeaucourt 2006]

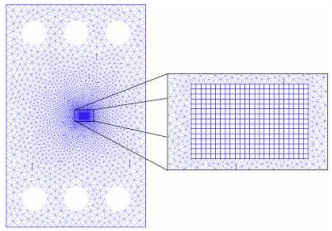
Scale effects and numerical discretization:

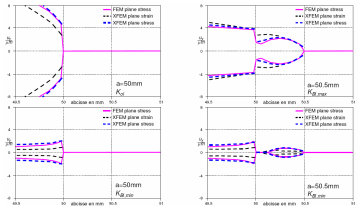
- *Some previous works show that we need to « capture » the following length scales:*
  - *Area influenced by the asymptotic behavior close tot the crack front*



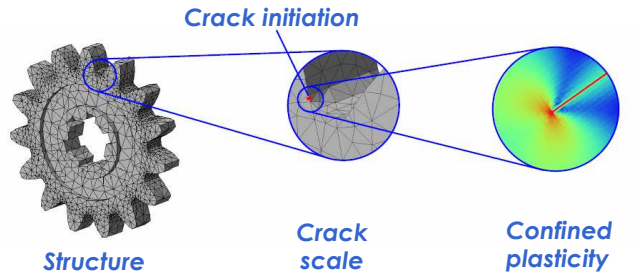


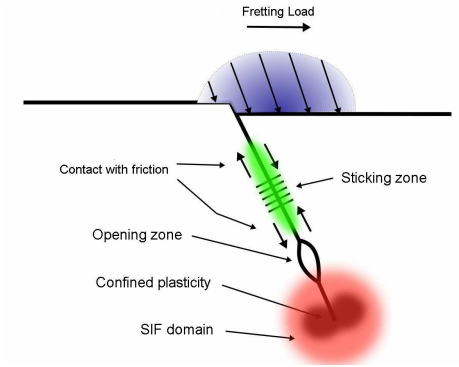
  - *Confined plasticity, closure effect*





  - *Localized non-linearities due to contact and friction along the crack faces*





→ *When several length scales occur, one may question the relevant spatial discretization needed to reach a given accuracy (with enrichment functions)*

OUTLINE

1 *Introduction of X-FEM+level sets for crack modeling*

2 ***An example of multi-scale approach coupled with X-FEM***

3 *Conclusions & perspectives*

Multi-scale strategies coupled with X-FEM

- *Agreement between the discretization of the structure and the geometric modeling of the crack (in presence of enrichments) ?*
- *Agreement between the discretization of the structure and the area influenced by the crack front singularity (in presence of enrichments) ?*
- *Agreement between the discretization of the structure and local non-linearities on the crack faces or / and in the bulk (in presence of enrichments) ?*
- *Which kind of strategy of enrichment to consider (topologic enrichment, geometric enrichment, enrichment of all scales) ?*

→ *Different numerical multi-scale strategies:*

- *Multi-grid strategies*
- *Micro-macro approaches*
- *Partition of unity*
- *…*

[Guidault P.-A., Allix O., Champaney L., Cornuault B., 2007] [Allix O., 2006] [Ladevèze O., Loiseau O., Dureisseix D., 2001]

[Chinesta & al. 2007] [Fish J., Yuan Z., 2005] [Ben Dhia H., Rateau G., 2005] [Qian D., Wagner G., Liu W., 2004] [Feyel F., 2003]

[Venner C., Lubrecht A., 2000] [Fish J., Pandheeradi M., Belsky V., 1995] [Parsons I., Hall J., 1990] [Brandt A., 1977]

Multi-grid methods

- *Fluid mechanics [Brandt 1977] [Hackbush et Trottenberg 1981] [Hackbush 1985]*
- *Application to linear solid mechanics [Parsons et Hall 1990]*
  Application to non-linear solid mechanics [Kacou et Parsons 1993]

$$\mathbf{U} \equiv \mathbf{P}\overline{\mathbf{U}} + \mathbf{U}_f = \mathbf{U}_g + \mathbf{U}_f$$

① *first relaxation step (smoothing)*

$$\mathbf{U}^k \rightarrow \widetilde{\mathbf{U}}^k = \mathbf{U}^k + \Delta\mathbf{U}_f^{k+1/2} \quad \textit{and} \quad \widetilde{\mathbf{R}}^k$$

② *restriction of the residual on the coarse scale*

$$\overline{\mathbf{R}}^k = \mathbf{P}^T\widetilde{\mathbf{R}}^k$$

③ *direct solving on the coarse scale (recursive for more than 2 grids)*

$$\overline{\mathbf{K}}\Delta\overline{\mathbf{U}}^{k+1} = \overline{\mathbf{R}}^k$$

④ *prolongation of the displacement on the fine scale*

$$\Delta\mathbf{U}_g^{k+1} = \mathbf{P}\Delta\overline{\mathbf{U}}^{k+1}$$

⑤ *correction*

$$\mathbf{U}^{k+1/2} = \widetilde{\mathbf{U}}^k + \Delta\mathbf{U}_g^{k+1} \quad \textit{and} \quad \mathbf{R}^{k+1}$$

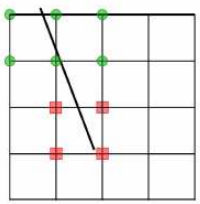
⑥ *last relaxation step*

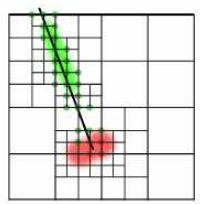
$$\mathbf{U}^{k+1/2} \rightarrow \mathbf{U}^{k+1}$$

An example of multi-scale strategy coupled with X-FEM

- *X-FEM with local mesh refinement for crack modeling*

Linear X-FEM modeling *Non-linear X-FEM modeling*

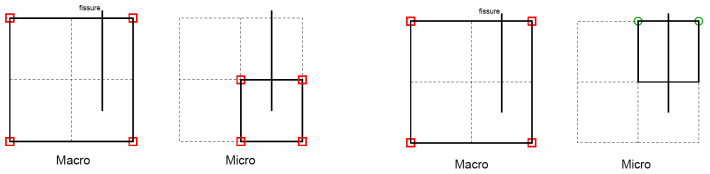




- *Localized multi-grid algorithm (L-MG) Full Approximation Scheme (FAS)*

- *Adapted prolongation and restriction operators (X-L-MG)*

  [Rannou J. & al. EJCM, 2007]



- *Is it needed to enrich all the grids?*

Different definitions of the prolongation operator

- *Nodal definition (collocation method)*

$$\mathbf{u}^f(\mathbf{x}_n) = \mathbf{u}^c(\mathbf{x}_n)$$

$$\mathbf{u}_n^f + \mathbf{a}_n^f H(\mathbf{x}_n) + \sum_{j=1}^{4} \mathbf{b}_{jn}^f \gamma_j(\mathbf{x}_n) = \mathbf{u}^c(\mathbf{x}_n)$$

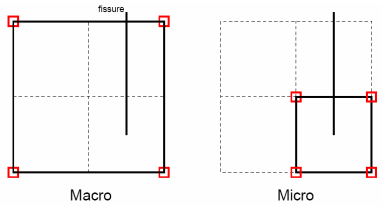
- *Variational definition (mortar method)*
  (unambiguously definition of the coefficients)

$$\int_\Omega \mathbf{u}^f(\mathbf{x})\mathbf{u}^{f*}(\mathbf{x})\,d\Omega = \int_\Omega \mathbf{u}^c(\mathbf{x})\mathbf{u}^{f*}(\mathbf{x})\,d\Omega \quad \forall \mathbf{u}^{f*}$$

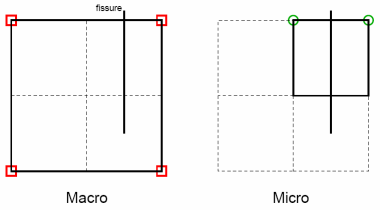
$$\mathbf{M}_{ff}\mathbf{U}_f = \mathbf{M}_{fc}\mathbf{U}_c \quad \Rightarrow \quad \mathbf{P} = \mathbf{M}_{ff}^{-1}\mathbf{M}_{fc}$$

An example of multi-scale strategy coupled with X-FEM
A nodal approach for the prolongation operator definition

- **Compatible enrichment**



- **Incompatible enrichment**



→ **One interpolates separately standard fields and enriched fields (in both cases)**

- $\vec{U}^g(\vec{x}) = \sum_{i \in \mathcal{N}} N_i^g(\vec{x}) \vec{u}_i^g + \sum_{i \in \mathcal{N}_{sing}} \sum_{j=1}^{4} N_i^g(\vec{x}) \gamma_j(\vec{x}) \vec{b}_{ji}^g$

- $\vec{U}^f(\vec{x}_i) = \vec{u}_i^f + \vec{a}_i^f H(\vec{x}_i)$ **with**

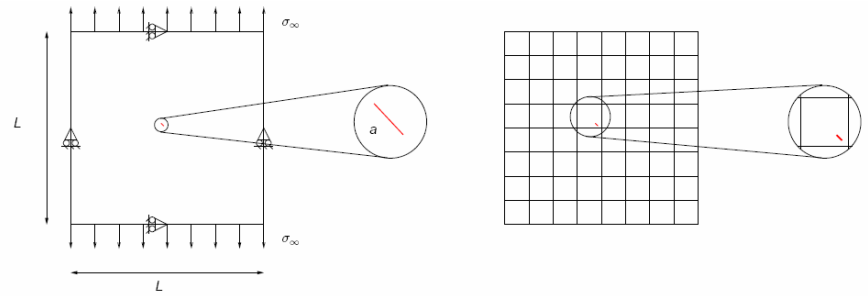
$$\begin{aligned} \vec{u}_i^f &= \sum_{j \in \mathcal{N}} N_j^g(\vec{x}_i) \vec{u}_j^g \\ \vec{a}_i^f H(\vec{x}_i) &= \sum_{j \in \mathcal{N}_{sing}} \left(N_j^g(\vec{x}_i) \sum_{k=1}^{4} \gamma_k(\vec{x}_i) \vec{b}_{jk}^g \right) \end{aligned}$$

→ **Local error due to prolongation step (high frequency error) immediately captured by relaxation step**

RANNOU J., GRAVOUIL A, COMBESCURE A., "A multigrid eXtended Finite Element Method for elastic crack growth simulation". European Journal of Computational Mechanics, VOL 16/2 - 2007 - pp.161-182

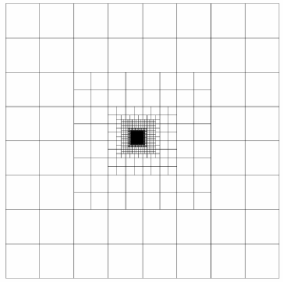
An example of multi-scale strategy coupled with X-FEM

- Crack in a quasi-infinite tensile plate



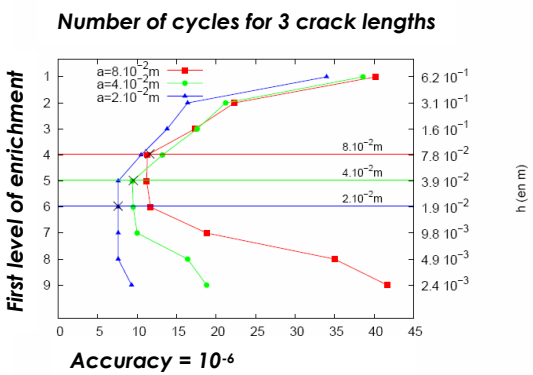
- Different grids used with the X-L-MG algorithm

9 grids (h1/h9 = 256)



l	*h(m)*	*enrichment*
1	$6,25.10^{-1}$	×
2	$3,12.10^{-1}$	×
3	$1,56.10^{-1}$	×
4	$7,80.10^{-2}$	✓
5	$3,90.10^{-2}$	✓
6	$1,95.10^{-2}$	✓
7	$9,77.10^{-3}$	✓
8	$4,88.10^{-3}$	✓
9	$2,44.10^{-3}$	✓

Element size (h) of the first enriched grid of the order of the crack length (a)

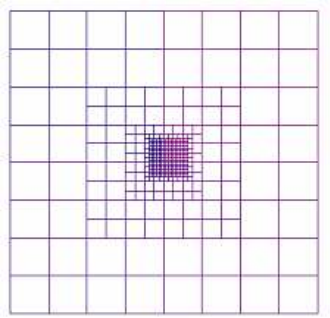


[Rannou J., Gravouil A., Baïetto M.-C, IJNME, 2008]

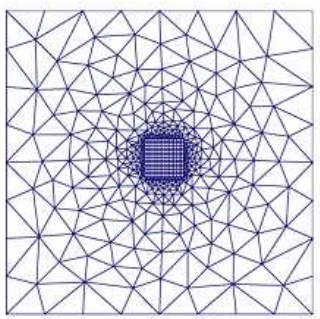
An example of multi-scale strategy coupled with X-FEM

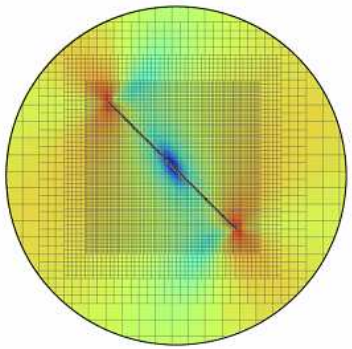
- *CPU time for both local multi-grid method and preconditioned conjugate gradient*

X-L-MG mesh

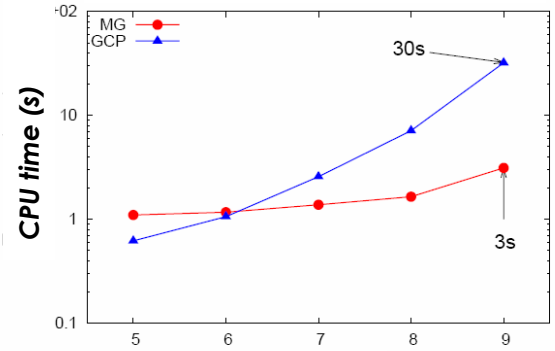


PCG mesh





CPU time for both methods



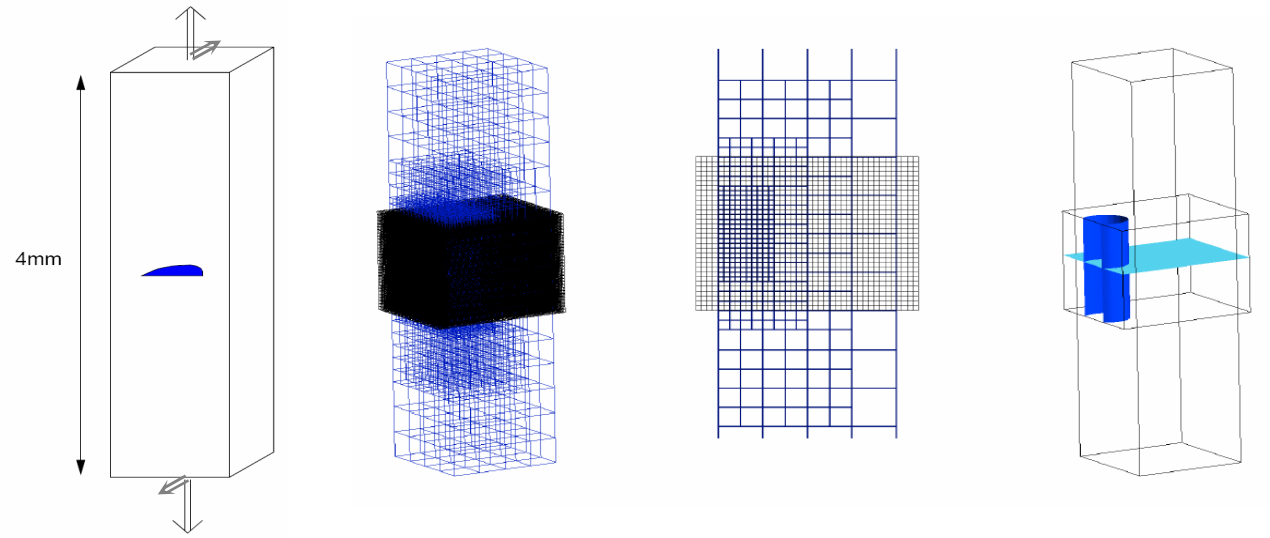
MG cycle number	collocation operator	mortar operator
1	$2.09\,10^{-01}$	$2.10\,10^{-01}$
2	$4.72\,10^{-03}$	$4.38\,10^{-03}$
3	$2.03\,10^{-04}$	$1.76\,10^{-04}$
4	$1.53\,10^{-05}$	$1.33\,10^{-05}$
5	$1.58\,10^{-06}$	$1.41\,10^{-06}$
6	$2.17\,10^{-07}$	$1.99\,10^{-07}$
7	$4.09\,10^{-08}$	$4.56\,10^{-08}$
8	$6.34\,10^{-09}$	$6.08\,10^{-09}$

- *CPU time lower and complexity with lower order compared to PCG (n versus $n^{1.5}$)*
- *Similar numerical convergence rate for collocation and mortar operators*

An example of multi-scale strategy coupled with X-FEM

- *Semi-circular crack in a 3D beam submitted to mode I and mixed mode*

Geometry and B.C. *Structure mesh and level set mesh* *Level sets*



→ *The geometrical modeling of the crack is independent of the structure discretization*

[Rannou J., Gravouil A., Baïetto M.-C., IJNME, 2008]

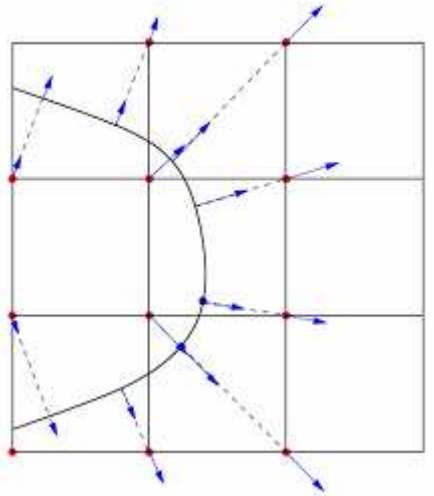
Definition of nodes along the crack front for SIF's calculation

- *Accurate definition of nodes from level sets*

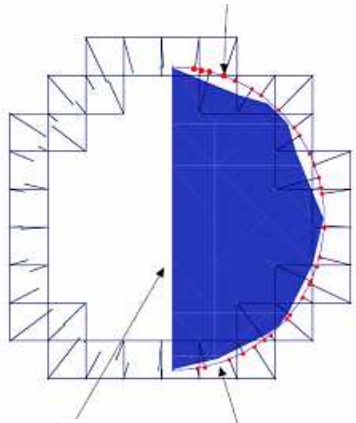
  $\underline{v} = -\psi\underline{\nabla}\psi - \varphi\underline{\nabla}\varphi$

- *Calculation of Stress Intensity factors on these nodes*

- *Extension of the front velocity to closest mesh nodes (no interpolation)*

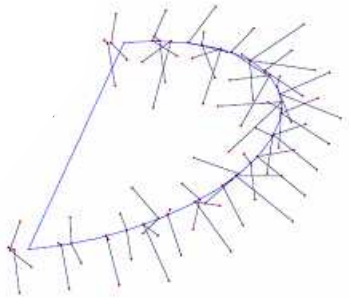


Nodes from gradients of level sets



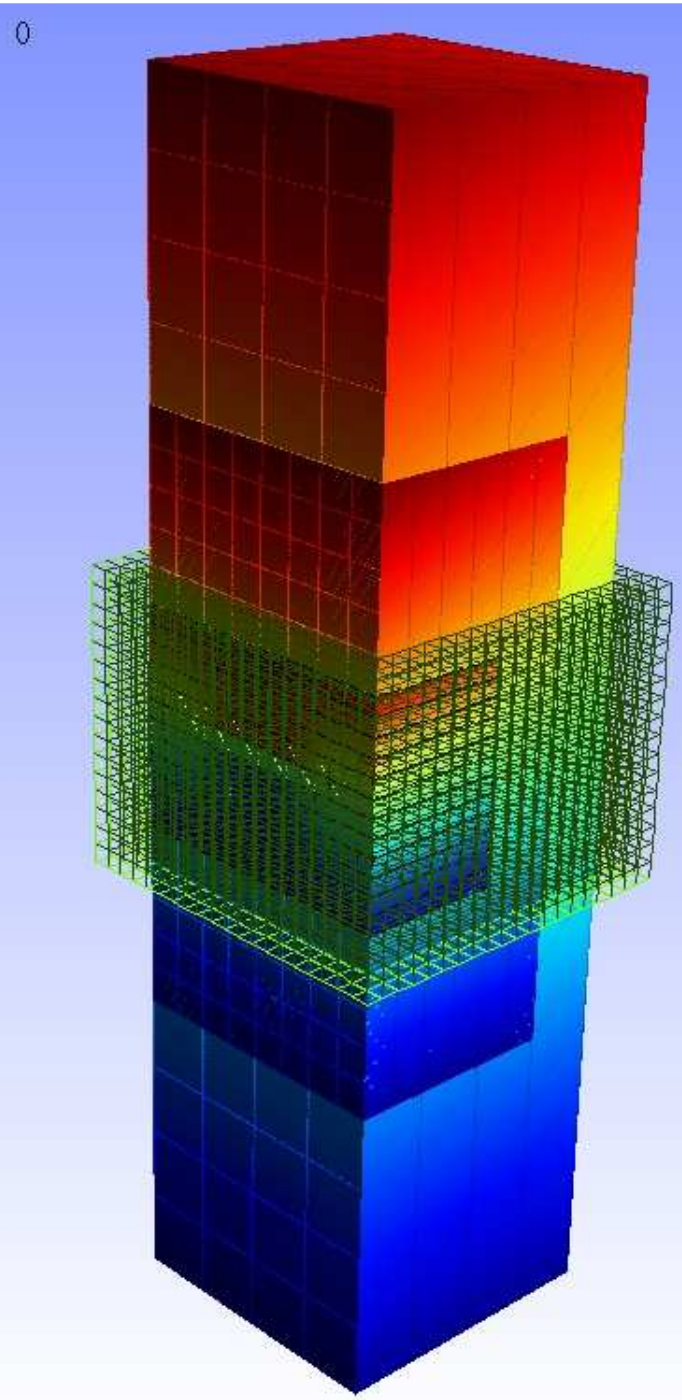
coarse interpolation of the level sets

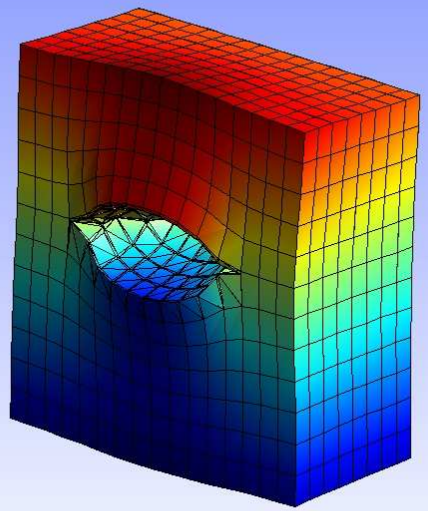
fine interpolation of the level sets

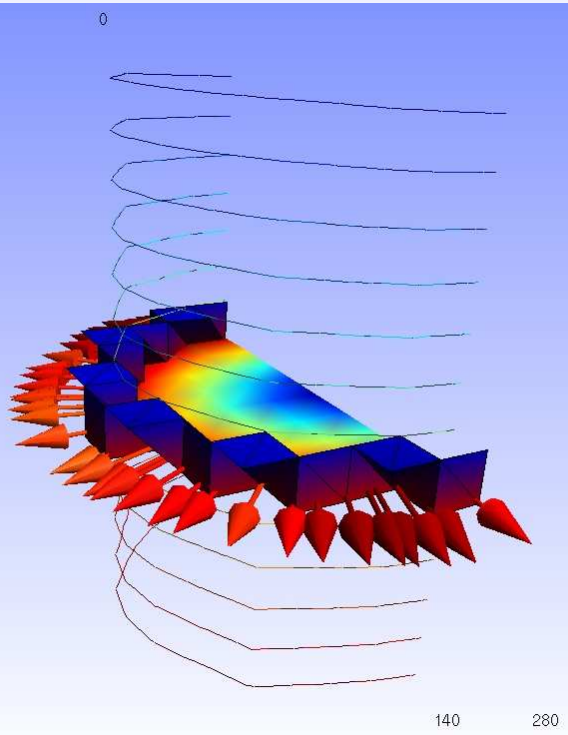


ELFE_3D

GMSH

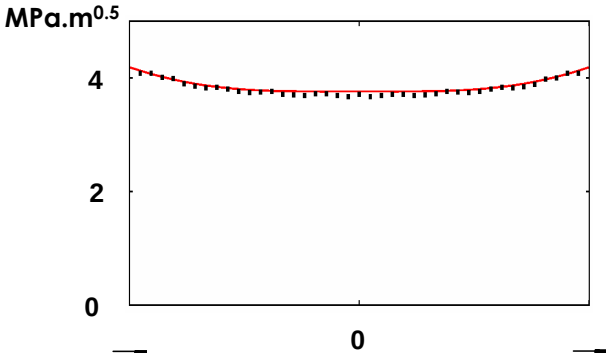






An example of multi-scale strategy coupled with X-FEM

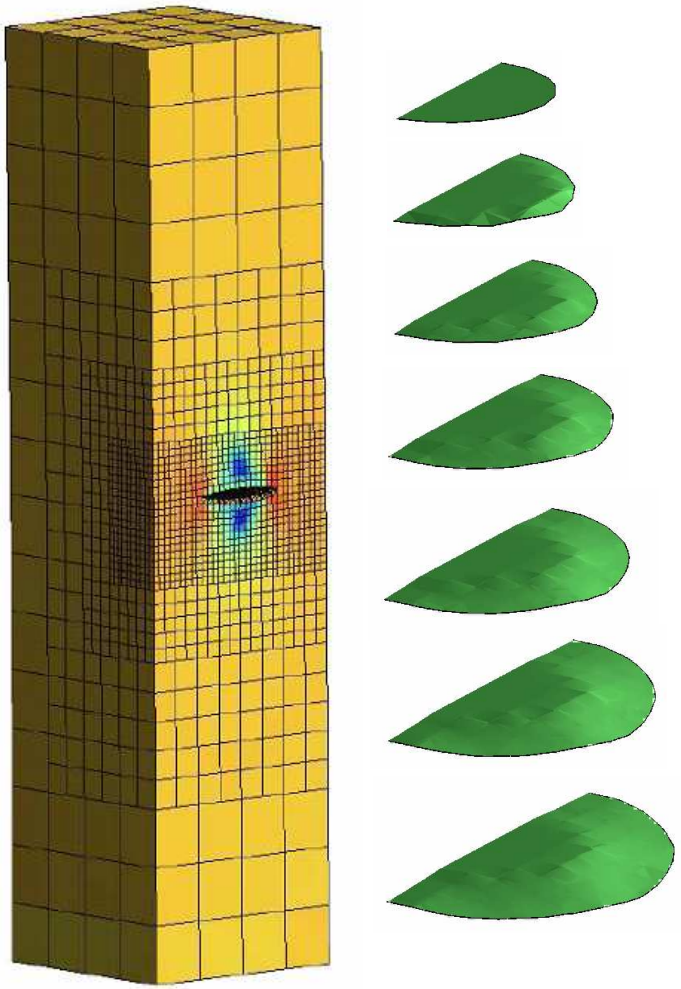
- **Comparison between numerical and analytical K_I in mode I (Raju and Newman)**



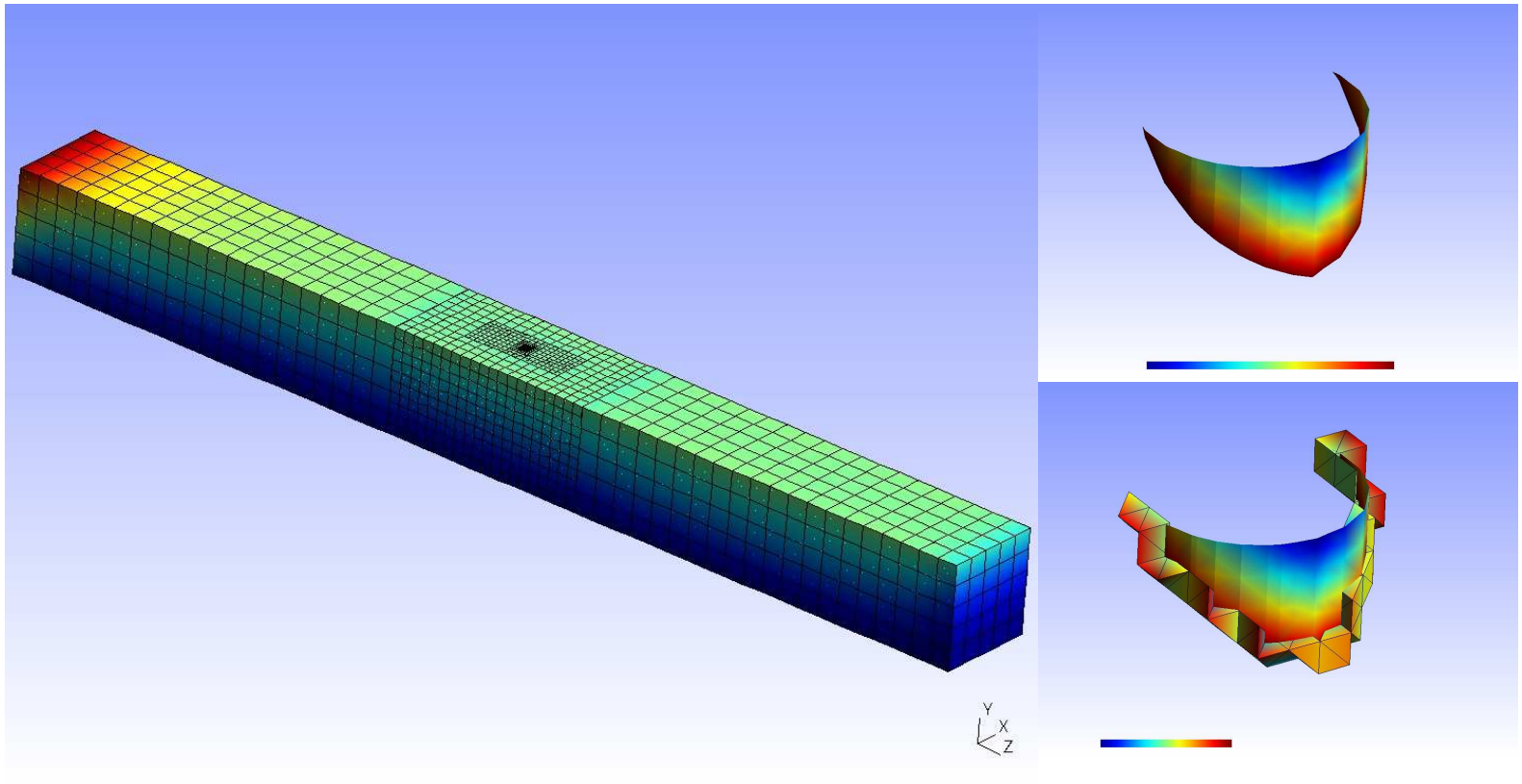
- **Deformed mesh and fatigue crack propagation in mixed mode**

level	h (m)	Nb dofs	Nb iterations	time (s)
0	$2.5.10^{-4}$	1344	528	0.59 *s*
1	$1.2.10^{-4}$	2124	140	4.45 *s*
2	$6.2.10^{-5}$	5262	75	1.61 *s*
3	$3.1.10^{-5}$	13524	40	0.71 *s*
	Nb dofs	22654	**CPU time**	7.4 *s*

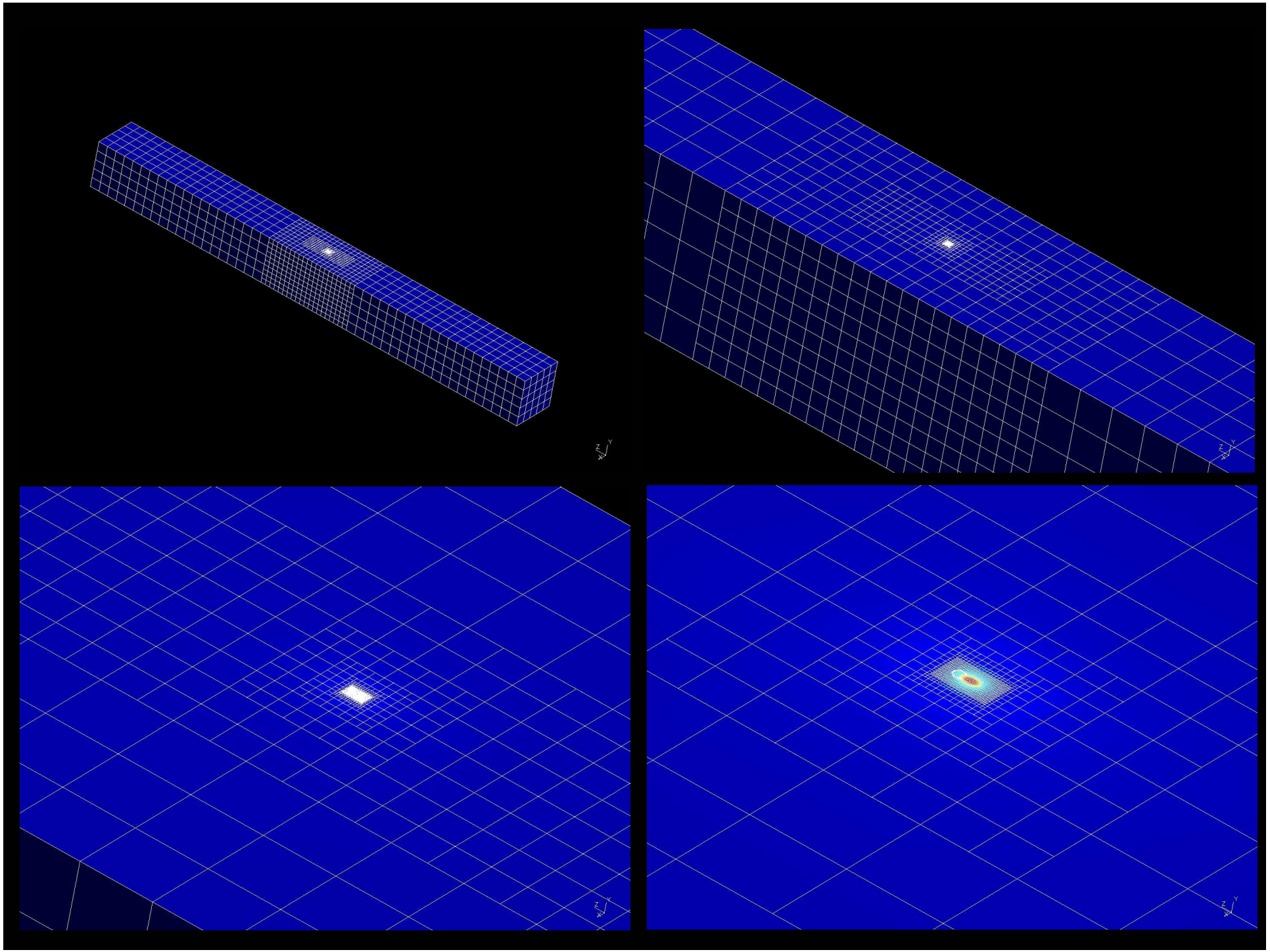
$h_0/a \approx 1 \rightarrow$ enrichment at all levels

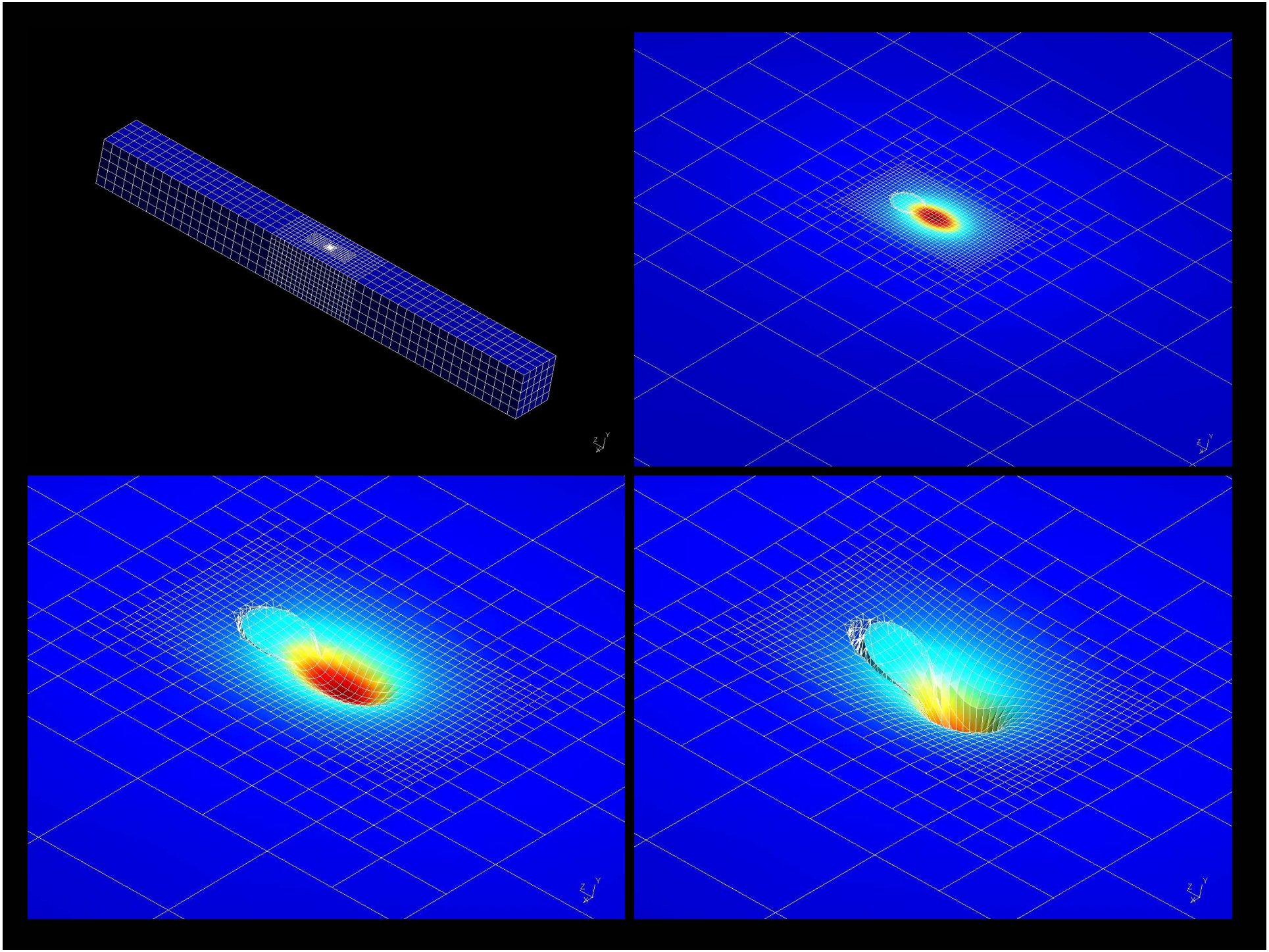


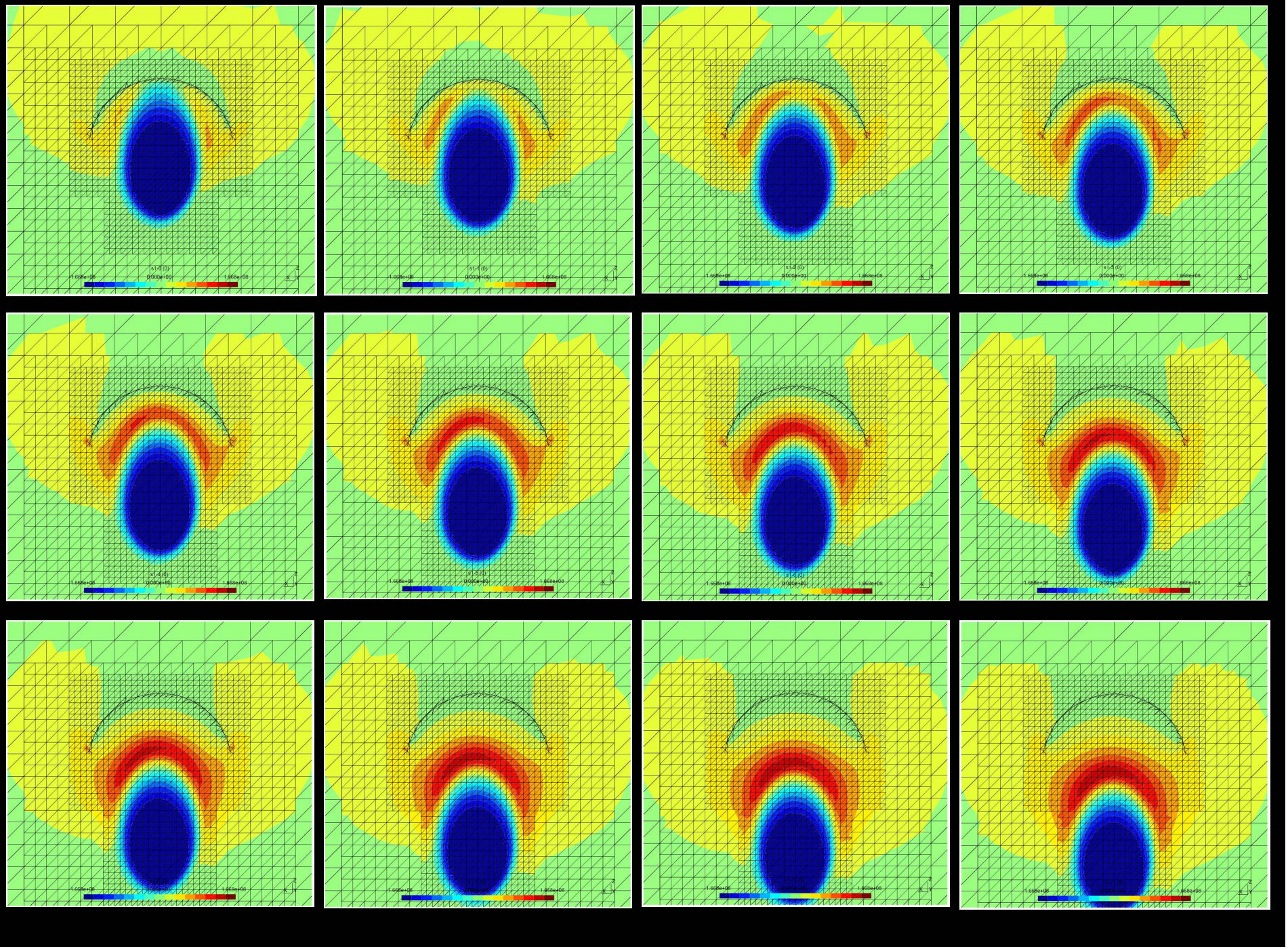
Application of X-L-MG to a non-planar crack with a moving normal and tangential Hertz load (Rannou J. 2008)



$h_0/a \approx 1 \rightarrow$ enrichment from level 5 to 9







OUTLINE

1 *Introduction of X-FEM+level sets for crack modeling*

2 *An example of multi-scale approach coupled with X-FEM*

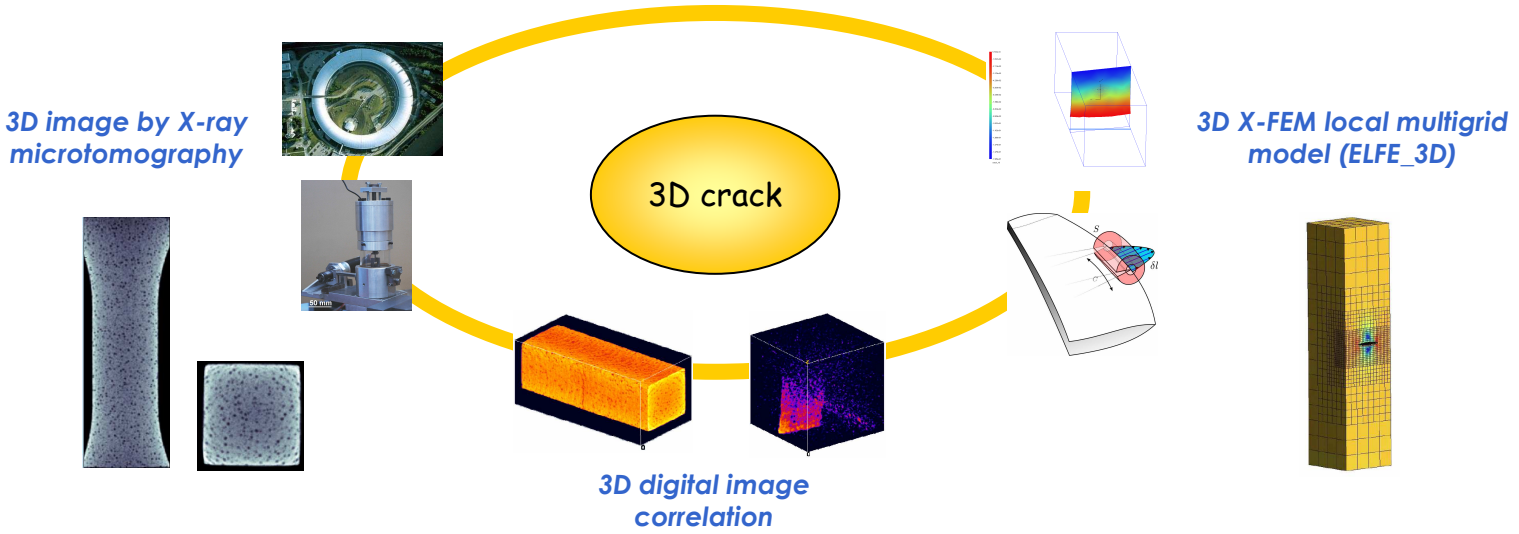
3 ***Conclusions & perspectives***

Main results

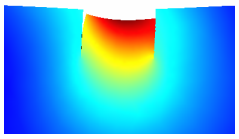
- *Accurate calculation of Stress Intensity factors for non-planar cracks (3D crack growth laws).*
- *One proposes a X-FEM local multi-grid strategy that ensures to capture in a unified approach different length scales which can occur in fracture mechanics.*
- *The crack is modeled on an independent local finite difference mesh adapted to the possible complexity of the crack shape during its propagation (+robust numerical integration of Hamilton Jacobi equations)*
- *Link between the convergence rate of the X-L-MG algorithm and the first enriched grid*
- *The introduction of local grids adapted to scale effects is not in contradiction with the non-remeshing property of the proposed method: indeed, the local grids are previously defined in the area of interest.*

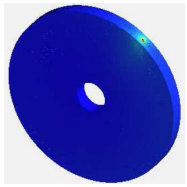
Perspectives

- *Automatic definition of local grids coupled with an adapted local error estimator*
- *Identification of 3D local crack growth law by the use of a X-FEM local multigrid strategy, 3D X-ray microtomography, and 3D digital image correlation (PROPAVANFIS Project, CETIM foundation, LMT / LaMCoS / MATEIS, France)*



- *Application to fretting, tribologic fatigue, extension to local nonlinear behaviors*





Step 1 - Initialiser $\psi_I^* = w_I = 0$ pour chaque nœud du maillage

Step 2 - Boucle sur les éléments finis

Calculer les quantités

$$\nabla\psi = \sum_{J=1}^{4} \nabla N_J \psi_J \qquad \overline{V} = \int_E V dx$$

et les coefficients suivants pour chaque nœud I du tétraèdre considéré:

$$K_I = \overline{V} \frac{\nabla\psi . \nabla N_I}{|\nabla\psi|} \qquad \delta\psi = \sum_{J=1}^{4} K_J \psi_J$$

$$\alpha_I = \frac{H\left(\frac{\delta\psi_I}{\delta\psi}\right)}{\sum_{K=1}^{4} H\left(\frac{\delta\psi_K}{\delta\psi}\right)} \quad \text{avec} \quad \delta\psi_I = H(K_I)\left(\sum_{L=1}^{4} H(K_L)\right)^{-1} \sum_{J=1}^{4} H(-K_J)(\psi_J - \psi_I)$$

ou H est la fonction de Heaviside.

$$\psi_I^* = \psi_I^* + \alpha_I \delta\psi$$
$$w_I = w_I + \alpha_I v$$

(v est le volume de l'élément fini E)

Step 3 - Boucle sur les nœuds du maillage pour l'intégration en temps

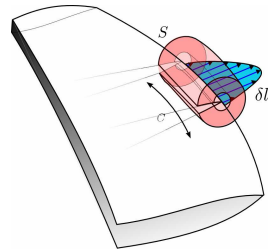
$$\begin{cases} \tilde{\psi}_I^{n+1} = \psi_I^n - \Delta t \dfrac{(\psi_I^*)^n}{w_I^n} \\ \psi_I^{n+1} = \dfrac{(\psi_I^n + \tilde{\psi}_I^{n+1})}{2} - \dfrac{\Delta t}{2} \dfrac{(\tilde{\psi}_I^*)^{n+1}}{\tilde{w}_I^{n+1}} \end{cases}$$

[Barth and Sethian 1998]

Stress Intensity Factors calculation

- *Dissipated energy for a virtual crack extension*

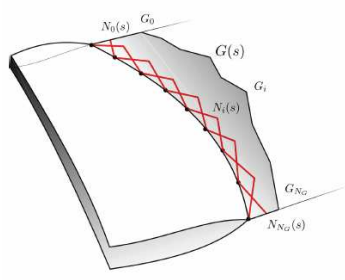
$$W_{diss} = \int_c G(s)\, \ldots ds \qquad \int_c \int_S \ldots \qquad P = W_{\ldots}(\ldots)$$



- *G(s) is discretized along the front with curvilinear shape functions:*

$$\ldots = \sum_{i=1}^{N_G} \ldots \qquad A_{ij} = \int_c N_j(s)\, \ldots ds$$

$$\ldots = \int_V \ldots \qquad \textit{[Parks et al., 2000]}$$



- *Diagonal matrix:*

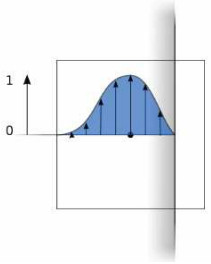
$$A^{lump}_{ii} = \sum_{j=1}^{N_G} A_{ij} = \int_c \sum_{j=1}^{N_G} N_j(s)\, \ldots ds = \int_c \ldots ds$$

- *Irwin formula*

$$\ldots = E\left[\ldots^2 \ldots^2\right] \ldots^2 \ldots$$

- *Interaction integral*

$$I_{int} = \int_V q_{i,j} \left[\ldots^{aux} \ldots^{aux} \ldots^{aux} \right] \ldots$$



- *Contact, friction, plasticity, dynamics*

$$J = \int_S \left(W \delta_{1j} - \sigma_{jk} u_{k,1} \right) q_{1,j}\, dV - \int_{\text{crack faces}} \ldots \left(\frac{\partial u_i}{\partial x_1} \ldots \frac{\partial u_i}{\partial x_1} \right) \ldots$$

[Combescure, Suo, 1986] [Moës N., Gravouil A., Belytschko T., IJNME 2002]

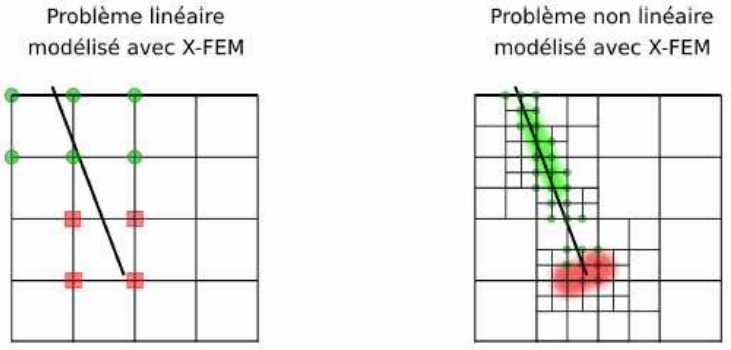
[Gosz & al. 1997,2002, Béchet 2005, Réthoré 2005, Elguedj 2006, Ribeaucourt 2006]

Un exemple de stratégie numérique multi-échelles

- ***X-FEM avec raffinement local du fait des non-linéarités***

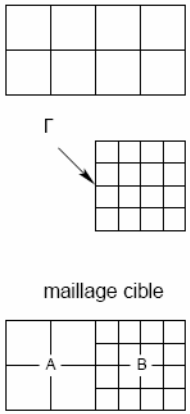
⇒ ***Eléments finis étendus multigrilles avec raffinement local pour la mécanique de la rupture***

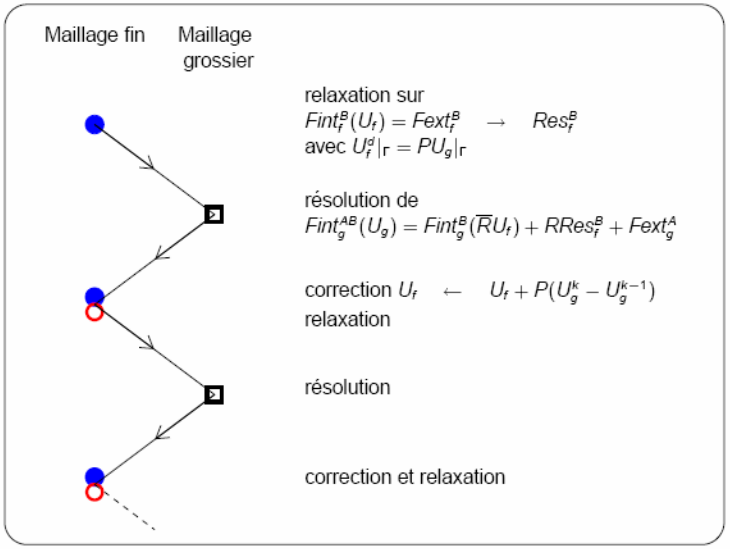
Algorithme multigrille localisé linéaire (MG-L-L) Full Approximation Scheme (FAS)



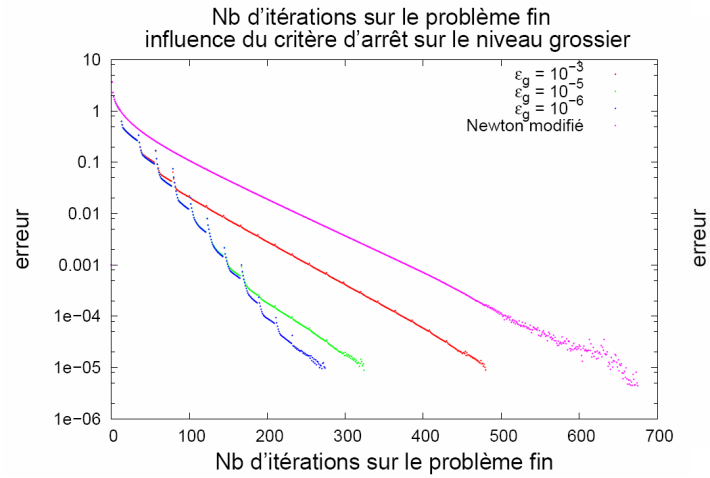
- ***Efficacité des solveurs itératifs pour la partie haute fréquence de la solution peu efficaces pour résoudre les composantes basses fréquences de la solution***

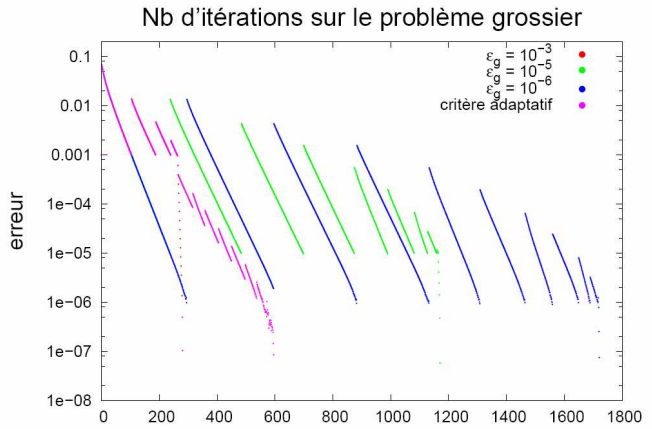
Chaque discrétisation est utilisée pour capturer une certaine gamme de fréquence en utilisant le solveur itératif de manière optimale





Exemple de stratégie numérique multi-échelles en non-linéaire

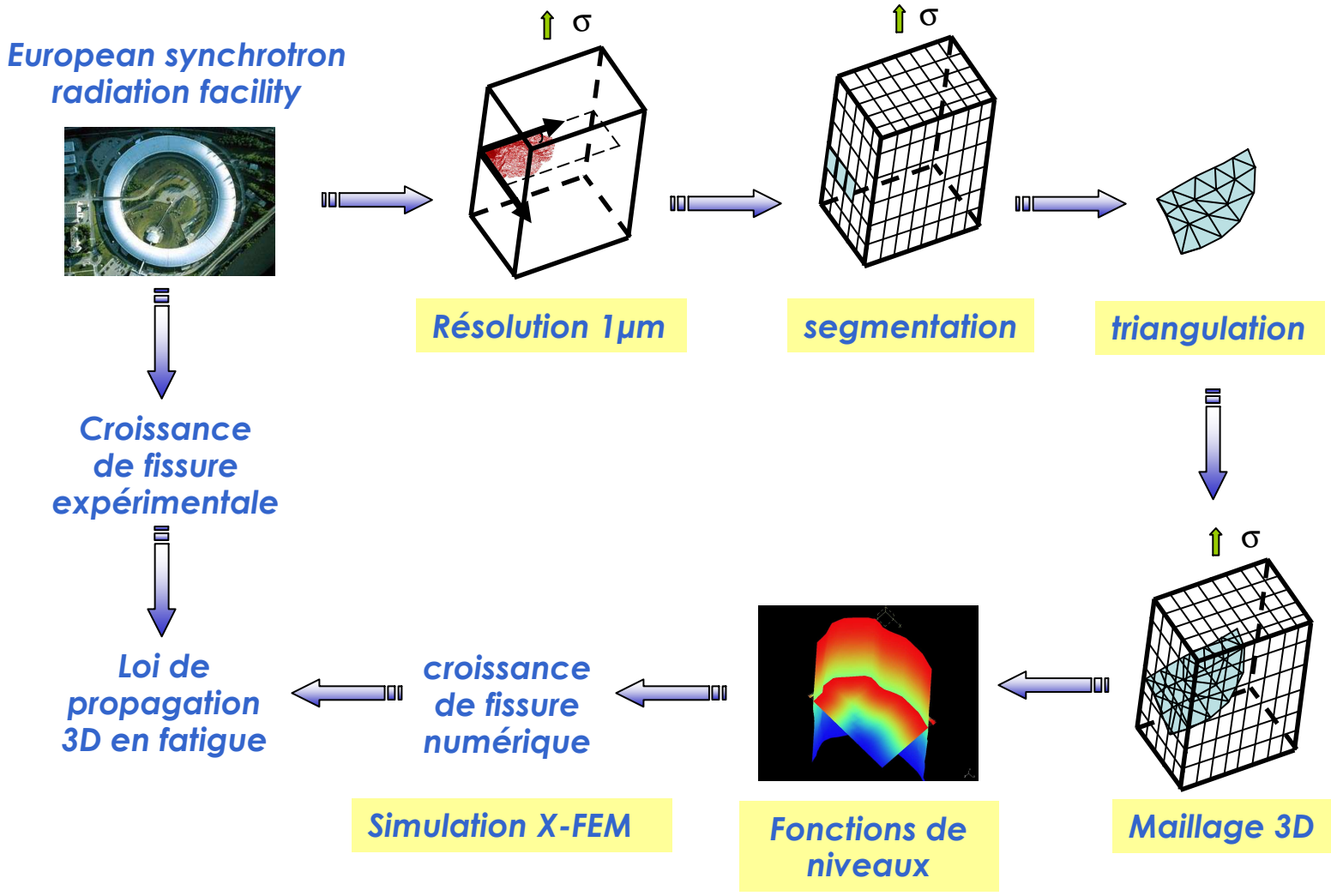




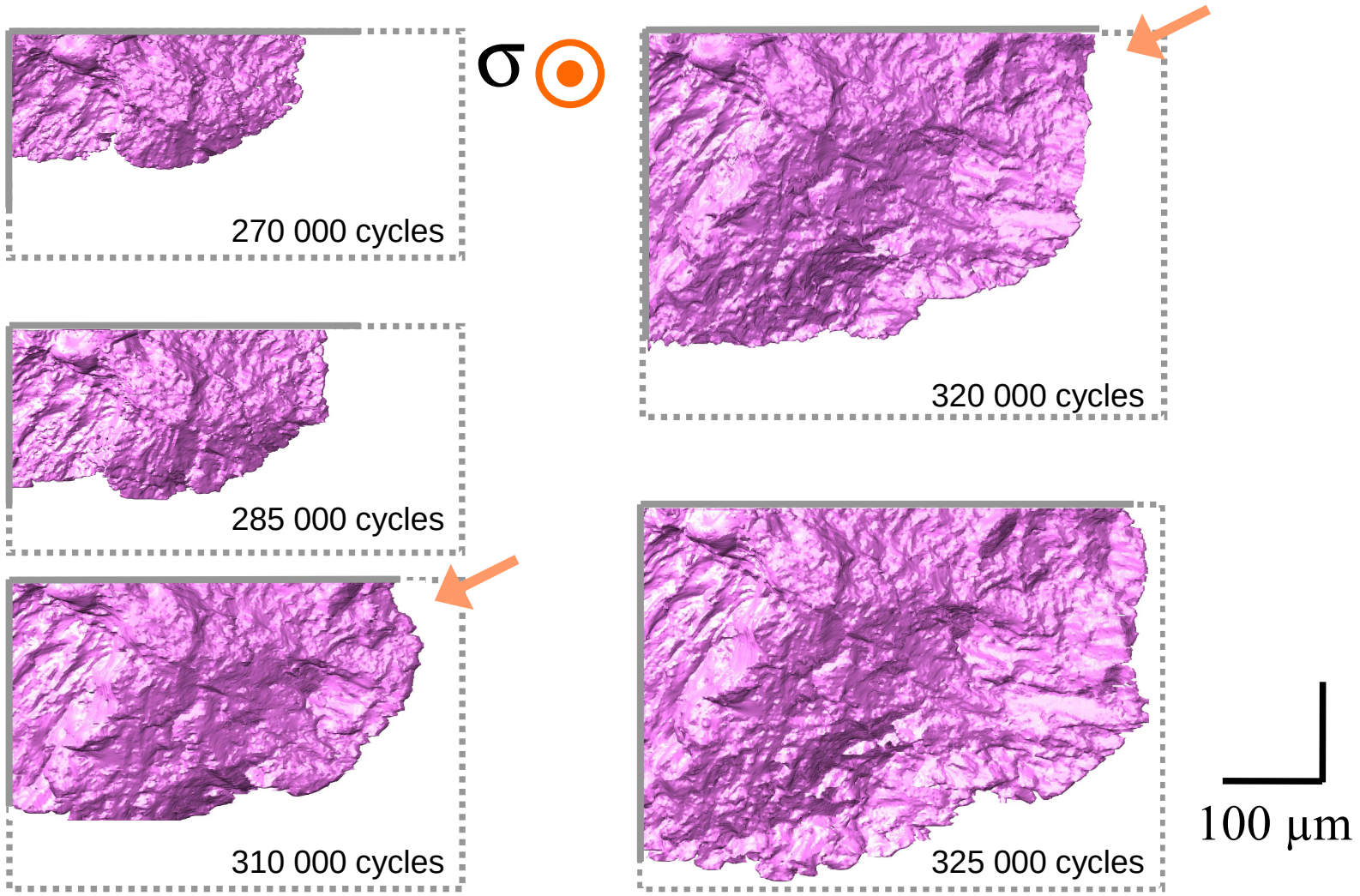
Critère d'arrêt sur le problème grossier

$$\varepsilon_g = \min\left(10^{-3}, C_f 10^{-2}\right)$$

Vers un couplage entre l'imagerie tridimensionnelle et X-FEM



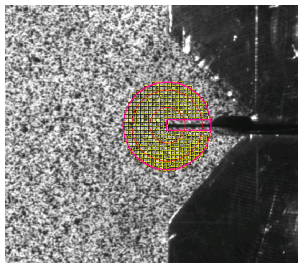
Microstructurally short cracks



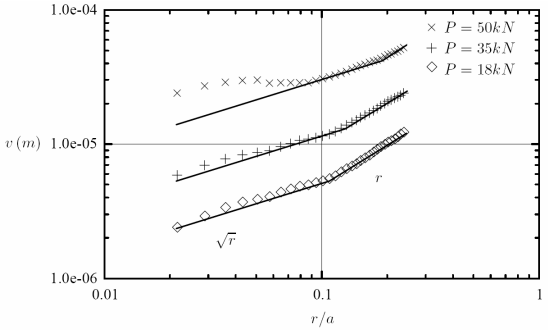
Fissuration en fatigue en mode mixte
Plasticité confinée – effet de refermeture

- *Mesure de champ par corrélation d'image (ICASOFT – MORESTIN F.)*

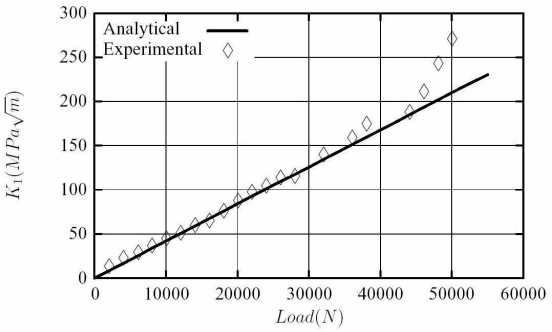
Résultats expérimentaux



Comportement asymptotique



FIC expérimentaux par techniques intégrales



[Réthoré J., Gravouil A., Morestin F., Combescure A., IJF, 2005]

- *Base d'enrichissement en présence de plasticité confinée*

$$\underline{r}_{n+1}\left\{\sin\frac{\theta}{2},\cos\frac{\theta}{2},\sin\frac{\theta}{2}\sin\theta,\cos\frac{\theta}{2}\sin\theta,\sin\frac{\theta}{2}\sin3\theta,\cos\frac{\theta}{2}\sin3\right\}$$

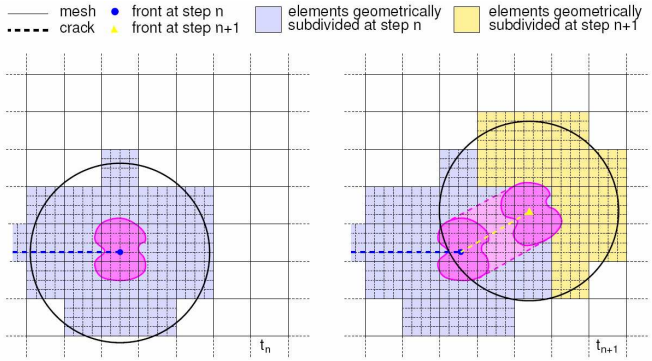
$$\underline{n}\left[\quad\right]$$ *(lié à l'écrouissage du modèle élasto-plastique)*

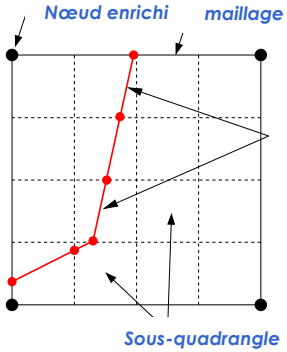
[Elguedj T., Gravouil A., Combescure A., CMAME, 2006]

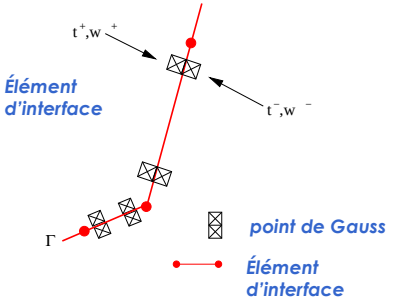
Fissuration en fatigue en mode mixte
Plasticité confinée – effet de refermeture

- *Stratégie de propagation de fissure en fatigue à 2 échelles (pas de remaillage ni de projection de champ)*

$$R_{sub} = \beta \times \frac{\pi}{16}$$







- *Modélisation du contact et du frottement*

$$\int_\Omega \sigma_n^{i} : \varepsilon(w^*)\, d\Omega = \int_{\partial_2\Omega} t_d \cdot w^*\, dS + \int_\Gamma \lambda_n^{i} \cdot w^*\, dS + \int_\Gamma w^* \cdot \left(\sigma_n^{i} t - \sigma_n^{i}\right) dS$$

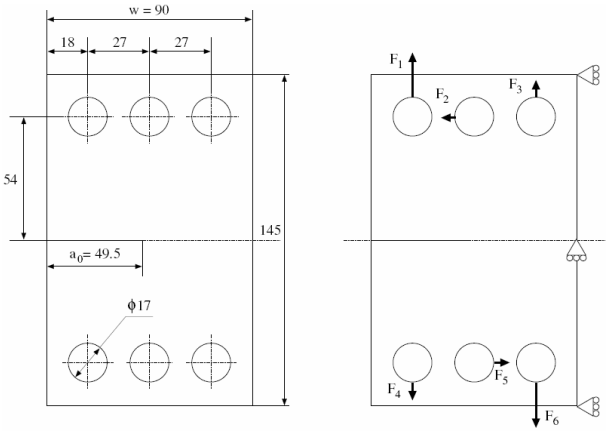
$$+ \int_\Gamma \left(t_{i-1} + \alpha w_{i-1}\right) \cdot w^*\, dS - \int_\Gamma \left(\lambda_i + \alpha w_i\right) \cdot w^*\, dS$$

[Elguedj T., Gravouil A., Combescure A., IJNME, 2007]

Fissuration en fatigue en mode mixte
Plasticité confinée – effet de refermeture

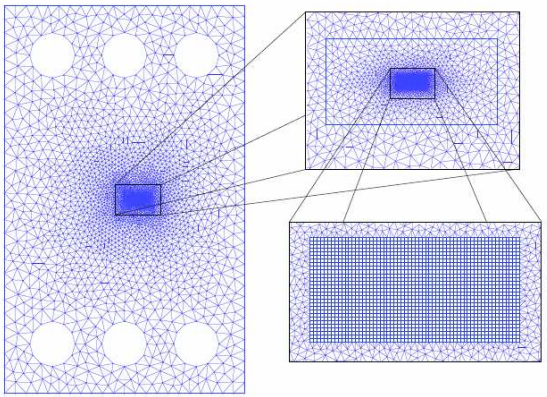
- **Éprouvette CTS: effet de surcharge**

L'éprouvette est soumise à une croissance de fissure totale de Δa=0.5 mm (20 pas pour FEM, 2 pas pour X-FEM), puis soumise à une surcharge de rapport 2.5 puis une croissance totale de Δa=0.5 mm

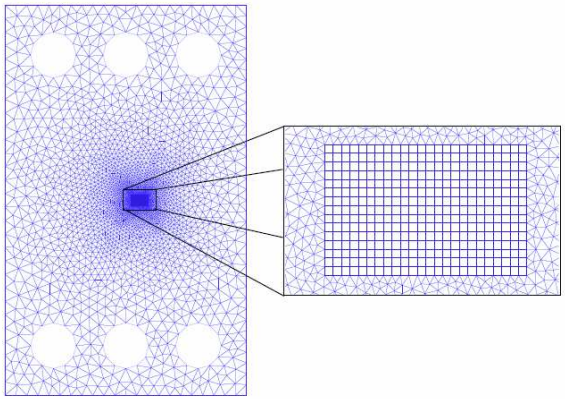


- **Comparaison FEM / X-FEM**

Maillage FEM



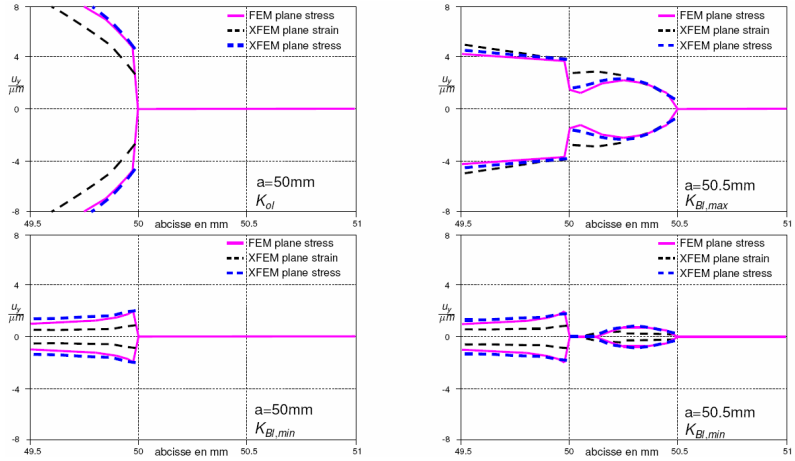
Maillage X-FEM (10 fois plus grossier)



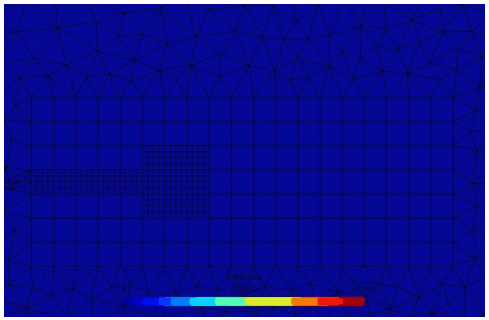
→ *[Sander M., Richard H.A., IJF, 2005]*

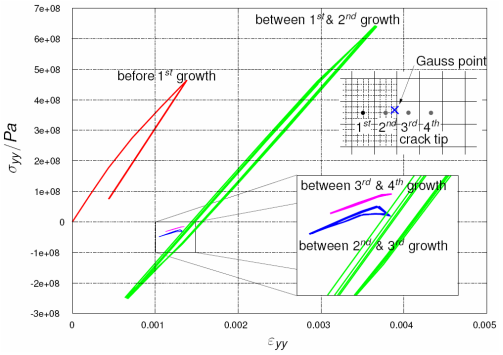
Fissuration en fatigue en mode mixte
Plasticité confinée – effet de refermeture

- **Comparaison du déplacement vertical au niveau des faces de la fissures entre les résultats de FEM et X-FEM**



- **Croissance de fissure dans une éprouvette CT sollicitée en mode I**





Fatigue de roulement

- *Etude du contact roue-rail*

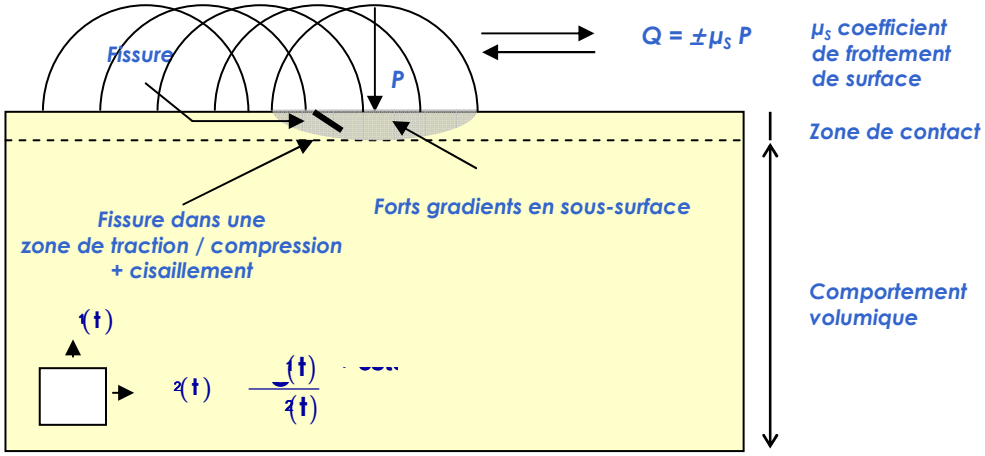
Fatigue de roulement

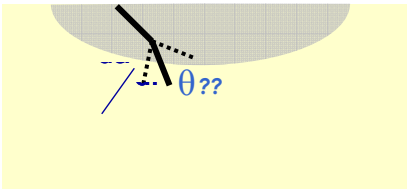
À chaque passage un cycle de chargement

Tension / compression + cisaillement

Fissuration

- *Modèle numérique bi-dimensionnel*



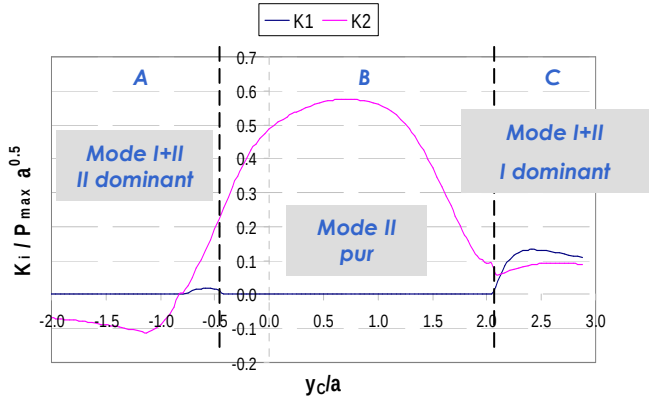


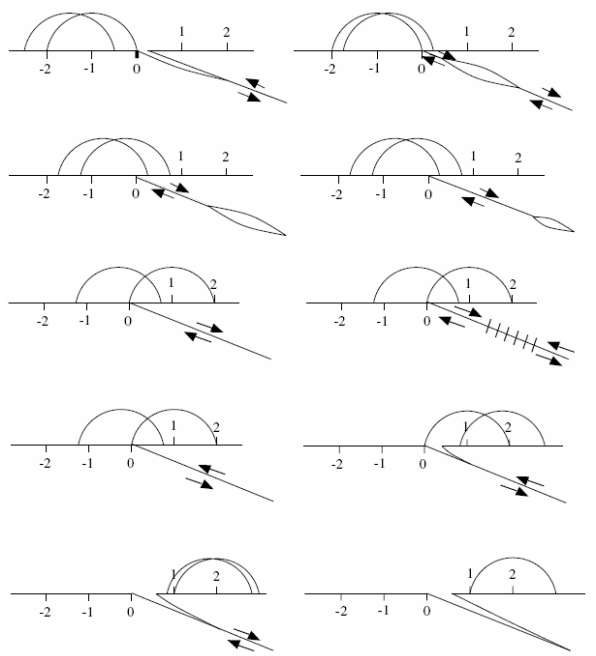
Le comportement de la fissure dépend fortement de sa longueur comparée à la zone de fort gradient

[Baïetto M.C. et Lamacq, ASME, J. of Tribol. 2002]

Fatigue de roulement

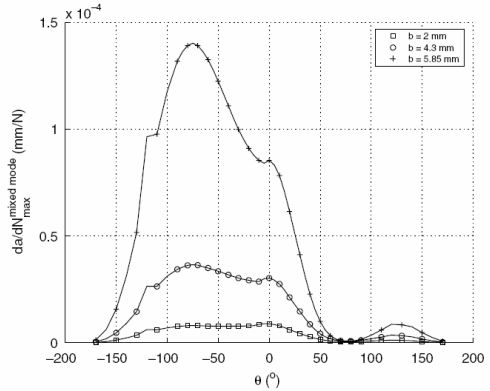
- *Formulation quasi-statique incrémentale X-FEM (méthode LATIN) (indicateur d'erreur normal et tangentiel découplé)*





- *Critère en fatigue de contact à la fin d'un cycle de chargement*

(on considère les quantités maximales pendant 1 cycle)



$$= 2.0.10^{9} \left(\Delta k_{2\,1max} + 0.772 \Delta k_{2\,2max} \right)$$

[Dolbow J., Moës N., IJNME, 2002] [Ribeaucourt , Baïetto, Gravouil, CMAME, 2007]

[Hourlier & al. 1982] [Amestoy 1983] [Lamacq V., Baïetto M.C., Vincent L., Tribol. Int., 1997]